The Fundamentals of

PHYSICS

Volume I

Force and Motion

Robert A. Cohen
Physics Department
East Stroudsburg University

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Acknowledgments

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Contents

Part A

Force and motion

1. The Natural State of Things

Puzzle #1: Suppose you are pushing a box along the floor. When you stop pushing, it slows down and stops. Why does it slow down?

Introduction

Each chapter of this book starts with a puzzle.

The puzzle typically describes a phenomenon and an associated question. Make sure you read the puzzle and think about it while reading the chapter. Also make sure you read the introduction to the chapter (this section). The puzzle and introduction give purpose to the chapter, so you know why the material is presented as it is. After all, if you don't know where you are going and why you are trying to get there, you are more likely to end up trying to memorize disconnected bits of information and interpreting the material as just a hoop you must jump through. You won't get anything useful out of the course doing it that way, and we don't want that.

❢ As you read each chapter, keep in mind the purpose of that chapter.

As for this chapter's puzzle, you might already have some ideas about why a box slows down when you stop pushing it. What we want to do in this chapter is to generalize what is happening so that we can not only predict when and how a box slows down but also the motion of *any* object, like objects that are dropped or thrown, or birds flying through the air and fish swimming through the water, or even the wind blowing in a hurricane. We may be *starting* with a box sliding on the floor but we have a much grander objective in mind – to predict and explain the motion of *all* objects.

1.1 The nature of interactions

The answer to the puzzle can be stated as follows: the box slows down because a force is exerted on the box. The big idea in this textbook is that forces are responsible for speeding up or slowing down an object.[i] Later in this part of the book we'll examine how we can predict how *much* an object speeds up or slows down when forces act on an object. For now, we want to focus on what it *means* for a force to act on an object.

In particular, a force acts on an object when the object interacts with another object. In the puzzle, a force is exerted on the box, slowing it down, because the box is interacting with the floor.

To get a sense of what it means for the box to interact with the floor, it helps to build a mental model[ii] (or abstract explanation) of what happens when objects interact. Mental models are really useful and we'll utilize them throughout physics to help us build a picture of how things are related and why. Indeed, one can argue that all of the equations and mathematics that one normally associates with physics are really just applications and interpretations of the mental models. Your mental models may not exactly match the ones I'm presenting here, but it is important to pay attention to the mental models you are using, and make sure they match the essential characteristics of the ones I present.

To understand what we mean by an **interaction**, consider two boxes sliding toward each other, as illustrated in part (a) of Figure 1.1. They stop as they collide, as each gets in the way of the other (illustrated in part b of the figure). If we could magnify what is happening where the two boxes touch (illustrated by the dashed box in part b, expanding to part c), we can imagine invisible tiny springs.[iii] The tiny springs act to keep the boxes from "meshing" into the other.

The key thing is that the springs represent how the two boxes are interacting. In real life, the tiny little springs don't exist. However, the mental model is

[i] Forces are also responsible for changing the direction of motion.

[ii] This is not meant to be a physical model, like a model airplane. Rather, it is more of an explanation that uses analogies and/or pictures to help us visualize what is going on.

[iii] The springs are being used to represent the interaction that is present. In volume II, we'll explore how this particular interaction is associated with the repulsion of electric charges. However, the interaction is similar, whether by electric charges or by springs, so imagining the interaction as tiny springs is sufficient for our purposes.

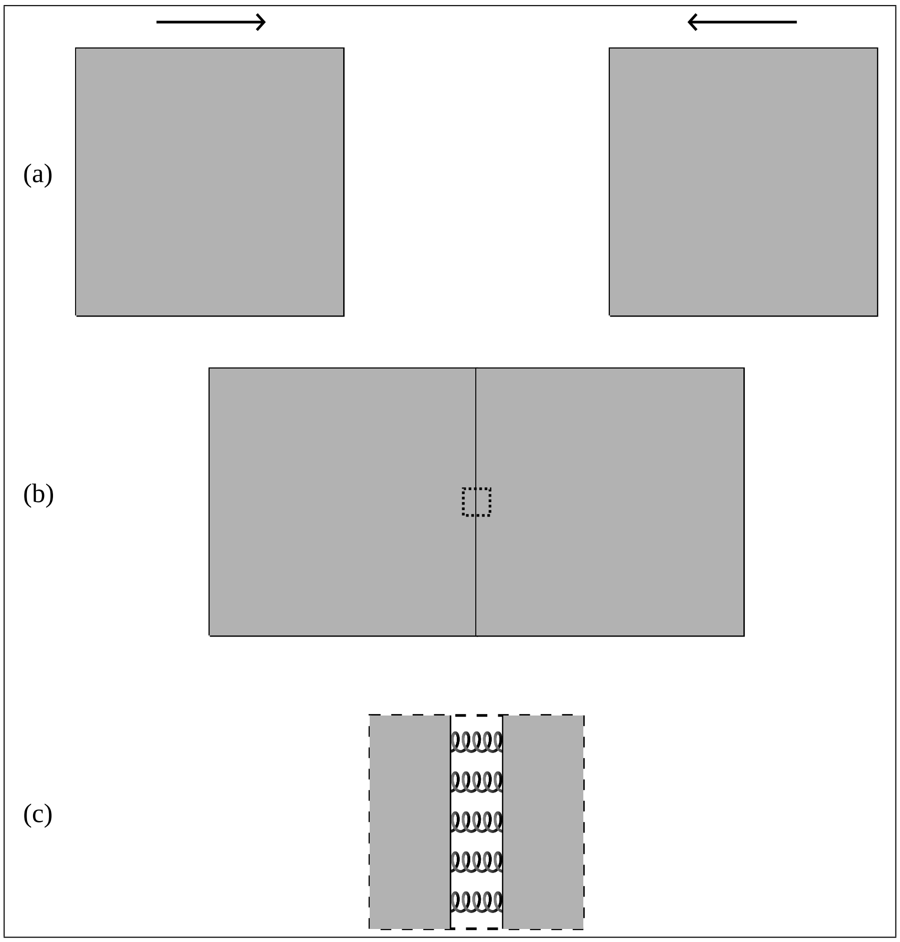

Figure 1.1: Left: An illustration of two boxes moving toward each other (part a) and colliding (part b), with their contact represented as tiny springs (part c).

still good, in that all objects are essentially covered with microscopic little springs that compress a tiny bit when something touches it, and in an attempt to expand the springs push on the two boxes, keeping them separate. The microscopic little springs are just the arrangement of molecules inside the material.

Do the boxes only interact when they touch?

That depends on what you mean by "touch." If you mean that the two objects get close enough to interact then, yes, touching is equivalent to interacting. And, for the situation illustrated in Figure 1.1, the two boxes don't interact until they touch.

❢ Many forces are only present when something is touching the object.

The same is true for when you throw something. While holding the object or actively in the process of throwing, you are interacting with the object. Once the object leaves your hand, though, you are no longer interacting with it.

✍ For the time being, we'll only consider two objects to be interacting if they are touching. However, in some cases, objects can interact with physically touching in this sense. For example, two magnets can interact even though they don't touch each other.

✓ *Checkpoint 1.1:* [iv] *Based on the mental model described above, in part (a) of Figure 1.1, before they touch, is the left box interacting with the right box?*

[iv] Checkpoints are problems provided throughout the text for *you* to continually evaluate your understanding of the readings. They are called "checkpoints" instead of "homework" or "practice" in order to stress the importance of active learning (sometimes called "active thinking") and self-assessment. In keeping with this self-assessment philosophy, an answer key to the checkpoints is available for you to check your understanding. Please use these checkpoints in the way they were intended. In other words, do not use the checkpoint as a *starting* point and then search through the text for the answer. Instead, read through the text and, as you read, compare what you are reading with what you already know. Then, use the checkpoint as a check.

1.2 Forces

A **force** refers to a pushing or pulling on the object due to its interaction with some *other* object.

For example, when a spring is compressed, it exerts a force on whatever is compressing it. This is because springs oppose compression and try to expand back to their original length.

In terms of our conceptual model of interactions, we can think of objects as being covered with invisible springs that compressed when the objects touch each other. This means that whenever two objects touch there is a force on each, due to that interaction.

Just as the invisible springs only compress when the boxes interact by touching, forces also only exist when two objects interact by touching.

Indeed, it is the *interaction* that is responsible for the force. For the situation with the two boxes, the interaction, which we visualize as tiny springs, push the boxes apart.

Language-wise, we can say that there is a force on each box due to their interaction with each other. There is a force on the left box due to this interaction, and there is a force on the right box due to this interaction.

Can we just say there is a force on the left box or a force on the right box?

Yes, you can, but only if we don't need to know what interaction the force is associated with or if it is obvious from the situation.

Can we say the right box exerts a force on the left box or the left box exerts a force on the right box?

While it is common to do so, for the time being it is better if you avoid that language, as it can reinforce a use of the term "force" that is inconsistent with the way we use it in physics. Remember, forces (as we are using the term) are due to *interactions* between objects – forces are not associated with specific objects.[v]

Why is it wrong to associate forces with specific objects?

[v] A notable example of such poor usage is the phrase "may the force be with you."

The reason we associate the force with the interaction, not the objects, is because the force only exists when the objects interact while the objects exist regardless of whether they interact. Also, as we will soon find out, an object's motion changes only while the object interacts with another object, so if you associate a force with a single object rather than an interaction between two objects, you may mistakenly think a force is acting when it isn't, and thus predict a change in motion that won't happen. Or, visa-versa, you may mistakenly think a force isn't acting when it is, and thus fail to predict the change in motion.

This is really important. If you are using the word "force" in a way that is different than the way we use it in physics then you'll just get confused because there will be a mismatch between your description of what is happening and our description.

✍ It isn't that our use the word "force" is correct and all other ways are wrong but rather that our way is better for the particular task at hand, which is to predict changes to an object's motion.

✓ *Checkpoint 1.2: Two boxes (as in Figure 1.1) slide toward each other and then collide. Before the boxes touch, is there a force on the left box due to an interaction with the right box?*

1.3 Law of force and motion

As mentioned in the previous section, an object's motion changes only while the object interacts with another object, and the force exerted during that interaction tells us how much the motion changes.

This idea is so important that the entire book will be focused on this relationship.

Why would we want to focus on that relationship?

The whole point of the textbook is to learn how to apply a few ideas to a large number of situations. This relationship (between how an object's motion changes and the forces that act upon the object) serves as a perfect context for learning this skill because it is so powerful, in the sense that it applies to lots of different situations. The change in a car's motion when the driver slams on the brakes, the arc of a ball while in the air (after being thrown), the oscillating motion of a leaf swinging in the wind, and the motion of a person falling on their face in a viral video are all examples of the same conceptual idea encapsulated in the relationship between how an object's motion changes and the forces that act upon the object.

> ✍ Consider the difference between *memorizing* the route between any two places, and just utilizing a map to *figure out* the route between any two places. Knowing how to read a map is much more powerful, if you can master it. For more details about the value of powerful ideas, see the section on "why physics?" in the supplemental readings.

