1. b (for an object moving in a circle, the net force must be directed toward the center of the circle)

2. b (for an object moving in a circle, the net force must be directed toward the center of the circle)

3. c (use the radius of the circular path the object, in this case the moon, is following)

4. a (Since the radius is 50 m, then one radian would correspond to an arc length of 50 m. An arc length of 10 m is only one fifth of that.)

5. (a) Use the universal law of gravitation ($F_g = Gm_1m_2/r^2$) with r being the distance from the center of the astronaut to the center of the Earth. Add the radius of the Earth to 350,000 m to get r then solve to get a force of 620 N.

   (b) For this, you can repeat the calculation from part (a) with r equal to the radius of the Earth or you can simply use mg, where g is 9.8 N/kg. I get a force of 690 N.

6. For the moon to move in uniform circular motion, it must have a speed equal to $2\pi r/T$. Converting the period into seconds (2.36×10^6 s), I get a speed of 1.02×10^3 m/s.

   Furthermore, the moon must be undergoing an acceleration (changing directions) at a rate equal to $2\pi v/T$ and directed toward the center of the circle. Plugging in the speed and the period (converted to seconds), I get an acceleration of 0.0146 m/s^2 (toward the Earth).

   From Newton's second law, the acceleration equals the net force per mass. In this case, the only force acting is the gravitational force (due to the moon's interaction with the Earth). That means the gravitational force per mass (of moon) must equal the acceleration (of the moon), 2.72×10^{-3} m/s^2.

   From Newton's universal law of gravitation, the gravitational force is

   $$F_\text{g} = G\frac{m_\text{earth}m_\text{moon}}{r^2}$$

   That means the gravitational force per mass (of moon) is:

   $$\frac{F_\text{g}}{m_\text{moon}} = G\frac{m_\text{earth}}{r^2}$$

Setting that equal to the acceleration (2.72×10^{-3} m/s^2), and plugging in 3.88×10^8 m for the radius of the circle (i.e., radius of the moon's orbit) and 6.67×10^{-11} Nm^2/kg^2 for the gravitational constant, I get a mass of 6.04×10^{24} kg. This is slightly more than the actual value because the center of the moon's orbit is not coincident with the center of the earth.

7. (a) The speed is equal to the circumference divided by how long it takes to go all the way around:

$$\begin{aligned} v &= (2\pi r)/T \\ &= 2\pi(6.37 \times 10^6 \text{ m})/(24 \times 3600 \text{ s}) \end{aligned}$$

which gives a speed of 463.2 m/s.

(b) The acceleration of an object in circular motion is directed toward the center of the circle and has a magnitude equal to:

$$\begin{aligned} a_c &= (2\pi v)/T \\ &= 2\pi(463.2 \text{ m/s})/(24 \times 3600 \text{ s}) \end{aligned}$$

which gives an acceleration of 0.0337 m/s^2 toward the center of the Earth (center of circular path in this case).

(c) From Newton's second law, the acceleration must equal the net force on the student divided by the student's mass. Solving for the net force, I get 2.36 N (toward the center of the circle).

(d) The only way the net force would be equal to the gravitational force is if the gravitational force was the only force acting. That means there would be no normal force. How is this possible?

This is only possible if the Earth is spinning so quickly that an object on the surface is technically "in orbit", without touching the ground. Since the gravitational force is equal to mg or 686 N, that means the net force is equal to 686 N. From Newton's second law, that means the acceleration is 686 N divided by the mass, or 9.8 m/s^2.

How can that be? That is only possible if the object is moving very quickly. Using $a_c = (2\pi/T)^2 r$, we can solve for the period (I get 5065.7 seconds). Then, using $v_t = 2\pi r/T$, one can find the velocity (I get 7.9×10^3 m/s). This is the speed an object must travel (at the earth's surface) to be "in orbit" (i.e., without falling down to the ground).