An introduction to the PHYSICS of WEATHER PREDICTION

Checkpoint Answers

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1. Weather Forecasting

Check Point 1.1: (a) 0900 EST, (b) 1400 GMT, (c) 0600 PST.

Check Point 1.2: The time is 0753 UTC (7:53 AM) on the 27th of the month, the temperature is 21°C and the dew point is 20°C.

Check Point 1.3: (a)

Check Point 1.4: The larger domain means that we'd need more grid points, and thus take longer to do the calculations, if the resolution wasn't changed.

2. Ideal Gas Law

Check Point 2.1: Yes. Technically, there is no such as an ideal gas but the air in the atmosphere is so close, we will consider it to be ideal.

$$P = \rho RT \tag{2.1}$$

Check Point 2.2: Yes, if the density increases by a greater fraction than the temperature decreases.

$$P = \frac{F}{A} \tag{2.2}$$

Check Point 2.3: Yes, each point has a pressure value. Even though the force exerted on the point is zero and the area of the point is zero, the ratio of the two still has a finite value.

Check Point 2.4: No. At a *single location*, isotropy means the pressure at a single point is the same upward and downward as well as horizontally. However, the air on one side of the paper is at a slightly different location as the other wide. As anyone familiar with trying to chase down a piece of paper that is being pushed around by the wind can attest, the pressure on one side of a piece of paper can be different than the pressure on the other side, leading to the paper being pushed toward the lower-pressure side.

$$\rho = \frac{m}{V} \tag{2.3}$$

Check Point 2.5: No. By dividing out the number of molecules and the volume, the temperature and density values are independent of the number of molecules and the volume.

Check Point 2.6: Yes, it has a particular non-zero density at that point. When dealing with a point of infinitesimal volume and infinitesimal mass, the ratio of the two, the density, is still finite.

Check Point 2.7: 1.01325×10^5 Pa, 101.325 kPa and 1013.25 mb

Check Point 2.8: 998.1 mb

Check Point 2.9: (a) 283.15 K, (b) 10 K

Check Point 2.10: 283K.

$$T_{\mathrm{C}} = \left(\frac{1^\circ\mathrm{C}}{1.8^\circ\mathrm{F}}\right)(T_{\mathrm{F}} - 32^\circ\mathrm{F}) \tag{2.4}$$

Check Point 2.11: It doesn't matter which reference you use. The end result might initially look different but it should simplify to the same relationship.

Check Point 2.12: Freezing is 32°F. Subtracting 32°F from 212°F gives 180°F.

Check Point 2.13: (a) About 1.2-1.3 kg/m^3, (b) about 1000 kg/m^3, (c) Density of water.

Check Point 2.14: The container of mercury would weigh much more, since its density is much greater. However, the container of mercury would be equal in weight to the weight of air if the container of air was as tall as the atmosphere.

3. What is in the Air?

Check Point 3.1: It will be closer to the gas constant for dry air, since only a small portion of moist air is water vapor.

Check Point 3.2: (a) Yes, (b) No

$$R = \frac{R^*}{\mu} \tag{3.1}$$

Check Point 3.3: Yes. If you divide the units of R^* by the units of μ you get the units of R.

Check Point 3.4: It means that each molecule is made up of two atoms.

Check Point 3.5: This is because the actual value incorporates gases other than nitrogen and oxygen. The next most prevalent gas is argon (Ar), which has a higher molar mass (39.948 u) than either nitrogen or oxygen. Thus, including that in the average will push the average higher. It doesn't push it much higher because argon only makes up a small portion of air.

Check Point 3.6: Mixing is more dominant. The top of Mount Everest is within the homosphere, well below the 100 km or so transition between the homosphere and heterosphere. Thus, the average molar mass of the air on top of Mount Everest is pretty much the same as the air down at sea-level.

Check Point 3.7: Dry air is air that has all of the gases except for water vapor.

4. Spatial Variations

$$P = P_0 e^{-z/H} \tag{4.1}$$

Check Point 4.1: Because if the pressure decreases with height exponentially, as in equation 4.1, then when $z = H$ the pressure will be equal to $P_0 e^{-1}$, which is P_0/e. And, since e is approximately 3 (actually 2.7 or so), that means the pressure at that height is approximately one-third P_0.