This particular relationship is so important that I'll give it a name: the **law of force and motion**.[vi] We'll provide a more specific statement of the law later. For the time being, it is sufficient to know that the law of force and motion states that an object's motion changes only while the object interacts with another object, and the force exerted during that interaction tells us how much the motion changes (speeds up or slows down, for example).

❢ Forces act to change an object's motion.

WHY IS THIS CALLED A LAW?

It is called a law because it describes the relationship between two measurable things. In this case, the law of force and motion describes the relationship between the forces acting on an object and the how the motion of that object changes (while the forces are acting on it).

ISN'T A LAW SIMPLY A THEORY THAT HAS BEEN PROVEN CORRECT?

No. This is a common misconception. If you search through the history of things called laws, like the *ideal gas law*, you'll never find a reference to them as a theory. Rather, laws are called laws because they describe relationships between quantities. For example, the ideal gas law describes how properties of gases (like pressure and temperature) are related. Theories are called theories because they are models that explain observations. For example, the

[vi]The law is often called Newton's second law, because it was the second of three laws that Newton mentioned in his *Mathematical Principles of Natural Philosophy.* Newton stated the law as follows (translated from the original Latin): *The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.* Notice how only forces *impressed* on the object influence its motion. An object's motion is <u>not</u> influenced by the forces it exerts on itself or other things.

kinetic-molecular theory is based upon using a model of individual molecules in motion to explain why gases behave the way they do.

Scientific theories and laws represent two ways of making testable predictions. A scientific law *describes* how observable quantities are related while a scientific theory *explains* why certain phenomena are observed to occur. For more information on laws and theories, see the supplemental readings.

Does "theory" mean "guess"?

To non-scientists, maybe. But not to scientists. To scientists, theories are models that are well-supported by observations. Some theories are better supported and have been more rigorously tested than some laws.[vii]

Can you prove a law or theory to be true?

No, you cannot prove a law to be true or prove a theory to be true. You can, however, *support* the law or theory. And, after many tests, the sheer amount of evidence may lead us to no longer have any doubt of its validity.

Indeed, some people may say a law or theory is proven if it has been supported to an extent where there is no longer any reasonable doubt about it. However, technically, we cannot prove a law or theory to be correct. And, certainly, in no case will a single piece of evidence be sufficient to prove a law or theory.

✓ *Checkpoint 1.3: According to the law of force and motion, if no forces are acting upon an object can the object be slowing down?*

1.4 Speeding up and slowing down

Most people already have a sense of motion – either you are moving or you are not, and you can move quickly or you can move slowly. As mentioned earlier, an object's motion changes (speeds up or slows down, for example) only while forces are acting upon it.[viii]

[vii]For example, Ohm's Law, a law discussed in volume II of this text, has been shown to fail in certain situations yet it is still called a law.

[viii]In chapter 2, we examine situations where there are *multiple* forces acting.

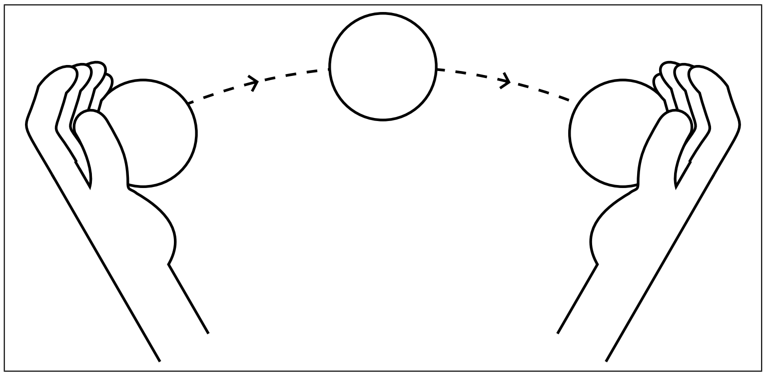

Figure 1.2: An illustration of a ball being thrown and caught.

If we can't "see" the microscopic springs on the surface of an object, how can we tell when a force is being exerted on an object?

Certainly, we can "feel" something when we touch an object, so it is easy to imagine the tiny invisible springs acting on us where we touch the object. And, for other objects interacting, like the two boxes in part (b) of Figure 1.1, we can put a force sensor between the two boxes, and the force sensor would indicate a force when they are right up against each other.

However, what about when we don't have a force sensor? How can we tell when a force is acting?

We have two ways, described by the two ideas discussed so far: (1) forces are exerted only when objects interact (usually by contact) and (2) an object's motion only changes if there is a force acting on it (law of force and motion).

To illustrate, consider the situation shown in Figure 1.2 (throwing and catching a ball).

In the illustration, the left hand throws the ball and the right hand catches it. According to the first idea, the ball only experiences a force while a hand touches it. That suggests there is a force exerted on the ball at the beginning of the ball's flight (while the left hand is in the process of throwing it) and there is a force on it at the end of the ball's flight (while the right hand is in the process of catching it).

Now let's see if this analysis is consistent with the second idea: that an object's motion only changes while there is a force acting on it. In order to throw a ball, the ball has to speed up. Conversely, to catch a ball, the ball has to slow down. Again, this is consistent with a force acting on the ball while the left hand is throwing it (and the ball is speeding up) and, again, while the right hand is catching it (and the ball is slowing down).

Even though we can't see the microscopic springs that would indicate whether a force is acting on the ball, we can still surmise that forces are acting based on whether something is touching the ball and/or whether the ball's motion is changing. Still, we can imagine that there are tiny invisible springs between the left hand and the left side of the ball that are being compressed while the left hand is touching the ball. Similarly, we can image that there are tiny invisible springs between the right hand and the right side of the ball while the right hand is touching the ball. In this way, the conclusions we make based on the two ideas is consistent with what our mental model would suggest.

What about when the ball is in the middle of its flight?

During that time, neither hand is touching the ball. Therefore, there is no force on the ball due to its interaction with either hand.[ix] In terms of our mental model, there would be nothing to compress the springs on either side of the ball.[x]

✓ *Checkpoint 1.4: In baseball, one player, called the pitcher, throws the ball to another player, called the catcher. Is there a force on the ball due to its interaction with the pitcher while the ball is in the air (between the pitcher and catcher, after the pitcher lets go of the ball)?*

Not only can we surmise when a force is acting on an object but we can also surmise the *direction* of the force. You might have noticed, for example, that the ball speeds up, moving rightward, while in contact with the left hand, where the force on the ball pushes it in its direction of motion (rightward).

[ix]As mentioned earlier, we need to be careful with the language. Some people might refer to the "force *of* the hand." This can be dangerous, as it seems to incorrectly imply that the hand "contains" a force that it applies to the ball. Since the hand does not contain a force, and the force only exists due to the hand and ball being in contact with each other, it is better to call it the "force *due* to the ball's interaction with the hand".

[x]The astute reader might notice that we are ignoring gravity. We'll consider the impact of gravity in Part C.

Conversely, the ball slows down, still moving rightward, while in contact with the right hand, where the force on the ball opposes its motion.

Indeed, the law of force and motion can be expanded beyond simply saying that an object changes its motion while a force acts on it. We can also say that an object speeds up while the force on it acts in the direction of the motion and, conversely, the object slows down while the force on it acts against that motion.

❢ To speed up an object, a force needs to act in the direction of the object's motion.

❢ To slow down an object, a force needs to act opposite the object's motion.

The key point is that the direction of the force, relative to the object's motion, determines whether the object speeds up or slows down. An object speeds up while the force exerted on it is in the same direction as its motion, and an object slows down while the force exerted on it is opposite its motion.[xi]

Example 1.1: At the moment an object is moving northward[xii], a northward-directed force (pushing the object northward) is being exerted upon it. While the force is acting on the object, is the object speeding up, slowing down or neither? Explain.

Answer 1.1: In this case the force on the object is in the same direction as the motion (both are northward). Consequently, the object must be speeding up, according to the law of force and motion.

✓ *Checkpoint 1.5: Suppose a westward-directed force (pushing the object westward) is acting upon an object that is already moving westward. What is happening to the object? Is it speeding up, slowing down, or neither? Explain.*

1.5 The meaning of constant

When an object's speed doesn't change, we say that the object's speed is **constant**. So, if I say that an object has a *constant* speed, I mean that the object is moving without speeding up or slowing down.

[xi] In chapter 20 we explore what happens when the force on an object is directed *perpendicular* to the object's motion.

[xii] There are four horizontal directions: north, east, south and west. North is opposite to south, and east is opposite to west.

The word “constant” can be used for any quantity, not just speed. For example, if the temperature is 10°C and it is staying at 10°C, we could say that the temperature is constant.

Notice that the word means more than just that it always has a value. It means that the value doesn’t change. For example, if an object is **at rest**[xiii]. then its speed is zero. If it remains at rest, then its speed would be constantly zero and so this would be a case where its speed is constant.

✍ Quantities which have values that may vary, like time, are called **variables**. Quantities with values that don’t change or vary are called **constants**. Some quantities, like the speed of light, never vary and are truly constant. Other quantities, like my mass, might be constant for a given problem but can be different at some later time.[xiv] For examples, see the constants section of the supplemental readings.

✓ *Checkpoint 1.6: In which of the following situations does the object have a constant speed?*
(a) A rock at rest on the ground.
(b) A ball that is rolling down a hill, speeding up as it does so.
(c) A bowling ball rolling across the floor without speeding up or slowing down.

1.6 Law of inertia

What is happening with the ball during the time when neither hand is touching the ball?

During the time period between when the left hand lets go of the ball and the right hand first touches it, neither hand is touching the ball and so there is no force on the ball due to the ball’s interaction with one hand or the other. Consistent with the law of force of motion, the ball neither speeds up nor slows down during that period.

If there is no force acting on the ball during this time, why does the ball keep going?

[xiii] The phrase “at rest” means the object is stationary, without moving.
[xiv] Often higher, to my chagrin.

The law of force and motion doesn't *explain why* things are a particular way. It simply *describes how* things are. Basically, it describes how there needs to be a force on an object to *start* it moving, but there doesn't need to be force on an object to *keep* it moving.[xv]

❢ Objects don't slow down on their own.

Between the time when neither hand is touching the ball, there is no force acting to speed it up or slow it down. Consequently, during that time, the ball continues to move without speeding up or slowing down.

> ✍ Remember that a force corresponds to a pushing or pulling on the object due to its interaction with some *other* object. Objects don't move "with" a particular force. Rather, forces are exerted *on* the object to make the object's motion change.

This idea, that the motion remains the same without a force to change it, is a consequence of the law of force of motion. However, many people consider it so important that they refer to it by a separate[xvi] name: the **law of inertia**.[xvii] The reason why people refer to it separately may be because people feel it is easier to comprehend the law of force and motion if the law is presented in two steps: one law describing what happens when there are no forces exerted on the object and a second law describing what happens when there is.[xviii]

The reason why it may be important to state the law of inertia is because of how easy it is to mistakenly think the opposite. In fact, during the time of Newton, a common but incorrect view was that the natural state of an object was to be at rest and so an object in motion was thought to require a force exerted on it to *remain* in motion. As pointed out by Newton, and described by the law of force and motion, the correct interpretation is that

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[xv] Unless there was a *second* force opposing the motion at the same time. Chapter 2 considers situations where multiple forces are acting at the same time.

