

1. b (From Newton's second law, the net torque is equal to the rotational inertia times the angular acceleration; so, the correct choice must correspond to the situation where the torque is maximum. Since the torque is defined as the tangential force times the distance to the axis, the correct answer is the one that has the force directed tangentially "around the axis" (the hinges in this case).)
2. a (The net torque must be zero; torque is the tangential force (the gravitational force on the masses) times the distance from the pivot; since the 50-g mass is 30 cm from the pivot, the 150-g mass must be one-third as far and on the opposite side)
3. b (In this case, the stick is off-center so it is like having a 50 g object placed 10 cm from the pivot; to balance that, you'd need to have the 100 g object 5 cm from the pivot on the opposite side)
4. The further the mass is from the rotation axis, the harder it is to change the object's rotation rate. The hoop has all the mass located a distance equal to the radius of the object from the center. For a disk, most of the mass is at a distance *less* than the radius from the center.

Thus, it requires more torque to stop the hoop from spinning.

5. (a) This part describes a situation where information about the motion is provided (constant angular acceleration, initial and final rotation rates) and asks about another aspect of the motion (angular displacement). This doesn't require Newton's second law (since force is not relevant). All we need is the definition of average velocity.

In this case, it started with a rotation rate of 10 rev/s and ended at zero. The average rotation rate is 5 rev/s. At this rate, over the course of 15 seconds, it must have made 75 revolutions.

- (b) This problem is specifying a change in rotation rate and asking about the net torque. The relationship between force and motion is given by Newton's second law. For rotation, that means we use angular velocity, torque and rotational inertia (instead of velocity, force and mass).

In this case, we are given the change in angular velocity. However, when using Newton's second law, we can't use ω equal to 10 rev/s.

Instead, we must convert this into radians. Since each revolution is 2π radians, that means that the disk is initially rotating at 62.83 rad/s.

Rotational inertia is defined as mr^2 . For a hoop, all the mass is located a distance equal to the radius of the object from the center. Consequently, the rotational inertia is the mass (4 kg) times the square of the radius (2 m). That gives a rotational inertia of $16 \text{ kg}\cdot\text{m}^2$,

From Newton's second law, the change in angular velocity (62.83 rad/s counter-clockwise, since it is slowing down) must equal the net torque (unknown) times the time (15 s) divided by the rotational inertia ($16 \text{ kg}\cdot\text{m}^2$). Solve for the net torque to get a value of 67.02 N·m.

A couple of common errors is to forget to convert the rotation rate into radians or to confuse torque, force, rotational inertia, etc., and end up solving for something else. Keeping track of your units can help you identify when you make that mistake.

6. There are three forces acting on the plank. One is the gravitational force on the plank (15 kg times $9.8 \text{ N/kg} = 147 \text{ N}$), acting at the center of gravity (which, since the plank is uniform, is in the middle of the plank, 1.5 m from each end). Another is the force exerted by the weight of the child (30 kg times $9.8 \text{ N/kg} = 294 \text{ N}$), acting at the location of the child (unknown). The third is the force of the pivot (unknown), acting at a point 1.0 m from one end.

We have two unknowns: the location of the child and the force of the pivot. We can get the force of the pivot by balancing the upward and downward forces. However, we aren't asked for the force of the pivot – we are asked for the location of the child. To get that, balance the torques. Since we don't know the force of the pivot, set the rotation axis at that location, which means that the torque of the pivot would be zero about that axis.

The torque due to the gravitational force on the plank about that axis is the force, 147 N, times the distance from where the force is acting to the axis of rotation. Since we've set the axis of rotation at the pivot, the distance is 0.5 m (from the middle of the plank to the pivot). That gives a torque of 73.5 Nm.

The torque due to the child is the force, 294 N, times the unknown distance. However, this torque must balance the other torque. Thus, we have

$$(294 \text{ N})r = 73.5 \text{ N} \cdot \text{m}$$

and, solving for r , we get a distance of 0.25 m. This is from the axis, remember, so since we set the axis as the pivot (for this problem), the child must be placed 0.25 m from the pivot or 0.75 from the end of the plank (on the short side of the pivot).