Check Point 4.2: No. The U.S. Standard Atmosphere represents an average state. At any given time, the actual temperatures will not be the same as what the U.S. Standard Atmosphere says is the average state.

Check Point 4.3: Less. The height of Everest is closer to the cruising altitude of planes than Earth's surface and thus the air there should have a density closer to what it is at the cruising altitude of planes (about a third or a quarter of the surface value of 1.2-1.3 kg/m^3).

$$\Gamma = -\partial T/\partial z \tag{4.2}$$

Check Point 4.4: That would be a situation of negative lapse rate (warming with height).

Check Point 4.5: Negative

Check Point 4.6: (a) Greater than 1.5 km (since the ratio of 800 mb to 600 mb is larger than the ratio of 1000 mb to 800 mb), (b) About 1.5 km (since the ratio of 800 mb to 640 mb is the same as the ratio of 1000 mb to 800 mb).

Check Point 4.7: Since one is 68°F and the other is 81°F, that means that there must be a point between them that is at 70°F and another point between them that is at 80°F. Given that, one would expect two isotherms, one for 70°F and one for 80°F. The isotherms have likely been "smoothed" so that it required too much of a kink to the 70°F and 80°F isotherms to have them include the 68°F and 81°F observations.

Check Point 4.8: Unless the surface happens to be at sea-level, the pressure on the map will typically be less than the pressure measured at the surface since most places on land are above sea-level.

Check Point 4.9: Large pressure gradient

Check Point 4.10: 0.006 mb/km (or 0.6 Pa/m)

Check Point 4.11: Large.

Check Point 4.12: The distance from 800 mb to 600 mb would be larger than the distance from 1000 mb to 800 mb but the distance from 800 mb to 640 mb would be the same, since the ratio of 800 mb to 640 mb is the same as the ratio of 1000 mb to 800 mb.

Check Point 4.13: About $-14°$C. Remember that the temperature reference lines are skewed.

5. Advection

Check Point 5.1: From the northeast (i.e., about 45°).

Check Point 5.2: 270 degrees

Check Point 5.3: 20 knots.

Check Point 5.4: 10 kts is faster than 10 mph, since a nautical mile is slightly longer than a statute mile.

Check Point 5.5: When the wind blows, the temperature could go up, down or stay the same, depending what the temperature is like in the region from which the air is coming. It feels colder if the air temperature is colder than our skin temperature, which is usually the case in winter.

Check Point 5.6: Warm advection means the wind is bringing in warmer air. Thus, we'd expect warmer temperatures.

Check Point 5.7: Warm advection

Check Point 5.8: The temperature tendency.

Check Point 5.9: $\partial T/\partial t$ is positive.

Check Point 5.10: In the central United States the temperature gradient is directed toward the south (i.e., warmer temperatures toward the south; typical of the northern hemisphere) and the wind is directed toward the north (a southerly wind). That corresponds to warm advection and positive temperature advection values.

Check Point 5.11: The fastest is 10 m/s. The slowest is 10 mph.

Check Point 5.12: Multiply the product (1 K/h) by the cosine of 45 degrees to get 0.71 K/h, the same as obtained from method 1.

6. Compression and Expansion

$$\frac{\partial T}{\partial t} = -\vec{v} \cdot \vec{\nabla} T + \frac{dT}{dt} \tag{6.1}$$

Check Point 6.1: dT/dt represents the change in air temperature following the air parcel. If the local change in temperature ($\partial T/\partial t$) is only due to the temperature advection ($-vec\nabla T$) then the change following the air parcel (dT/dt) must be zero.