[xvi] In the same way, Boyle's law, Charles' law, Gay-Lussac's law, and Avogadro's law all follow from the more general *ideal gas law*.

[xvii] Newton stated the law as follows (translated from the original Latin): *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*

[xviii] Indeed, this may be why Newton introduced the law of inertia prior to the law of force and motion, which is why many people refer to it as **Newton's first law**. Despite the convention of assigning Newton's name to the law, the essence of the law of inertia, however, was not original with Newton – a version of it was stated at least as far back as 1638 (by **Galileo Galilei**).

an object slows down (or speeds up, or changes directions) only when there is a force exerted on the object.

Looking back, however, we can see why people would think that a force was needed to keep an object moving with a constant velocity. As we know, a force is required to *start* an object in motion so people may have had difficulty distinguishing between *starting* the motion and *maintaining* the motion.

Second, in most cases there are forces acting against an object's motion (like friction), so a second force needs to be applied in order to counter friction. Back then, people didn't know about friction and so surmised that the natural state of an object was to be at rest.[xix] We'll explore friction in the next section.

❢ An object's motion does not change during the time when there is no force acting on the object.

In any event, now we know better. A force is needed to *start* an object moving and to *stop* an object, but no force is needed to *keep* an object moving with the same velocity (unless there is already a force, like friction, acting that needs to be countered).

> ✍ This is a really important point, so I want to repeat it. There needs to be a force on an object to *start* it moving, but there does not need to be a force on an object to *keep* it moving unless, at the same time, there is an *additional* force (associated with additional object) acting to slow it down. If there is no other force acting on it to slow it down or speed it up, the object will continue to move without speeding up or slowing down.

❢ An object's inertia is its tendency to stay in motion (if it is moving) or stay at rest (if it is at rest).

Why is this called the law of inertia?

I refer to this idea as the law of inertia because an object's **inertia** is what we call the object's tendency to continue doing what it is doing (i.e., staying at rest or moving in the direction it is moving). It requires a force to *start* moving but, once moving, its own inertia will keep it going.

Is an object's inertia considered a force?

No. There are several reasons why an object's inertia is not considered a force.

❢ An object's inertia is not considered to be a force exerted on the object.

First, if we allowed inertia to be considered a force, we couldn't treat objects at rest and objects in motion the same way (as in the law of inertia).

[xix] In chapter 2, we explore situations with multiple forces acting.

Second, as mentioned earlier, the word "force" refers to actions imposed upon an object due to interactions between the object and other objects (more will be said on this in chapter 5). Consequently, we'll say that an object cannot exert a force upon itself or, more precisely, an object can't influence its own motion without interacting with *other* objects.[xx] Since inertia is only associated with the object itself and not an interaction with *another* object, it isn't considered a force.

✍ Distinguishing between force and inertia is like distinguishing between roads and rivers on a map. Yes, they both look like lines on a map. However, if you want to figure out the route from one place to another, you cannot treat roads and rivers as though they are the same thing.

Example 1.2: For each of the following situations, use the law of inertia (or law of force and motion) to determine whether the force on the object is zero or not:
(a) An object remains at rest.
(b) An object *starts* to move northward.
(c) An object is moving northward, without speeding up or slowing down.
(d) An object is moving northward and slowing down.

Answer 1.2: The force on the object is zero in cases (a) and (c) because the object's motion (speed and direction) is not changing in those cases. The force on the object is not zero in cases (b) and (d) because the motion is changing (i.e., speeding up or slowing down) in those cases.

✓ *Checkpoint 1.7: I ride my bike up the side of a hill, slowing down as I do. During this time, is my inertia considered one of the forces acting on me?*

1.7 Friction and drag

WHAT ABOUT THE AIR?

[xx]Certainly, there can be forces on one part of the object due to that part interacting with other parts of the same object, but the motion of the whole is unaffected.

Realistically, unless the object is in outer space (where there is no air or anything else), there will always be *something* that touches the object. For the ball's flight through the air, there is a force on the ball due to its interaction with the air, which acts to slow down the ball. For a fish swimming through water, there is a force on the fish due to its interaction with the water, which acts to slow down the fish. We call this **drag**.

In many cases, the drag force is **negligible**, which means it is so tiny that it doesn't noticeably affect things and so we can neglect it. For the ball moving through the air, the drag force is probably negligible.

How do you know the drag force due to the air[xxi] is probably negligible?

We can observe how birds glide horizontally through the air (without flapping) and see that they do not experience any significant slowing. In comparison, fish quickly come to a stop when they stop swimming. That suggests that the drag force due to water is *not* negligible.

To understand what is going on, it helps to construct a mental model of how air and water act to slow down objects. I'll examine air here, but the same analysis can be done with water.

Air is made up of lots of tiny little molecules that are bouncing around. When a molecule hits an object, there is a force exerted on the molecule and object. The force associated with each molecule that hits may be small, but it can be significant when we have lots of molecules.[xxii] There is a force pushing on the object due to its interaction with the air. However, since the air is all around the object, the force due to the air pushes on all sides of the object.[xxiii] As we'll discuss in chapter 2, when there is just as much force in the direction of the object's motion as opposite the motion, the object

[xxi] The drag due to air is sometimes called **air resistance**.

[xxii] One can use a syringe to demonstrate that air exerts a force. Take the syringe and push the plunger all the way in (to push all of the air out of the syringe tube). Then, plug up the end of the syringe (so no air can get in). If you try to pull the plunger out, you will find that you need to exert a force to keep the plunger from jumping back in. This is because the air on the outside is pushing the plunger in and there is no air inside the syringe pushing the plunger out.

[xxiii] You can demonstrate this with the syringe. Simply orient the syringe discussed in the previous footnote two different ways. First do the experiment with the tip of the syringe down and the plunger pulled up. Then do the experiment with the tip of the syringe up and the plunger pulled down. You'll find the force exerted due to the air is the same.

neither speeds up nor slows down and there is no drag.[xxiv] However, as you are probably aware, the drag can be significant when the wind is strong (air moving quickly) or the object is moving quickly through the air. In that case, the force due to the air in front of the object, pushing the object backwards, can be significantly greater than the force due to the air behind the object, pushing the object forward. It is this difference that produces the drag, which opposes the motion. As mentioned earlier, for small speeds, typical of situations we'll be studying, the drag is small enough to be ignored.

❢ Drag is insignificant unless the object is very light or is moving very quickly.

✍ A crucial part of solving problems is being able to tell what is relevant and what isn't. It would be useful to try an experiment with and without air present to determine in which situations it is or is not relevant.

How is water different?

Water has a much higher density than air, which means there are many more molecules that can impact the surface of the object and result in a force. As a result, the drag in water can be significant, even for small speeds.

✓ *Checkpoint 1.8: For objects at rest or moving slowly through the air, why is it valid to ignore the force due to their interaction with the air?*

A force similar to drag is **friction**, which exists when objects slide along a surface. Indeed, it is friction that makes the box in the puzzle slow down. Consistent with the two ideas we've mentioned, the box is slowing down, which suggests that a force is acting on the box, and the box is touching the floor, which suggests the force is due to the floor.

Friction, like drag, opposes the direction of the sliding. And, like drag, it is sometimes significant (as with the box sliding along the floor) and sometimes negligible. For example, ice is very slippery. If you push a box across ice and then let go, the box will glide across the ice without slowing down much. This suggests that ice is nearly **frictionless**.

Another way to minimize friction is to use wheels. For example, if you are in a

[xxiv] The air would still act to "compress" the object but it wouldn't act to change the object's motion. For **thermodynamics**, such compression might become relevant.

store and see a wayward[xxv] shopping cart[xxvi] rolling by itself,[xxvii] chances are it continues rolling, suggesting that friction is small, until it hits something (since a force is needed to *start* or *stop* moving but not to *keep* moving).

✓ *Checkpoint 1.9: Suppose two cars, one moving at 50 mph and the other moving very slowly, are both put in neutral. The fast car slows down while the slow car continues to roll for a long time. Assuming both are on level ground, in which case is drag more negligible and why?*

Summary

This chapter examined how an object's motion changes when a force is exerted upon it.

The main points of this chapter are as follows:

- As you read each chapter, keep in mind the purpose of that chapter.
- Many forces are only present when something is touching the object.
- Forces act to change an object's motion.
- To speed up an object, a force needs to act in the direction of the object's motion.
- To slow down an object, a force needs to act opposite the object's motion.
- Objects don't slow down on their own.
- An object's motion does not change during the time when there is no force acting on the object.
- An object's inertia is its tendency to stay in motion (if it is moving) or stay at rest (if it is at rest).
- An object's inertia is not considered to be a force exerted on the object.
- Drag is insignificant unless the object is very light or is moving very quickly.

[xxv] By wayward, I don't mean that the cart's motion isn't predictable or that the cart can make conscious decisions about its actions. I just like the word. I actually imagine that the store is a K-mart. In Waymart, Pennsylvania. A wayward Waymart K-mart cart.
[xxvi] Also known as a grocery cart, buggy, basket, carriage or wagon.
[xxvii] Boy, there sure are a lot of footnotes in this chapter!

Frequently Asked Questions

If there was no force on an object, does that mean the object is stationary?

No. If the object is already moving when the force was removed, the object will continue to move.

If something is moving, is there a "force of the motion" that keeps the object moving?

No. What carries the object forward is its inertia. See section 1.6 for more.

Why don't we call an object's inertia a force?

The law of force and motion is based on a precise meaning of force. If you confuse "force" with "inertia" or "effect", you won't be able to apply the law appropriately. Only use the word "force" for the pushes and pulls that are applied via interactions the object has with *other* objects.

Is inertia measurable?

We don't assign a value to an object's inertia (although an object's momentum, defined in chapter 27, is related).

To keep an object moving with a constant speed and direction, the law of force and motion says that the force on it must be zero. What happens if the force is not zero?

It depends on the direction of the force. If the force is in the direction of the motion then the object speeds up. If opposite the motion, the object slows down.

If you continue to apply a force in the direction of motion, will the object continue to speed up?

Yes. See section 1.4.

How long do we have to wait between when the force is applied and when we see the change in motion?

There is no delay. The object's motion changes *while* the force is applied. There is no "time delay" between when the force is applied and the object "realizes" that it has got to do something about it.

This is an important point because this is not easily seen when observing the motion of real objects. In fact, cartoons tend to reinforce the incorrect notion that there is a delay. In some cartoons, for example, a character runs over the edge of a cliff and "hangs" there a couple of seconds before falling straight down.[xxviii]

In reality, as soon as the character leaves the ground, the character would be pulled downward.

Terminology

Air resistance	Drag	Galileo Galilei	Negligible
At rest	Force	Inertia	Newton's first law
Constant	Friction	Law of force and motion	Thermodynamics
Constants	Frictionless	Law of inertia	Variables

Additional problems

Problem 1.1: State the law of force and motion in words.

Problem 1.2: Can there ever be a situation where an object slows down without any force being exerted upon it? If so, when? If not, why not?

[xxviii] One could say that the cartoon introduces a time delay in order to allow the character (and the viewers) to recognize the "gravity" of the situation.

2. Multiple Forces

Puzzle #2: If you push a box across the floor, will it speed up, slow down or just move with a constant speed?

Introduction

Although not stated explicitly, *two* forces are acting on the box in the puzzle: the force associated with you pushing it, and the friction as it slides across the floor. This is different from what we examined in chapter 1, where we only considered one force acting on the object.

> ✍ Remember that forces are exerted on an object due to its interaction with *other* objects. If an object is interacting with several other objects simultaneously, there will be multiple forces exerted on it, one force for each interaction.

Fortunately, the physics when multiple forces are acting simultaneously is pretty much the same as the physics with single forces, namely that each force acts to either speed up or slow down (or change the direction of the object's motion). And, if the forces are doing opposite things, then the object's motion changes in a way that depends on which force is greater.

To do this, we first need to quantify the value of forces. Thus, before illustrating the process, we'll first quantify forces, in terms of values and units.

2.1 Units

Most quantities are made up of a number and a unit. For example, if I buy three apples then the quantity "3 apples" is made up of a number "3" and a unit "apples." Similarly, if I spend three hours studying then the quantity "3 hours" is made up of a number "3" and a unit "hours."

The unit depends on the quantity, but there can be different units for the same quantity. For example, time can be expressed in several different units, including seconds, minutes, hours, months and years. Length can be also be expressed in different units, including feet, yards, miles and meters.

In physics, and most sciences, we use the set of units called the **International System of Units** (or Système International d'Unités), abbreviated as **SI** units (see supplemental readings). For example, the SI unit of time is the second and the SI unit for length is the meter.

The **newton** is the SI unit of force. The name of the unit is in honor of Isaac Newton. By convention, unit names are lower-case, even if the unit name is a person's name.[i]

Before using a unit, it is a good idea to get a feel for how much of the quantity is represented by the unit. Most people already have a feel for how long one second is, but unless you are a physicist you probably don't have a good feel for what a newton is.

To get a feel for what a newton is, grab an apple and hold it in your hand. To hold something you need to exert an upward force on it. Otherwise, it will fall to the ground. It turns out that you need to exert an upward force of roughly one newton to hold an apple.

In comparison, you need to exert an upward force of around 20 newtons to hold a 2-liter bottle of soda, about 70 newtons to hold a 15-lb bowling ball, and around 200 newtons to hold a 45-lb child.

✓ *Checkpoint 2.1: Suppose we were to guess the amount of force necessary for a cat to push a small coin off the top of a bookcase. Would the force be closer to one newton or closer to one hundred newtons?*

Units are typically abbreviated by a single letter. For example, we use "s" for seconds and "m" for meters. The same is true for the newton. However, since a newton is named after a person, the convention is to use a capital letter "N" for the abbreviation rather than a lower-case letter "n", even though the unit name itself is not capitalized.[ii] For more information on unit abbreviations,

[i] Degrees Celsius and degrees Fahrenheit are exceptions because there the name describes the unit (degrees) and is not the unit itself.

[ii] Be careful not to confuse it with the abbreviation for North, also indicated as "N".

see the supplemental readings.

✓ *Checkpoint 2.2: What is the unit abbreviation for the newton?*

2.2 Magnitude and direction

Whereas a number and a unit are all that is needed to describe a quantity like time, it turns out that a quantity like force also has a *direction*.

In other words, we need *three* things to describe the quantity of a force: a number, a unit *and* a direction. For example, consider the situation discussed before where we need to exert an upward force of 1 N on an apple to hold it up.

In this scenario, the quantity "1 N upward" is made up of a number "1" and a unit "N" and a direction "upward." All forces have all three parts.

Indeed, if someone asks you for the force, they are asking you for all three parts: the number, the unit and the direction. If you only provide the number and unit then you are not being complete.

✍ The only exception is when the force is zero. In that case, it has no direction (or at least the direction is meaningless).

Sometimes, however, we want to convey just the number and unit portion. We call that portion the **magnitude** of the quantity.[iii] In the example, the magnitude of the force is "1 N."

❢ Unless zero, a force has both a magnitude (number and unit) and a direction.

You can only leave off the direction if only the magnitude is being requested or it is clear from the context that only the magnitude is needed.

What is the direction of a force?

The direction is the which way the force is pushing (or pulling) the object. Upward is just one direction the force can be. Vertically, a force can be upward or downward. There are also horizontal directions, like westward, northward, eastward and southward. There are also directions in between all

[iii] The term has other meanings in science as well (e.g., stellar magnitude and earthquake magnitude) and you need to be careful to distinguish between them based on context.

of these, like northeastward. Sometimes we can use forward and backward, or rightward and leftward, although these depend on one's perspective.[iv]

✓ *Checkpoint 2.3: A person pushes on a car. A force sensor indicates that the force exerted (due to the person) on the car is 300 N eastward.*
(a) What is the magnitude of the force exerted on the car (due to the person)?
(b) What is the direction of the force exerted on the car (due to the person)?

2.3 The net force

To answer the question posed in the puzzle, we must first identify the directions of the forces that are present. In particular, we need to distinguish between the forces acting in the *same* direction as the object's motion (propelling it forward and making it speed up) and those acting *opposite* the direction of motion (retarding its motion and making it slow down).

For example, suppose we have two forces acting: one in the direction of motion and one opposite the direction of motion. One force is acting to speed it up and the other is acting to slow it down. Whether the object speeds up or slows down depends on which force has the *greater* magnitude.

For example, suppose one force is 10 N eastward and the other is 20 N westward. The westward one has a greater magnitude and thus that one has a greater influence on the object's motion. The eastward force still has an impact, but its impact is to decrease the impact of the westward force.

Thus, we can't totally neglect the force with the smaller magnitude. We must consider both together and instead consider the combined force or what is often called the force **imbalance**.

The force imbalance is the part that remains after the forces cancel each other out somewhat. We do this by comparing the force pushing one way to the force pushing the opposite way. The "leftover" amount – the imbalance – is obtained by subtracting the smaller from the larger.[v]

[iv] For example, rightward to one person will be leftward to someone looking in the opposite direction.

[v] One can think of a two-pan mass balance. What matters is the *difference* between the two pans, not how much *total* is in the two pans.

We'll call the combined amount the **net force**.

The word "net" is being used in same way it is used in the business world, as with "net income," where the word is used to represent what is left over after you subtract out expenses. In this usage, the word "net" does not mean "safety net" or "fish net stockings".[vi] The net force, then, is basically the *unbalanced* force.

Just as forces have a direction as well as a magnitude, so does the net force. The net force direction is in the direction where more force is acting. I illustrate what that means in the next example.

❢ The net force direction is in the direction where more force is directed.

Example 2.1: Two forces are exerted on an object. One force is 15 N eastward. The other force is 20 N westward. What is the magnitude and direction of the net force on the object?

Answer 2.1: In this case the forces do not balance. The net force represents the amount of the imbalance. Subtract the 15 N from the 20 N to get 5 N. Since the westward force is more than the eastward force, the net force is directed westward.

It is very important to recognize that we are defining the term "net force" in a very particular way. For example, a trash compactor exerts a lot of force on the garbage it is compacting but according to our definition the net force on the garbage is zero, since the forces push inward on the trash on all sides.

The reason we define it this way is because our objective is to predict how the object's motion changes, not how much it will be squashed.[vii]

✓ *Checkpoint 2.4: Two forces are exerted on an object. One force is 12 N eastward. The other force is 20 N westward. What is the net force acting on the object?*

WHAT HAPPENS IF THERE ARE MORE THAN TWO FORCES ACTING ON THE OBJECT?

[vi] Didn't think you'd find the phrase "fish net stockings" in a physics book, did you?

[vii] Not that there is anything wrong with predicting how much an object will be squashed – it is just not what we are focusing on at the moment.

If the object has more than two forces acting upon it, add up all of the forces in one direction and compare that to the sum of all the forces in the opposite direction.

For example, consider a tug-of-war game, with two people pulling on ropes that are connected to a big ball, one person on each side of the ball.

If each person pulls with the same force magnitude, like 50 N each, then they counter each other exactly and we say the forces are balanced and there is zero net force acting on the ball.

However, suppose two people are pulling against one person. If the two people pull with a *total* force magnitude of 50 N then there would still be zero net force acting on the ball. For example, one of those forces could be 20 N and the other could be 30 N, which together would equal 50 N and balance the 50 N exerted by the single person.

Example 2.2: Suppose we have four forces acting on an object. Two are pulling to the right with magnitudes equal to 30 N and 40 N, while two are pulling to the left with magnitudes equal to 20 N and 50 N. What is the net force acting on the object?

Answer 2.2: We have a total of 70 N pulling to the right (add together 30 N and 40 N). We also have a total of 70 N pulling to the left (add together 20 N and 50 N). Since the magnitudes are equal and they are opposite in direction, the forces balance and so the net force is zero.

✓ *Checkpoint 2.5: Suppose an object at rest has five forces acting on it, two acting upward and three acting downward. If the magnitudes of all five are not identical, is it possible for the net force acting on the object to be zero? If not, why not? If so, provide an example with numbers (i.e., provide the magnitudes).*

2.4 Equilibrium

As described by the law of force and motion, we know that an object speeds up when a force is exerted in the same direction as the object's motion and, conversely, an object slows down when a force is exerted opposite the direction of motion.

When more than one force is acting, the same holds true but we need to look at the *net* force acting on the object.

In other words, an object speeds up when the *net* force acting on the object is in the same direction as the object's motion and, conversely, an object slows down when the *net* force acting on the object is opposite the direction of motion.

WHAT IF THE NET FORCE IS ZERO?

If the net force is zero then the object's motion doesn't change, just as we described in section 1.6. Understanding such a situation is so important that it is given a name: **equilibrium**.[viii] In other words, when the net force acting on the object is zero, the forces are balanced and we say that the object is in equilibrium.

The idea of equilibrium is used throughout the sciences. In biology, for example, if you have the same concentration on both sides of a membrane, we can think of the situation as being in equilibrium. If the concentration on opposite sides is different then the situation is no longer in equilibrium, and the solute will migrate across the boundary. In chemistry, equilibrium is indicated by "$\rightleftharpoons$", as in liquid $\rightleftharpoons$ vapor, where there is just as much liquid evaporating to vapor as there is vapor condensing to liquid.[ix]

The idea of equilibrium is similar[x] for motion. When forces are balanced, we have equilibrium. When that happens, the object's velocity doesn't change. If you change the forces, that can move it out of equilibrium and the velocity will change. To reach equilibrium again the forces would once again need to

[viii] Hence the name of this section.

[ix] In comparison, we use an arrow to indicate the process from reactants to products. For example, evaporation would be indicated as liquid $\rightarrow$ vapor.