Check Point 6.2: It implies that the air parcel's temperature won't change at all. The only way for the parcel's temperature to change is if it expands or contracts and that will only happen if the surrounding air is at a lower or higher pressure.

$$dT = \frac{1}{c_\mathrm{p}\rho} dP \tag{6.2}$$

Check Point 6.3: According to equation 6.2, dT has the same sign as dP since both c_p and ρ must be positive. So, if the pressure decreases, the temperature must decrease as well.

$$T = T_0 \left(\frac{P}{P_0}\right)^{R/c_\mathrm{p}} \tag{6.3}$$

Check Point 6.4: If the pressure decreases, the new pressure P is less than the original pressure P_0. That means that the ratio P/P_0 is less than one. When that ratio is raised to any power greater than zero, the value remains less than one. So, the new temperature must be some fraction of the original temperature. This means the temperature must decrease also.

Check Point 6.5: Temperature needs to be in kelvin. Pressure can be in either millibars or pascals because the units will cancel out either way.

$$dU = dQ - dW \tag{6.4}$$

Check Point 6.6: If the expansion is adiabatic, dQ is zero. If it expands, it does work on the environment and so dW is positive. According to the equation, that means that dU is negative. Since dU represents the internal energy (temperature), that means the temperature decreases.

$$c_{\mathrm{p}} dT = \frac{dQ}{m} + \frac{dP}{\rho} \tag{6.5}$$

Check Point 6.7: The definition of work ($dW = Fdx$), the definition of pressure ($P = F/A$) and the definition of heat capacity ($C = dQ/dT$) and the definition of specific heat ($c = C/m$).

$$dU = dQ - PdV \tag{6.6}$$

Check Point 6.8: First cancel the $-mR_d T$ on both sides then solve for dQ and divide by m. One can then use the definition of density to replace V/m by $1/\rho$.

$$dU = mc_{\mathrm{v}} dT. \tag{6.7}$$

Check Point 6.9: The quantity dQ doesn't *always* equal $m_{\mathrm{v}} dT$ or $m_{\mathrm{p}} T$. Rather, it depends on whether the volume or the pressure is held constant.

$$c_{\mathrm{p}} d(\ln T) = Rd(\ln P). \tag{6.8}$$

$$\ln\left[\frac{T}{T_0}\right] = \ln\left[\left(\frac{P}{P_0}\right)^{R/c_{\mathrm{p}}}\right] \tag{6.9}$$

Check Point 6.10: Because the expression $c_{\mathrm{p}} \frac{dT}{T} = R\frac{dP}{P}$ contain variables (T and P) that don't remain constant as the pressure (and temperature changes).

Check Point 6.11: (a) c_{p} is bigger (by an amount equal to R).

7. Radiation

Check Point 7.1: Nothing. Light is an example of electromagnetic radiation.

Check Point 7.2: Yes. Indeed, it travels fastest in a vacuum and almost as fast in air.

Check Point 7.3: Blue.

$$v = f\lambda \tag{7.1}$$

Check Point 7.4: Assuming the speed stays the same, the wavelength will decrease.

Check Point 7.5: This is so the radar can see "through" the clouds in order to get to the precipitation.

Check Point 7.6: Ultraviolet.

Check Point 7.7: The low cloud will be warmer and, as such, will emit more infrared radiation.

$$P = A\sigma T^4 \tag{7.2}$$

Check Point 7.8: No. The surface area of Earth (around 5×10^{14} m^2) is much bigger than the surface area of a human (around 2 m^2).

Check Point 7.9: The Sun's irradiance at its surface is larger.

Check Point 7.10: It decreases in a way that is inversely proportional to R^2, as the energy spreads out in space.

$$\lambda_{\text{peak}} = \frac{2897\mu\text{m} \cdot \text{K}}{T} \tag{7.3}$$

$$F_\lambda = \frac{c_1}{\lambda^5 \left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]} \tag{7.4}$$

Check Point 7.11: The irradiance in W/m^2 per μm would be larger in number since a μm is larger than a nm and thus the "bin size" is larger.

8. Radiation Balance

Check Point 8.1: Around 7:30 am

Check Point 8.2: About 70%. The rest is reflected back out to space.

Check Point 8.3: (a) The cross-section is πR^2, where R is the radius of Earth, (b) The surface area is $4\pi R^2$, so the surface area is four times as great as the cross-section.

Check Point 8.4: Because the radiation emitted to space does not come from the surface but rather throughout the atmosphere, which has a colder temperature than Earth's surface.