[x] One difference is that in the biology and chemistry examples, a situation that is out of equilibrium will eventually lead to equilibrium, where that is not necessarily the case in physics. For example, with concentrations, the migration of solute will lead to the concentrations becoming equal again (a concept called **Le Chatelier's principle**). Similarly, with evaporation, more liquid evaporates than vapor condenses but eventually a new equilibrium state may be reached where the two processes are once again balanced. With forces, however, the forces don't necessarily become balanced just because the object speeds up or slows down.

be balanced, and the velocity would once again become constant but at a different value than before.

Notice that equilibrium does *not* mean the object is necessarily at rest. All it means is that the object's motion isn't changing.

In other words, if an object is initially at rest, a net force of zero would keep it at rest. We'd need a force imbalance in order to get the object moving. However, once in motion, if the net force became zero again then, from that point onward, we'd have equilibrium and the object would continue to move with the same speed and direction from then on.

✍ To distinguish between these two cases of equilibrium, we call one a case of **static equilibrium** (i.e., equilibrium while stationary) and the other a case of **dynamic equilibrium** (i.e., equilibrium while moving). In this usage, the word "static" means stationary.[xi]

To illustrate, consider the following scenario:

> Suppose I am pushing a table across a floor at a constant speed. If the friction acting on the table (against the motion) is 50 N, how hard do I have to push on the table (in the direction of motion) in order to *keep* it sliding at the same speed, without slowing down, speeding up or changing directions?

Although a force imbalance is needed to *start* the table moving, once the table is already moving, the forces have to balance (i.e., I have to push with the same magnitude as friction). Since the friction force has a magnitude of 50 N, that means I have to exert a force with magnitude 50 N.

✓ *Checkpoint 2.6: Consider the table as before. We have already shown that we need a force of 50 N to keep the table sliding at a constant speed. Suppose we push with a magnitude greater than 50 N, so that the table speeds up to a different speed, at which point we return our force to 50 N magnitude. If the friction acting on the table (against the motion) remains at 50 N, how hard do I have to push on the table (in the direction of motion) in order to keep it sliding at the new speed, without slowing down, speeding up or changing directions? Why?*

[xi] Notice that the first five letters of both static and stationary are "stati".

2.5 Applying the law of force and motion

According to the law of force and motion, an object speeds up when the net force acting on the object is in the same direction as the object's motion and, conversely, an object slows down when the net force acting on the object is opposite the direction of motion.

Applying the law is pretty straightforward as long as you keep in mind that the net force is the combined effect of all of the forces acting on the object (i.e., the force imbalance) and that a force imbalance means the object's motion is changing. In other words, you can't use just one of the forces that is acting on the object. You need to consider *all* of the forces acting on the object and, from those, determine the *net* force.

Example 2.3: For each of the following situations, determine whether the forces on the elevator balance or not (i.e., is the net force zero or not):
(a) An elevator remains at rest on the ground floor.
(b) The elevator *starts* moving upward.
(c) As the elevator moves upward, without speeding up or slowing down.
(d) As the elevator approaches the next floor, it slows to a stop.

Answer 2.3: (a) Forces are balanced, because an object at rest remains at rest only if the net force on it is zero.
(b) Forces are not balanced, because if an object is speeding up (getting faster and faster), that means its motion is changing, which requires a force imbalance (upward, in the direction of motion).
(c) Forces are balanced, because the object's motion (speed and direction) is not changing.
(d) Forces are not balanced, because if an object is slowing down, that means its motion is changing, which requires a force imbalance (downward, opposite the direction of motion).

✓ *Checkpoint 2.7: Two forces are exerted on an object that is moving westward. One force is 10 N eastward. The other force is 20 N westward. What happens to the object? Does the object speed up, slow down, or neither? Explain.*

As described in chapter 1, one can get an object to stop and turn around by applying a force opposite the object's motion. The same is true when more than one force acts, but we need to use the net force.

Example 2.4: A child is playing with a yo-yo (a spinning toy that hangs from a string). The yo-yo is dropped and the child uses the string to pull the yo-yo back up. As the yo-yo is reversing direction (from going down to coming back up), what is the direction of the force on the yo-yo?

Answer 2.4: In this case the net force is in opposite the initial direction. Since the yo-yo is initially going downward, the net force on it must be upward.

✓ *Checkpoint 2.8: A rock is thrown up in the air. After being let go, it goes up in the air, slows down, and then comes back down.*
(a) As the rock is going up and slowing down, what is the direction of the net force acting on it?
(b) As the rock is reversing directions (at the top), what is the direction of the net force acting on it?
(c) As the rock is coming back down and speeding up, what is the direction of the net force acting on it?

IF AN OBJECT CONTINUES TO CHANGE ITS VELOCITY AS LONG AS THE FORCE IMBALANCE IS PRESENT, HOW DO WE GET A MOVING OBJECT TO STOP?

To have the object stop and stay stopped, we have to remove the force imbalance at exactly the moment that the object stops. Otherwise, the object will start moving in the direction of the force imbalance.

This sounds harder than it really is.

For example, let's suppose we have a box that is sliding across a floor. We expect the box to slow down and stop.

The reason the box slows down is because of friction between the floor and the box. The friction force opposes the motion, making the box slow down.

However, the friction only acts if the box is sliding. At the moment the box stops, the friction stops acting, leaving the box at rest on the floor.

✓ *Checkpoint 2.9: I jump off a chair and land on the ground.*
(a) As I fall, I am moving downward. When I hit the ground, I slow down. According to the law of force and motion, is the net force on me zero during the time I am slowing down? If not, what must be the direction of the net force on me?
(b) I eventually come to rest on the ground and remain at rest. According to the law of force and motion, is the net force on me zero while I am at rest on the ground? If not, what must be the direction of net force on me?

2.6 Drag and terminal velocity

As mentioned in section 1.7, fluids like air and water exert a force on objects that move through them. This force is called **drag**.[xii] When a ball flies through the air, for example, there is a force on the ball due to the air, which opposes the motion of the ball. Similarly, for a fish swimming through water, there is a force on the fish due to the water, which opposes the motion of the fish. We call this force **drag**.

Drag is significant for things moving through water but for objects moving through air it is usually small enough that we ignore it. That is, it is small if the object isn't moving very fast. The faster an object moves, the greater the drag.[xiii]

How fast does an object have to move for drag to be significant?

It depends.

[xii] It can also be called **resistance**.

[xiii] The drag tends to be proportional to the speed, with the specific relationship commonly referred to as **Stokes' Law**, after George Gabriel Stokes (1819-1903), an Irish physicist and mathematician. The relationship holds if the fluid is incompressible, there are no other objects nearby that would affect the flow pattern, the motion of the object is constant, the object is spherical and rigid and the air velocity right at the object's surface is zero. If these conditions aren't met, the relationship between the drag and the speed tends to follow a different relationship, commonly called **Newton's resistance law**, which states that drag is proportional to the *square* of the speed).

We can ignore the drag when walking at a leisurely pace (with no wind). However, on a windy day the drag from the wind can be significant, and can push over trash cans and make it difficult to walk. If you've ever put your hand outside the window while sitting in a moving car, you probably experienced the air drag on your hand. For a car driving down the street at 35 mph, the drag is significant, and it gets stronger at higher speeds, which is why driving at 75 mph is not as fuel efficient as driving at 55 mph.[xiv]

What happens when drag becomes significant?

To answer that, I can apply the law of force and motion.[xv]

Let's suppose an object starts at rest and a constant northward force is applied to it. Initially, since the object is at rest, drag will be very small and we can ignore it. Given the northward force, there will a force imbalance and the object will speed up.

As the object speeds up, though, the drag becomes greater and greater. At some point, we can no longer ignore it in comparison to the constant northward force that is being applied.

As long as the drag (opposing the motion) is smaller in magnitude than the northward force (pushing it forward), there is still an imbalance and the object continues to speed up. The only difference is that the object will not speed up as quickly as it did initially, in the absence of drag.

At some point, though, the object will reach a speed where the drag becomes so large that it has a magnitude *equal* to that of the northward force (but opposite in direction). At that point, we have balanced forces and the net force on the object is zero.

Since the object is *already* moving, a zero net force in this situation means the object continues to move but just no longer speeds up. We say that it has reached its **terminal speed** or, alternatively, its **terminal velocity**.[xvi]

❢ As an object speeds up, the drag increases, leading to a smaller change in velocity. When the object no longer speeds up, we say it has reached its terminal velocity.

Does the object then slow down?

[xiv] This is why, to minimize fuel use, the United States imposed a national speed limit of 35 mph in 1942 (until 1945) and a limit of 55 mph in 1974 (until 1995).

[xv] One of the advantages of the law of force and motion is that we can predict what will happen without having to do multiple trials.

[xvi] The terminal speed or velocity is just a fancy phrase meaning the final speed or velocity. Speed and velocity are very similar. The difference is discussed in section 3.3.1.

No. In order to slow down the net force would have to be southward (i.e., the drag would have to have a magnitude that is *greater* than that of the propelling force). Since the forces balance, the object neither speeds up nor slows down.

CAN WE PREDICT THE TERMINAL VELOCITY?

Yes, it is possible, although we won't do it, mostly because the drag will be negligible for most of the situations we'll be examining and, in fact, we'll be ignoring drag for the most part. However, even in cases when drag is important, it won't be necessary for us to calculate the terminal velocity value. It is sufficient, for us, to just realize that the terminal velocity will be larger when the propelling force is larger (since it has to go faster to produce sufficient drag to oppose that propelling force), will be larger for low-drag situations (like going through air) than high-drag situations (like going through water), and will be larger for more aerodynamic objects (since the drag will be less).

DOES THE OBJECT REACH ITS TERMINAL VELOCITY INSTANTANEOUSLY?

No, the object doesn't reach the terminal velocity instantaneously, as it takes some time to reach a speed where the drag exactly balances the propelling force.

CAN THE DRAG EVER BE GREATER THAN THE MAGNITUDE OF THE PROPELLING FORCE?

For that to happen, either the propelling force would have to decrease or the object would have to be moving faster than the terminal speed.

An example of this would be a meteor that happens to be moving through empty space, where drag is minimal, and thus can be moving very, very quickly. When it reaches the atmosphere, it suddenly experiences a significant drag, where the terminal speed is much less than the meteor's speed. At that point, the meteor slows down.

As for ordinary objects, the only way for an object to reach a speed greater than the terminal speed is if it is thrown or launched with an initial speed greater than the terminal speed (like a bullet being shot out of a rifle).

✍ If the object is moving faster than the terminal speed then the object will slow down until it reaches the terminal speed, where the drag once again becomes equal in magnitude to the propelling force.

✓ *Checkpoint 2.10: Suppose we roll a ball across the floor such that it rolls across the floor at 3 mph.*
(a) After we let go of the ball, is there a propelling force on the ball pushing it forward?
(b) After we let go of the ball, do you expect the drag to be significant?
(c) After we let go of the ball, do you expect the ball's speed to decrease or stay the same?

Summary

This chapter introduced the concept of a net force and how that is related to changes in the object's motion.

The main points of this chapter are as follows:

- Unless zero, a force has both a magnitude (number and unit) and a direction.
- The net force direction is in the direction where more force is directed.
- As an object speeds up, the drag increases, leading to a smaller change in velocity. When the object no longer speeds up, we say it has reached its terminal velocity.

By now you should be able to use the term net force and distinguish that term from similar, but not applicable uses.[xvii]

Frequently Asked Questions

If two people push on me from opposite directions, I feel a lot of force on me. How can the net force on me be zero?

The net force does *not* refer to the total amount of force pushing on you. It instead refers to the amount of force imbalance that is present.

[xvii] For example, by "net force" we mean the "unbalanced force". We don't mean "the force due to the object's interaction with a net".

The reason it is defined this way is because we are trying to predict the object's motion, not how squashed it will get.

Does static equilibrium have something to do with static electricity?

Only in the sense that both have to do with stuff that is stationary. For static equilibrium, the object is stationary. For static electricity, the electric charges are stationary (relatively).

Does "no net force" mean there are no forces acting on an object?

No. No net force means that the forces are balanced.

How can an object be moving without a force imbalance?

While a force imbalance is needed to *start* an object moving, it is not needed to *keep* an object moving.

What happens if the force imbalance changes while the object is moving?

As long as the force imbalance remains in the same direction as the object's motion, the object will continue to speed up. It just may not speed up as quickly (if the magnitude of the force imbalance is less) or it may speed up more quickly (if the magnitude of the force imbalance is greater).

Conversely, as long as the force imbalance remains opposite the direction of the object's motion, the object will continue to slow down. It just may not slow down as quickly (if the magnitude of the force imbalance is less) or it may slow down more quickly (if the magnitude of the force imbalance is greater).

Terminology

Drag	Le Chatelier's principle	SI
Dynamic equilibrium	Magnitude	Static equilibrium
Equilibrium	Net force	Stokes' law
Imbalance	Newton's resistance law	Terminal speed
International System of Units	Resistance	Terminal velocity

Additional problems

Problem 2.1: For each of the following, describe a real-life situation, if possible, for each of the following cases. If it is not possible, explain why.
(a) The net force is exerted in the same direction as motion of the object.
(b) The net force is exerted opposite the direction of object's motion.

Problem 2.2: An object has three forces acting on it: 5 N northward, 10 N northward and 20 N southward. What is the magnitude and direction of the net force on the object?

Problem 2.3: Suppose the net force on an object is zero. If it is moving during this time, will it (a) slow down, (b) speed up, or (c) neither speed up nor slow down?

Problem 2.4: As I ride my bicycle down a hill, I apply my brakes in such a way that I move with a constant speed down the hill. During this time, when my speed and direction is constant, is the net force exerted upon the bicycle zero? Explain how the law of force and motion can be used to answer this question.

Problem 2.5: For each of the following situations, identify whether the net force exerted on the object must be zero according to the law of force and motion. If it is, explain why? If is isn't, identify the direction of the net force. In each case, the object in question is underlined.
(a) A sled, initially at rest, is given a push to get it moving.
(b) By pushing a book horizontally against the wall, one can keep it at rest (without it falling or sliding down the wall).
(c) A person pushes against a large box that sits on the floor but, due to friction, the box stays put.
(d) An elevator that starts moving upward just after the doors close.
(e) An elevator that is sitting at a floor with the doors open.
(f) You, standing in an elevator, waiting for the doors to close.
(g) A person pushes a large box across the floor such that it moves with a constant speed along a straight line.

Problem 2.6: For each of the following situations, identify whether the net force exerted on the object (as it moves) is zero (balanced) or not zero (imbalance).
(a) A toy car that moves with a constant speed (and direction).
(b) A fan cart that speeds up while moving in a constant direction.

(c) A bicycle that is being ridden up up a hill while slowing down.
(d) An elevator moving downward between floors at a constant speed.

Problem 2.7: (a) An object is at rest. Suppose two forces are exerted upon it, equal in magnitude but opposite in direction. Does the object remain at rest? If so, why? If not, in which direction does the object start to move?
(b) An object is set in motion. Once it is in motion, suppose two forces are exerted upon it. The two forces are equal in magnitude but opposite in direction. Does the object speed up or slow down? Why or why not?

Problem 2.8: Suppose you are seated on a plane that is cruising at a steady speed of 250 m/s (about 560 mph) in a constant direction. According to the law of force and motion, is the net force on you zero? Why or why not?

3. Force and Motion Equation

Puzzle #3: How do we predict how much the object speeds up or slows down when forces are acting on it?

Introduction

From chapters 1 and 2, we know that an object's motion changes while a force imbalance is present, and we know when it will speed up and when it will slow down. However, we have not yet described how one goes about predicting how *much* the object speeds up or slows down during a given force imbalance. In fact, we haven't even quantified how fast an object moves.

We'll address these issues in this chapter by providing the law of force and motion in equation form.

3.1 Letter abbreviations

Before getting to the force and motion equation, we first need to be comfortable with how various quantities are indicated in the equation. Quantities are things like the net force acting on an object and the change in an object's motion.

It turns out that the net force is *not* the only thing that influences how much an object's motion changes. Certainly, the net force determines whether the motion changes or not, but there are two other quantities, in addition to the net force, that determine *how much* the motion changes. Those two other quantities are the object's mass and the time during which the force imbalance is present. In an equation, each quantity is represented by a letter abbreviation. In this section, I introduce the letter abbreviations for all three quantities: the net force, the mass, and the length of time.

3.1.1 The net force ($\vec{F}_{\text{net}}$)

In equations, we'll use the letter "F" to represent the value of a force.

To illustrate, suppose we have a force that has a value of 10 N rightward. We could then write:

$$\vec{F} = 10 \text{ N rightward}$$

Since we are also using letter abbreviations for units (like newtons), it can become confusing to know which letters stand for units and which stand for quantities (which, in turn, consist of a number *and* a unit, and possibly even a direction). To make it clearer which letters represent quantities and which represent units, I'll follow the convention of using *italic* font for quantity abbreviations and roman font for unit abbreviations. Notice, for example, how I use italic font for the F in the expression above, whereas I use the roman font for the N in the expression.

> ✍ This convention (*italic* font for quantity abbreviations and roman font for unit abbreviations) is especially helpful when using the same letter for both a quantity and a unit. For example, the quantity "mass" will be indicated by "m" while the unit "meter" will be indicated by "m".

There is nothing special about what letter we use to represent a quantity or unit. However, we typically try to use a letter that makes sense and won't be confused with an abbreviation we are already using for another quantity or unit.

For more information on quantity and unit abbreviations, see the supplemental readings.

Why does the F have a little arrow on top of it?

The arrow is included to indicate that the quantity also has a direction.[i] Some quantities, like force, have a direction (like upward and downward) while other quantities, like mass do not. Consequently, as you'll see in the next section, we don't include the arrow with the abbreviation for mass.

Does the direction of the arrow indicate the direction?

[i] Actually, the standard from the National Institute of Standards and Technology (NIST) is to use the bold-face font (e.g., **F**). I am using the arrow notation since that is easier to write by hand. I also think the arrow notation makes it a little more obvious that the value includes a direction.

No. It only means that there is *some* direction. The arrow is always drawn pointing to the right and placed on top of the letter abbreviation. The actual direction could be anything (like upward or downward).

WHAT IF WE WANT TO INDICATE JUST THE MAGNITUDE OF THE FORCE?

If you want to indicate just the magnitude of the force (i.e., the numerical value and the unit), the convention is to surround the force abbreviation by vertical bars. For example, if the force has a value equal to 10 N rightward, we could indicate the magnitude as follows:

$$|\vec{F}| = 10 \text{ N}$$

However, it is also common to just write the letter without the arrow on top if one wants to indicate just the magnitude.

❢ $\vec{F}_{\text{net}}$ is used to represent the net force.

Remember that the law of force and motion states that the object's change in motion is associated with the *net* force exerted on it.[ii] To distinguish between the net force and the individual forces that may be acting, I'll use subscripts. For example, if the net force was equal to 10 N rightward, we would indicate the quantity as follows:[iii]

$$\vec{F}_{\text{net}} = 10 \text{ N rightward}$$

In comparison, to indicate the friction force, I'll use $\vec{F}_{\text{f}}$.

✓ *Checkpoint 3.1: Suppose a force is equal to 2000 N northward.*
(a) What is $\vec{F}$?
(b) What is $|\vec{F}|$?

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[i] When the bold-face font is used to indicate quantities that have a direction, the magnitude would be indicated as $|\mathbf{F}|$) or just F.

[ii] Recall from chapter 2 that the net force takes into account all of the forces acting on the object. The net force acting on the object is the same as the force imbalance on the object.

[iii] An alternate way of indicating the net force is via the $\Sigma\vec{F}$. This is because the Greek letter Σ (sigma) is used in mathematics to indicate summations, and the net force is the sum of all the forces acting on the object.

3.1.2 The mass (m)

As mentioned earlier, the impact of the net force on the motion depends on more than just the net force. It also depends upon the mass of the object.

To see why, suppose you exert the same force on two objects of very different masses, like a car vs. a shopping cart. It is a lot easier to stop a rolling shopping cart than a rolling car. This is because the car has a much greater mass. Massive objects will not experience as great a change as light objects (for the same force exerted on it).

Because of this, the force and motion equation will also include the mass of the object, and so we'll need an abbreviation for the mass. Since the word "mass" begins with letter "m", we will use m for mass. For example, if the mass of the object is one kilogram then:

$$m = 1 \text{ kg}$$

There are a couple of things I want to point out about the unit we are using for mass.

The SI unit of mass is the **kilogram**, and it is abbreviated as kg (in roman font because it represents a unit). It is a two-letter unit abbreviation, rather than a single letter abbreviation, for reasons that will be discussed shortly.

Most people in the United States are familiar with the pound as a unit of mass, and know their mass in pounds. To get a sense of how much a kilogram represents, consider that a one-liter bottle of water has a mass of one kilogram. Consequently, your mass is much more than one kilogram. In the United States, most male adults have a mass between 62 and 125 kilograms while most female adults have a mass between 50 and 116.5 kilograms.[iv]

✓ *Checkpoint 3.2: A common unit of mass in the United States is the pound, abbreviated as lb. Using an online converter (like Google), determine your mass in pounds and kilograms.*

[iv] This is according to *Anthropometric Reference Data for Children and Adults: United States, 2011-2014*, from Series 3, Number 39, of *Vital and Health Statistics*, August 2016, U.S. Department of Health and Human Services. Adults are defined as 20 years of age or older, and "most" is defined as 90%.

The reason the kilogram is abbreviated with two letters, rather than one, has to do with the fact that the SI system is metric. A **metric system** of units is a system of units that is based on factors of ten.

✍ Sometimes the SI system is referred to as the **metric system** and, in most cases, it is okay to use both terms to mean the same thing. Technically, however, SI is just one example of a metric system.

With the metric system, we use prefixes to indicate multiples of ten of a base unit. For mass, the base unit is the gram, which is abbreviated as "g". The "kilo-" prefix represents one thousand, so one kilogram is equivalent to one thousand grams. For more information on the metric prefixes, see the supplemental readings.