Check Point 8.5: The surface temperature.

Check Point 8.6: The radiation emitted by the atmosphere would increase, and since that would increase the amount absorbed by Earth, Earth's temperature would also increase.

Check Point 8.7: The sky would be dark (as it would absorb the visible and then re-emit in the infrared). There would also be a lot of sunburn and skin cancer.

Check Point 8.8: The radiation emitted by the atmosphere.

$$(1-\alpha_{\mathrm{SW}})(1-a)S_{\mathrm{avg}} + F_{\mathrm{atmos}} = F_{\mathrm{sfc}} \tag{8.1}$$

$$S_{\mathrm{avg}} = aS_{\mathrm{avg}} + F_{\mathrm{atmos}} + (1-\alpha_{\mathrm{LW}})F_{\mathrm{sfc}} \tag{8.2}$$

$$F_{\mathrm{sfc}} = S_{\mathrm{avg}}\frac{(2-\alpha_{\mathrm{SW}})(1-a)}{(2-\alpha_{\mathrm{LW}})} \tag{8.3}$$

Check Point 8.9: Colder

Check Point 8.10: Advection (convection) and evaporation.

Check Point 8.11: The value of $(2-\alpha_{\mathrm{LW}})$ decreases and since that term is in the denominator, F_{sfc} and the associated surface temperature both increase.

Check Point 8.12: The value of $(2-\alpha_{\mathrm{SW}})$ decreases and since that term is in the numerator, F_{sfc} and the associated surface temperature both decrease.

Check Point 8.13: The value of $(1-a)$ decreases and since that term is in the numerator, F_{sfc} and the associated surface temperature both decrease.

Check Point 8.14: F_{sfc} is proportional to S_{avg} so surface temperature would increase.

9. Vertical Balance

$$\frac{\partial P}{\partial z} = -\rho g \tag{9.1}$$

Check Point 9.1: Yes. The vertical pressure gradient $\partial P/\partial z$ has to be negative because the density ρ and gravitational field g must both be positive. A negative vertical pressure gradient means that the pressure P decreases with height z.

$$P_{\text{top}} = P_{\text{bottom}} \exp\left(-\frac{g}{RT_{\text{avg}}}\Delta z\right) \tag{9.2}$$

$$\ln\left(\frac{P_{\text{top}}}{P_{\text{bottom}}}\right) = -\frac{g}{RT_{\text{avg}}}\Delta z \tag{9.3}$$

Check Point 9.2: Because if the layer is large (i.e., z_{top} is much above z_{bottom}) there is no single value of temperature that is appropriate for the entire depth of the layer.

$$H = \frac{RT_{\text{avg}}}{g} \tag{9.4}$$

Check Point 9.3: It increases, which means the height at which the pressure falls to $1/e$ of the surface value is higher.

$$M = \frac{A}{g}P_0 \tag{9.5}$$

Check Point 9.4: They are proportional. If the mass goes up, so does the surface pressure.

$$\Delta P = \frac{Mg}{A}. \tag{9.6}$$

Check Point 9.5: The pressure at the top must decrease, such that the difference in pressure between the top and bottom increases. In other words, the mass above 1 km must have decreased.

$$P_0 = P_{\text{trop}} \exp\left(\frac{g}{RT_{\text{avg}}} z_{\text{trop}}\right) \tag{9.7}$$

Check Point 9.6: The surface pressure should decrease. According to equation 9.7, a positive temperature advection should correspond to a lower surface pressure.

Check Point 9.7: The height increases. If P_{top} and P_{bottom} remain the same then Δz must increase if T_{avg} increases.

Check Point 9.8: The plotted pressure will typically be greater than that observed, since the plotted pressure corresponds to sea-level and most surface observing stations are above sea level.

$$\frac{\partial \rho}{\partial z} > 0 \quad \text{if} \quad \frac{\partial T}{\partial z} < -\frac{g}{R}. \tag{9.8}$$

$$\frac{\partial \rho}{\partial z} = -\frac{\rho}{T}\left(\frac{\partial T}{\partial z} + \frac{g}{R}\right) \tag{9.9}$$

Check Point 9.9: No. The vertical density gradient $\partial\rho/\partial z$ is typically negative but it can be positive if the vertical temperature gradient is very strongly negative (i.e., large positive lapse rate).