Whether you use the gram or the kilogram depends on two things. If you are using the force and motion equation (to be introduced in section 3.2) and all of the other quantities are using the SI units (like seconds and newtons) then you want to use the kilogram, since the kilogram is the SI unit for mass. That will make all the units work out.

On the other hand, if you are not using the force and motion equation and are just trying to convey to someone what an object's mass is, you can use whichever is easier. A penny, for example, has a mass of about 3 g, which is equivalent to 0.003 kg. In order to avoid having to write (or say) the zeros, most people would just give the mass of a penny in grams. Similarly, the mass of a two liter bottle of soda is about 2000 g or 2 kg. Again, most people would just give the mass of a two-liter bottle of soda in kilograms, since that involves less zeros.[v]

Why is the kilogram the SI unit and not the gram?

It turns out that the newton, which is based on the kilogram not the gram, is consistent with the units used in electricity and magnetism (which are discussed in volume II of this textbook). In other words, since the kilogram-based newton can be used with the relationships in electricity and magnetism, We have selected the kilogram as the SI unit so that it, too, can be used with all of the other units.

[v]An alternate way to avoid the use of zeros is to use **scientific notation**, which incorporates the zeros as powers of ten. For example, 2000 can be written as 2×10^3. For more information on scientific notation, see the supplemental readings.

✓ *Checkpoint 3.3: The mass of a new pencil is about 5 g. Why is the mass given in units of grams instead of kilograms?*

3.1.3 The length of time (Δt)

How much an object's motion changes depends not only on the net force exerted on it and the mass of the object but also how long the net force is acting. The longer the force is applied, the greater the change in the object's motion. If the force is applied only for a moment, the object's motion won't change much at all (unless the net force is very large and/or the object's mass is very small; see previous section).

We will use the letter t to represent the time. For example, suppose the time has a value equal to ten seconds. We could indicate this as follows:

$$t = 10 \text{ s}$$

where I've used the letter "s" to represent the unit of "seconds," which is the SI unit for time. A second is about the time from one heartbeat to the next. As before, notice that the time abbreviation "t" is italic while the unit abbreviation "s" is not.

✍ As with mass, time does not have a direction so the abbreviation for time (t) is not written with an arrow on top.

For the law of force and motion, it is the *length* of time that is important, rather than a *particular* time like the time on a clock (e.g., 9 pm). To indicate that we want the total amount of time that the net force is present, we'll use a Δ before the t, as in Δt.

WHAT IS Δ?

That is the upper-case Greek letter "Delta" (Δ). In mathematics, Δ is used to represent a change. We use the Δ before the t to make it clear that we want the total amount of time that the net force is present, rather than an individual time.

DO WE NEED TO KNOW THE STARTING AND ENDING TIMES, OR TOTAL AMOUNT OF TIME?

The quantity Δt represents the total amount of time so, as long as we know that time, we don't need to know the starting and ending times. However, and this is important, we *do* need to know the starting and ending times for the purpose of determining the net force that is acting during the time in question. In other words, $\vec{F}_{\text{net}}$ only includes the forces that are acting *during* the time indicated by Δt. We are predicting the change in motion that occurs while $\vec{F}_{\text{net}}$ is acting, so forces that act prior to the Δt time interval are *not* incorporated into the $\vec{F}_{\text{net}}$ value.

For example, suppose throw a ball. As we discussed before, the ball no longer interacts with my hand once the ball leaves my hand. If we are using the force and motion equation to predict the ball's motion during a time interval that starts *after* the ball leaves my hand then $\vec{F}_{\text{net}}$ does *not* include the force on the ball due to my hand.

✓ *Checkpoint 3.4: Starting at 10 am, the net force on an object is 10 N eastward. This lasts until 11 am. What is* Δt*?*

3.2 Force and motion equation

We have now identified the three things that impact how much an object speeds up or slows down: the net force $\vec{F}_{\text{net}}$ acting on the object (due to its interaction with other objects), the mass of the object m and the amount of time that the net force is applied Δt. Now that we have mathematical symbols to represent these three things, we can write down the mathematical equation that shows *how* they impact the amount the object speeds up or slows down:[vi]

$$\Delta\vec{v} = \frac{\vec{F}_{\text{net}}\Delta t}{m} \tag{3.1}$$

The left side of the expression represents how much the object speeds up or slows down, and we'll explore how to quantify $\Delta\vec{v}$ in the section that follows

[vi] As mentioned in chapter 1, most people refer to this relationship as **Newton's second law** because it based on the second of the laws that Newton introduced in his Principia. As written here, the equation should technically use the *average* net force and so it should have $\vec{F}_{\text{net,avg}}$ instead of $\vec{F}_{\text{net}}$, but at the moment we don't need to worry about the difference since we are only considering forces that are constant.

this one. In this section, I want to examine why the right side is the way it is.

Before I do, I first want to point out that the equation is "labeled" with a number in parentheses. Not all equations will be labeled in this way. I'll only label those equations that I'll need to refer back to later.

Each label consists of two numbers. The first number indicates the chapter the equation appears in. The second number indicates the order of the equation within the chapter. In this case, I'll refer to the force and motion equation as equation 3.1, since it is the first equation labeled in chapter 3.

I also want to point out that this expression does not *replace* the law of force and motion – it just expresses the relationship in a mathematical way. We still need to recognize, as the law of force and motion states, that an object's motion changes only while it interacts with other objects. In other words, the natural state of objects is for them to maintain their state of motion. Slowing down, or any other change in its motion, requires an interaction with another object.

This second point is really important. Just because we have a mathematical relationship does not mean we have to use it. Indeed, we typically don't use it unless we have to, and then only after interpreting the problem in terms of the general law of force and motion, which then tells us whether we need the equation. So, don't short-cut the process by going straight to the equation. Only use the equation after interpreting the situation with the law of force and motion.

3.2.1 Algebraic arrangements

You probably noticed that the force and motion equation is a relationship between *four* quantities. More than likely, you've only seen relationships between two quantities, like x and y, in your math classes, so let's examine what it means to have a relationship between four quantities.

Typically, when we have a relationship between four quantities, we measure three of the four quantities and then use the relationship to determine the value of the fourth. You might think, based on what we've discussed so far and based on how the equation is written, that we use the relationship to find the change in motion ($\Delta\vec{v}$) given the net force ($\vec{F}_{\text{net}}$), the time (Δt)

and the mass (m). However, which three we measure, and which one we determine from the relationship, depends on the situation. For this reason, you may want to rewrite the equation with a different arrangement of the quantities (or may encounter the equation with a different arrangement of the quantities).

To illustrate, let's first consider some arrangements where we keep the left side as is and simply rearrange the right side. For example, the right side could be written in the following ways, all equivalent to one another:

$$\frac{\vec{F}_{\text{net}}\Delta t}{m} = \frac{\vec{F}_{\text{net}}}{m}\Delta t = \frac{1}{m}\vec{F}_{\text{net}}\Delta t = \vec{F}_{\text{net}}\Delta t\frac{1}{m}$$

In each case, $\vec{F}_{\text{net}}$ and Δt are in the numerator (top of fraction) and m is in the denominator (bottom of fraction). You should be comfortable with these different representations, even though for consistency I'll tend to write it the first way.

Now that you've seen how we can rearrange the right side, let's consider some ways where the quantities appear on different sides. The simplest such rearrangement is when we flip the left and right sides, as follows:

$$\frac{\vec{F}_{\text{net}}\Delta t}{m} = \Delta\vec{v}$$

There is nothing wrong with writing the expression this way, as it is equivalent to equation 3.1.

Using algebra, we can rearrange even further. With algebra, one can do any action (like multiplication, division, addition and subtraction) as long as you do the same to both sides of the equation.

For example, we could multiply both sides of equation 3.1 by m. To see why we might want to do this, let's first rewrite equation 3.1 as follows, using what we know from above:

$$\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t\frac{1}{m}$$

Then, let's multiply the left side by m and the right side by $m/1$ (which is the same thing as m):

$$m\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t\frac{1}{m}\frac{m}{1}$$

The right side now has m in both the numerator and the denominator, and since m/m is equal to 1, regardless of what the mass value happens to be, we say the two m's "cancel", leaving us with the following:

$$m\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t$$

Notice that the end result is that we have essentially "moved" the m from the right side to the left side and, at the same time, switched it from the denominator to the numerator. This "movement" is a consequence of multiplying both sides by m.

Again, there is nothing wrong with writing the force and motion equation this way (with $m\Delta\vec{v}$ on the left) instead of what we had before (with just $\Delta\vec{v}$ on the left). Use whichever version you think best reflects the law of force and motion.

✓ *Checkpoint 3.5: Which of the following ways are equivalent to the force and motion equation 3.1, in the sense that it can be obtained from equation 3.1 via algebraic manipulation? For each, explain why it is equivalent or not.*
(a) $\Delta t = (\vec{F}_{\text{net}}/m)\Delta\vec{v}$
(b) $\Delta\vec{v}/\Delta t = \vec{F}_{\text{net}}/m$
(c) $\vec{F}_{\text{net}} = m\Delta\vec{v}/\Delta t$

3.2.2 Influence of net force

When given a relationship with four quantities, it can be difficult to visualize how the four quantities are related. The simplest way to do this is to keep all quantities the same except for two. We can then use the expression to see how those two are related. We can choose any pair to keep constant, allowing us to see how the remaining two are related.

In this section, I'll use the equation to examine how the change of motion is related to the net force. In the sections that follow, I'll examine how other pairs of quantities are related. Keep in mind that each case is not a separate idea. Rather each case is just a consequence of the force and motion equation. We are only looking at two quantities at a time to simplify our analysis.

So, we know that the change in motion depends upon the force you exert (i.e., how hard you push and the direction of your push). This is why the right side includes the force imbalance ($\vec{F}_{\text{net}}$) exerted on the object.

We also know, or at least should suspect, that the greater the magnitude of the force imbalance, the greater the change in the motion. In fact, the two quantities are **directly proportional**.

WHAT DOES IT MEAN TO BE DIRECTLY PROPORTIONAL?

Two quantities are directly proportional when a change in one quantity corresponds to a proportional change in the other quantity. We'll examine exactly what this means in chapter 4. For now, it is sufficient to simply recognize that if we compare two situations, the change in motion will be larger in the situation with the greater force imbalance.

Mathematically, two quantities are directly proportional if they appear on opposite sides of an equation (as in this case) and both are in the numerator (as in this case) or both are in the denominator. This is illustrated here by using boxes to highlight $\Delta\vec{v}$ and $\vec{F}_{\text{net}}$.

$$\boxed{\Delta\vec{v}} = \frac{\boxed{\vec{F}_{\text{net}}}\Delta t}{m}$$

If the two quantities are on the *same* side of an equation then they are directly proportional if one is in the numerator and the other is in the denominator.