10. Lapse rates

Check Point 10.1: No. We assume there is no mixing between the rising parcel and the air around it, so the temperature may be very different.

$$\Gamma_{\mathrm{d}} = \frac{g}{c_{\mathrm{p}}} \tag{10.1}$$

Check Point 10.2: It would warm up to 18.5°C. If you bring the air down from 1 km above the surface, that air will warm up 10 K (due to adiabatic warming at 10 K/km). Consequently, it will no longer be 6.5°C cooler but rather 3.5°C warmer than the surface air.

Check Point 10.3: (a) The temperature of a dry parcel (which dry adiabat it follows depends on its initial temperature).

$$\theta = T\left(\frac{1000\ \mathrm{mb}}{P}\right)^{R/c_{\mathrm{p}}} \tag{10.2}$$

Check Point 10.4: (a) Both are 273 K, (b) The temperature cools to 263 K (cools by 10°C) but the potential temperature remains equal to 273 K.

Check Point 10.5: The entire layer has the same potential temperature (a lapse rate of 10 K/km).

11. Stability

Check Point 11.1: Technically, warm will *accelerate* upward only if it is warmer than the surroundings. Only then will the density of that air be less than the density of the surroundings (assuming the same pressure). And, that means the net force is upward. An upward net force means its acceleration must be upward.

$$\vec{F}_{\text{buoyancy}} = g(\rho - \rho')\hat{k} \tag{11.1}$$

Check Point 11.2: Because if the air parcel (of density ρ') is less dense that its surroundings (of density ρ) then the difference $(\rho - \rho')$ will be positive.

Check Point 11.3: If the land warms up, so will the air, which means the 1000-500 mb thickness increases. That pushes up the height at which the pressure is 500 mb, causing a horizontal pressure gradient (and leading to wind at that level directed from the land to the sea).

Check Point 11.4: Being colder, it will descend to around 790 mb.

Check Point 11.5: (a) Cool. (b) Unstable equilibrium, because if warmer than the environment the air parcel will continue to rise, away from its initial position.

Check Point 11.6: The potential temperature is the same throughout the layer.

Check Point 11.7: (a) The air parcel would have cooled about 10°C(cooling at the adiabatic lapse rate), (b) The air parcel would be 3.5°C cooler than the air around it since the lapse rate associated with the air around it is 6.5 °C/km, (c) stable.

12. Wind Direction

Check Point 12.1: It stays the same (or decreases slightly due to friction). The angular velocity (rotation rate) increases.

Check Point 12.2: It is in the same direction as Earth's rotation in both the northern and southern hemispheres.

Check Point 12.3: Toward the east. Colder air is typically toward the poles. So, at upper levels the lower pressure would be toward the poles. Air flows cyclonically around a low, so that would mean the air is flowing in the same direction that Earth spins (toward the east).

Check Point 12.4: No.

Check Point 12.5: The rotation rate of Earth is roughly twice as slow, as it is equivalent to roughly one revolution per day, whereas the hour hand rotates two revolutions per day.

Check Point 12.6: (a) 7.292×10^{-5} s^{-1}, (b) Zero.

13. Wind Speed

Check Point 13.1: The wind is blowing eastward (positive î component) and southward (negative ĵ component), so that v (the ĵ component of the wind) is negative and u (the î component) is positive.

Check Point 13.2: The sine is greater. The sine gives the ĵ component, which has to be bigger than the î component in this case because the direction is closer to the ĵ direction than the î direction.

Check Point 13.3: Toward the east (i.e., about 0°).

$$V_{\mathrm{g}} = \frac{1}{f\rho}\nabla P \tag{13.1}$$

$$(2\Omega_{\mathrm{e}} u \sin\phi) = -\frac{1}{\rho}\frac{\partial P}{\partial y} \tag{13.2}$$

$$f = 2\Omega_{\mathrm{e}} \sin\phi. \tag{13.3}$$

Check Point 13.4: Yes, it depends upon latitude. It is greater near the poles and zero at the equator.

Check Point 13.5: It increases.

$$V_{\mathrm{g}} = \frac{g}{f}\nabla_p Z \tag{13.4}$$

$$\left.\frac{\partial P}{\partial x}\right|_z = \left.\frac{\partial P}{\partial z}\right|_x \left.\frac{\partial z}{\partial x}\right|_P \tag{13.5}$$

Check Point 13.6: It would be likewise toward the east (higher pressure means that a pressure level is higher up in the atmosphere.

$$f_{\mathrm{c}} = \frac{v^2}{r} \tag{13.6}$$

$$v_{\text{low}} = v_{\text{g}} - \frac{v^2}{fr}. \tag{13.7}$$

$$v_{\text{high}} = v_{\text{g}} + \frac{v^2}{fr}. \tag{13.8}$$

Check Point 13.7: The Coriolis force is larger. The pressure gradient force is directed away from the high pressure center. Consequently, the Coriolis force has to be greater in magnitude in order to have a net force directed inward toward the high pressure center.

Check Point 13.8: Somewhat toward low pressure.

14. Humidity

Check Point 14.1: No. Water vapor is invisible.

Check Point 14.2: $-10°C$

$$q = \frac{m_v}{m_T}. \tag{14.1}$$

Check Point 14.3: Because it is a ratio of two masses and comparing grams of water vapor to kilograms of air gives a number of order 1.

$$r = \frac{m_v}{m_d} \tag{14.2}$$

$$q = \frac{r}{1 + r} \tag{14.3}$$

Check Point 14.4: The mixing ratio will be larger. This can be seen from equation 14.3 or by recognizing that the dry air is less than the total air, so the ratio of water vapor to dry air must be bigger than the ratio of water vapor to the total air.

Check Point 14.5: (a) about 3.6 or 3.7 g/kg, (b) 2 g/kg, (c) between 5.5 and 6 g/kg

Check Point 14.6: Oxygen, which is typically around 20% of the air, whereas water vapor is at most 3-4% of the air.

Check Point 14.7: 23.38 mb (2.338 kPa)

Check Point 14.8: e

$$\epsilon = \frac{R_d}{R_v} \approx 0.622 \tag{14.4}$$

$$\rho = \frac{P - e + \epsilon e}{R_d T} \tag{14.5}$$

$$R = R_{\mathrm{d}} \frac{P}{P - e + \epsilon e} \tag{14.6}$$

Check Point 14.9: Only P, e and T are measured.

$$\mathrm{RH} = \frac{e}{e_{\mathrm{s}}} \tag{14.7}$$

Check Point 14.10: (a) The relative humidity is 100%, and the vapor pressure and saturation vapor pressure are both 23.38 mb. (b) The saturation vapor pressure is still 23.38 mb and the vapor pressure is half of that (11.69 mb).

Check Point 14.11: (a) The saturation vapor pressure would be 1013 mb, the typical sea-level pressure, (b) The boiling point would go down, since the temperature wouldn't have to be as high to create a vapor pressure equal to the air pressure. Consequently, the water can't get as hot and it will take longer to cook food.

15. Clouds and Precipitation

Check Point 15.1: (a) Cloud condensation nuclei; around one million, (b) around one million

Check Point 15.2: The more CCN, the more droplets that are produced for a given drop in mixing ratio, which means the size of each drop must be smaller (as there is less water available for each drop).

Check Point 15.3: The mixing ratio remains the same as does the potential temperature.

Check Point 15.4: The relative humidity of the initial parcels was 100%. The relative humidity of the combined parcel is 105% (i.e., 17.8/17.0).

Check Point 15.5: (a) Typical cloud droplets have diameters between 1 and 100 μm in diameter whereas raindrops range in size between 1 and 5 mm in diameter (a factor of about 100), (b) One process is the lowering of the saturation vapor pressure around large droplets (because of the lower curvature of its surface). The other process is the collision between droplets and the resulting coalescence.

Check Point 15.6: The melting of ice crystals, as the cloud is likely below freezing.

Check Point 15.7: Because impurities in the tap water can act as ice nuclei, initiating freezing once the temperature falls below 0°C.

Check Point 15.8: When the rain freezes before it hits the ground, it falls as ice pellets or sleet. When the rain freezes after it hits the ground, it is called freezing rain.

Check Point 15.9: Both are essentially frozen raindrops, but hail has repeated layers of water being frozen, as the drop repeatedly gets pushed up into a colder layer.

$$v = \frac{2}{9}\frac{r^2 \rho_{\mathrm{water}} g}{\eta} \tag{15.1}$$

Check Point 15.10: Zero, the gravitational force is balanced by the drag force.

$$v = \sqrt{\frac{8rg}{3C}\left(\frac{\rho_{\text{water}}}{\rho_{\text{air}}}\right)} \tag{15.2}$$

Check Point 15.11: Rain drops would fall since 50 cm/s is less than their fall speed (between 3.8 m/s and 12 m/s), whereas cloud droplets would rise since 50 cm/s is greater than their fall speed (between 0.03 mm/s and 30 m/s).

16. Latent Heating and Cooling

Check Point 16.1: Absorbed; it takes energy to break a bond. This cools down the air.

Check Point 16.2: The process of evaporating the sweat.

Check Point 16.3: The wet-bulb temperature, unless the air is saturated (in which case they are the same).

Check Point 16.4: Since 6 mb is very close to the saturation vapor pressure at 0°C, the air will cool at the moist adiabatic lapse rate, which is about 6°C/km. Thus, the temperature will be −6°C. If the air was totally dry, it would cool by 10°C. If the air had a relative humidity between 0 and 100% then it would cool at the dry adiabatic rate until it saturates and then it would cool at the moist adiabatic rate.

Check Point 16.5: (a) A vapor pressure of 6 mb is roughly what is needed for saturation a 0 °C and so the temperature decreases but not as quickly as it would if it were dry; (b) The potential temperature increases because the latent heating causes the temperature to be higher than it otherwise would be; (c) the equivalent potential temperature stays the same.

Check Point 16.6: This would mean that the equivalent potential temperature would have to be less than the coldest equivalent potential temperature above it. Looking at the sounding, the coldest equivalent potential temperature occurs around 500 mb (look at the moist adiabatic curves), which is at 18°C. Thus, if the surface was at 1000 mb, the answer would be 18°C. Since the surface is at around 950 mb or 960 mb, that temperature looks to be around 16°C, based upon the skew-T.

Check Point 16.7: Less stable, as condensation during rising will warm the parcel, making it even more buoyant.

Check Point 16.8: Because it is really the heating per mass of water that changes state.

Check Point 16.9: The amount of energy required to sublimate 1 gram of ice, as the bonds are stronger in the ice.

Check Point 16.10: The amount of energy required to evaporate 1 gram of ice.

$$dT = \frac{1}{c_\mathrm{p}} \frac{L\ dm_\mathrm{v}}{m_\mathrm{T}}. \tag{16.1}$$

$$dT = \frac{L\ dq}{c_\mathrm{p}} \tag{16.2}$$

Check Point 16.11: L refers to the latent heat of vaporization (or fusion or sublimation) for water associated with its change in state, dq refers to the change in specific humidity, and c_p represents the specific heat of the air at constant pressure.

A. Lists of Abbreviations

B. Reference Information

C. Simulating the diurnal cycle

$$\frac{dT}{dt} = \frac{F}{c \cdot h \cdot \rho} \tag{C.1}$$

D. Equations of Motion

$$\frac{dv_r}{dt} = \frac{v_t^2}{r} \tag{D.1}$$

$$\frac{d(rv_t)}{dt} = 0 \tag{D.2}$$

$$\frac{du}{dt} = -\frac{uw}{a} + \frac{uv\tan\phi}{a} - 2\Omega w\cos\phi + 2\Omega v\sin\phi \tag{D.3}$$

$$\frac{dw}{dt} = \frac{v^2}{a} \tag{D.4}$$

$$\frac{dv}{dt} = -\frac{vw}{a} \tag{D.5}$$

$$\frac{du}{dt} = -2\Omega w\cos\phi + 2\Omega v\sin\phi - \frac{uw}{a} + \frac{uv\tan\phi}{a} \tag{D.6}$$

$$\frac{dv}{dt} = -(\Omega^2 a\sin\phi\cos\phi) - (2\Omega u\sin\phi) - \frac{u^2\tan\phi}{a} - \frac{vw}{a} \tag{D.7}$$

$$\frac{dw}{dt} = (\Omega^2 a\cos^2\phi) + (2\Omega u\cos\phi) + \frac{u^2+v^2}{a} \tag{D.8}$$

$$(du_*/dt)\hat{\imath} + (dv/dt)\hat{\jmath} + (dw/dt)\hat{k} = -u_*(d\hat{\imath}/dt) - v(d\hat{\jmath}/dt) - w(d\hat{k}/dt). \tag{D.9}$$

E. Molar masses

Check Point E.1: (a) A neutron is an atomic particle with no charge. A nucleon is any atomic particle that happens to reside in the nucleus of the atom. That means a nucleon can be either a neutron or a proton.
(b) Protons and electrons have opposite charge. Thus, in order for an atom to be neutral, it must have equal numbers of each. A neutral nitrogen atom, then, must have seven electrons.

Check Point E.2: (a) No. (b) Yes. (c) Yes (it is called an **ion**)

Check Point E.3: The mass of the electrons alone is equal to $8m_e$, which is $7.287590608 \times 10^{-30}$ kg. Compare that to the mass of protons and neutrons, which is $8m_p + 8m_n + 8m_e$) or $2.6787679426 \times 10^{-26}$ kg. Divide to get 0.0272%. This is a very small fraction and we can ignore it if we only need to get the mass within 0.1% or so.

Check Point E.4: The determination of the mole is simply a matter of convenience so that we don't need to worry about very tiny or very large numbers. However, once determined, there can be many other parameters that depend upon it. Consequently, it shouldn't be changed once it has been agreed to.

Check Point E.5: Technically, when energy is released, the mass decreases, and when energy is absorbed, the mass increases. However, the mass remains essentially unchanged in all three cases since the energies involved are very small compared to the masses involved.

$$E = mc^2 \tag{E.1}$$

Check Point E.6: When the individual nucleons come together some energy is released. Because of the mass-energy equivalence ($E = mc^2$) that energy shows up as a smaller mass of the nucleus.

Check Point E.7: Since the molar mass is approximately 40 g/mol, that means there are probably 40 nucleons in the nucleus. Since 18 of them are protons that means that there must be 22 neutrons.

Check Point E.8: Each relative abundance represents a fraction. For example, 0.25 represents 25% of the total. Thus, the entire amount should correspond to 1.0000 or 100%.

F. Atmospheric Evolution

$$v_e = \left[\frac{2Gm_{\text{earth}}}{r}\right]^{1/2} \tag{F.1}$$

$$\Delta\left(\frac{1}{2}mv^2\right) = F_{\text{avg}}\Delta x \tag{F.2}$$

$$F_g = G\frac{m_{\text{earth}}m}{r^2} \tag{F.3}$$

$$F_{\text{avg}} = \int \frac{F(x)dx}{\Delta x} \tag{F.4}$$

$$\frac{1}{2}mv^2 = Gm_{\text{earth}}m \times \frac{1}{r} \tag{F.5}$$

Check Point F.1: (a) The force of gravity on an object is equal to the object's mass times 9.8 N/kg only if the object is near the surface of Earth.
(b) To escape the gravitational pull of Earth, the molecule has to move significantly away from Earth and the gravitational force on the molecule must decrease to zero. That is a significantly different than the value at sea-level.
(c) One divided by an infinitely large number equals an infinitely small number.

Check Point F.2: Neither gas is being lost in a significant way, although hydrogen, being lighter, is more likely to reach the escape velocity and thus be depleted.

Check Point F.3: Burning fossil fuels moves carbon from one form (hydrocarbon) to another (gaseous carbon dioxide).