✓ *Checkpoint 3.6: Suppose an object's motion changes by 5 m/s when the object experiences a net force of magnitude 10 N for 3 seconds. For the same object, would you expect the change of motion to be greater or less if the magnitude of the net force is less (for the same amount of time)?*

3.2.3 Influence of time period

The analysis in the previous section (discussing how the net force and change in motion are related) assumes that we are using the same time period and same mass.

Let's now consider the case where we again apply the equation twice but with the *same* net force (and same mass) but with different time periods.

We know that the change in motion depends on the length of time that the imbalance exists. That is why the right side includes the Δt. Also notice that the Δt is in the numerator, just like $\Delta\vec{v}$ (but on the opposite side of the equation).

❢ The longer the force is applied, the greater the object's change in velocity.

$$\boxed{\Delta\vec{v}} = \frac{\vec{F}_{\text{net}}\boxed{\Delta t}}{m}$$

This means that $\Delta\vec{v}$ and Δt are directly proportional. In other words, an object experiencing a particular net force for *twice* as long will experience a change in velocity that is *twice* that of the other.[vii] This is consistent with what was discussed in section 3.1.3.

Notice how the time period, like the net force, both influence the change in motion in the same way. That means a large net force exerted for a short time ($\vec{F}_{\text{net}}\Delta t$) can produce the same change in motion as a small net force exerted over a long time ($\vec{F}_{\text{net}}\Delta t$). That is the reason why cars have airbags. When a car crashes, the motion of the car stops, along with all of the human occupants of the car. The quicker that happens, the larger the force that is needed, and a large force can do quite a bit of damage to the internal structure of a human being. A front dashboard is not very forgiving whereas an airbag is designed to provide some "give", extending the time over which the motion stops, meaning a smaller force is needed, which is not as damaging.

✓ *Checkpoint 3.7: I have two identical objects and the same force (not zero) is exerted on each but for different amounts of time. Object A experiences the force for 5 seconds. Object B experiences the force for 10 seconds. Which object experiences the greater change in velocity?*

3.2.4 Influence of mass

Whereas the net force and time are in the numerator, the mass is in the denominator.

$$\boxed{\Delta\vec{v}} = \frac{\vec{F}_{\text{net}}\Delta t}{\boxed{m}}$$

[vii]For this reason, the longer the barrel of a gun, the faster the bullet speed (all other things being equal, which is rarely the case).

This is consistent with the mass being **inversely proportional** to the change in motion.

What does it mean to be inversely proportional?

Two quantities are inversely proportional if one decreases by the same proportion the other increases, and visa-versa. We'll examine exactly what this means in chapter 4. For now, it is sufficient to simply recognize that if we compare two situations, the change in motion will be larger in the situation involving the object with the smaller mass.

Notice how the mass influence the motion in a way that is opposite that of the time period and net force. That means a large net force exerted on a high-mass object can produce the same change in motion as a small net force exerted on a low-mass object (when exerted for the same length of time). Similarly, exerting a force on a high-mass object for a long time can produce the same change in motion as the same force exerted on a low-mass object for a short time.

✓ *Checkpoint 3.8: Suppose the same force is exerted upon both a heavy bowling ball and a light Ping-pong ball for the same length of time. As a result, the bowling ball experiences a change in motion of 1 m/s. Would the Ping-ping ball experience a greater change than 1 m/s or less?*

3.3 The change in velocity ($\Delta\vec{v}$)

In this section, we'll explore how we quantify the change in motion, which we indicate as $\Delta\vec{v}$ in equation 3.1.

3.3.1 Velocity vs. speed

The reason the equation of force and motion uses $\Delta\vec{v}$ to represent the change of motion (how much the object speeds up or slows down) is because, technically, it represents the object's change in *velocity*.

Outside of physics, velocity and speed are typically used interchangeably. In physics, however, they mean slightly different things. For the time being, the distinction isn't important but getting the difference straight helps us to understand why the equation uses $\Delta\vec{v}$ to represent the change in motion.

❢ Velocity is *both* the speed (how fast) *and* the direction of motion.

Basically, **speed** is how fast an object is moving while **velocity** is how fast *and* the direction the object is moving.

For example, suppose a police officer gives you a speeding ticket. That would be because your *speed* was too fast. On the other hand, suppose you were going the speed limit but moving the wrong way down a one-way street. You'd still get a ticket, but it wouldn't be because your speed was wrong. The ticket would instead be because you were moving in the wrong *direction*.

In physics, we'd say that your speed was fine in the second example (going the wrong way) but your *velocity* was not. Your velocity was in the wrong direction. It may take some time to get used to this usage, as most people outside of physics use the terms speed and velocity interchangeably.

WHICH ONE IS IN THE FORCE AND MOTION EQUATION?

It turns out that a force imbalance can change the direction of motion as well as the speed, so technically the force and motion equation gives the change in *velocity*, and that is why the equation uses $\vec{v}$ as the letter abbreviation. The arrow on top is to remind us that it includes a direction, just as we do for force (see section 3.1.1).

As mentioned earlier, at this point we are focusing only on how forces change an object's speed, and so will use the force and motion equation only to predict how much an object speeds up or slows down. As such, for the time being, we don't need to distinguish between the speed and velocity to use the equation. I've only introduced the distinction here to explain why we are using v as the abbreviation in the expression.

✍ In physics we use the word **vector** to describe a quantity, like the velocity and the net force, that includes a direction.[viii] In comparison, a **scalar** is a quantity, like speed and mass, that does not include a direction. For more information about the difference between vectors and scalars, see the supplemental readings.

[viii]Don't confuse this use of the term "vector" with the term used in biology, where it is used to describe an organism or agent that transmits a pathogen or other such material. While both have to do with movement, they do not mean the same thing.

Example 3.1: Suppose an object is moving at 10 m/s eastward.
(a) What is the value of $\vec{v}$?
(b) What is the value of $|\vec{v}|$? Recall from section 3.1.1 that the absolute value symbols to indicate that we only want the magnitude.

Answer 3.1: (a) $\vec{v}$ is the object's velocity, which is 10 m/s eastward.
(b) $|\vec{v}|$ is the object's speed, which is 10 m/s.

✓ *Checkpoint 3.9: Suppose an object is moving at 20 mph northward.*
(a) What is the value of $\vec{v}$*?*
(b) What is value of $|\vec{v}|$*?*

3.3.2 The importance of change

As mentioned in section 3.1.3, the upper-case Greek letter "Delta" (Δ) is used to represent a change. Consequently, the change in velocity is indicated as $\Delta\vec{v}$.

The presence of the Δ on the left side is a crucial part of the law of force and motion. After all, the law does *not* predict the velocity of the object. Rather, it predicts the *change* in the object's velocity, like how much it speeds up or how much it slows down. The Δ helps reinforce the idea that the forces are associated with a *change* in the object's velocity, as discussed in chapters 1 and 2.

✓ *Checkpoint 3.10: An object starts at rest at 10 am and then speeds up, reaching a velocity of 20 m/s westward at 11 am.*
(a) What is $\Delta\vec{v}$*?*
(b) Would your answer be any different if the object started with a velocity of 5 m/s westward at 10 am, instead of being at rest at 10 am? If so, in what way? If not, why not?

3.3.3 Simultaneity

It is important to remember that Δt indicates both the time during which the force is applied *and* the time during which the velocity changes. This is because the motion changes only when the force imbalance is present. The motion doesn't change while no force imbalance exists.

IS THERE ANY DELAY BETWEEN WHEN THE FORCE IMBALANCE IS PRESENT AND WHEN THE VELOCITY CHANGES?

❢ The change in object's velocity occurs while the forces are not in balance.

No. The change in motion occurs *while* the force is being applied. There is no delay.

✍ In physics the word "force" may be used differently than how it is sometimes used in ordinary life outside of physics. Outside of physics class you might say that an instructor forced you to pull an all-nighter because of the amount of homework that was assigned. That usage differs from the physics usage in two ways. First, in the ordinary usage, the all-nighter follows the assignment of the homework whereas in the physics usage the change in velocity occurs simultaneously with the application of the force. Second, in the ordinary usage, there isn't an equivalent force on the instructor by you whereas in the physics usage both interacting objects experience the same force, an idea explored in chapter 5.

✓ *Checkpoint 3.11: Suppose an object experiences a force for three seconds. During which of the following time intervals does the object experience a change in velocity?*
(a) Prior to when the force is present.
(b) While the force is present.
(c) After the force has ended and is no longer present.
(d) During more than one of the above time intervals.

3.3.4 Units

I've used units of meters per second (abbreviated as m/s) to indicate how fast an object is moving because the meter and the second are the SI units for length and time, respectively.

As we've done with the other SI units, let's examine the meter and get a sense of what a meter is.

A meter is a little more than a yard (three feet). Consequently a velocity of 1 m/s means that the object is moving one meter every second. Stand up now and try to move at a rate of 1 m/s. This may be a little awkward, depending on where you are at the moment[ix], but it is important so that you get a feel for just how fast 1 m/s is.

Do you think you can move at 10 m/s? That would be pretty fast for a person. However, a car can easily go 10 m/s. In fact, on a highway where the speed limit is 65 mph (i.e., 65 miles per hour), cars typically go about 30 m/s (the actual conversion is 65 mph = 29.05 m/s; see problem 3.3).

Why use meters per second instead of miles per hour?

The reason we are using meters per second is because meters and seconds are part of the International System of Units (SI; see section 2.1).[x]

Summary

This chapter introduced the equation form of the law of force and motion, written as $\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t/m$.

The main points of this chapter are as follows:

- $\vec{F}_{\text{net}}$ is used to represent the net force.
- The longer the force is applied, the greater the object's change in velocity.
- A light object will experience a greater change in velocity than a heavy object (for the same applied force and length of time).
- Velocity is *both* the speed (how fast) *and* the direction of motion.
- The change in object's velocity occurs while the forces are not in balance.
- The force and motion equation ($\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t/m$; equation 3.1) predicts how much an object's velocity will change while there is a force imbalance exerted on it.

[ix] Watch out for strollers, railings and the like.

[x] For speed limits, countries that use SI tend to use kilometers per hour instead of meters per second. The advantage of meters per second is that we don't have to convert when using speed with other quantities (like force, mass and time). This is discussed further in section 4.1.

By now you should be able to interpret the force and motion equation as the mathematical representation of the law of force and motion.

Frequently Asked Questions

Isn't velocity the same thing as speed?

To non-scientists, the terms may mean the same thing. To scientists, though, velocity is not only the speed of the motion but also the direction.

For example, if a ball is thrown upward with an initial speed of 3 m/s, we would say that the ball's initial velocity is "3 m/s upward."

What does "mph" mean?

It is shorthand for miles per hour (mi/h). See page 57.

How can I tell when a letter is an abbreviation for a unit and when it is an abbreviation for a quantity?

I'll follow the convention of using *italic* font for quantity abbreviations and normal font for unit abbreviations.

Terminology introduced

Constant	Inversely proportional	Speed
Conversion factor	Kilogram	Vector
Directly proportional	Metric system	Velocity
Final time	Newton's second law	
Initial time	Scalar	

Problems

Problem 3.1: How do we distinguish between the unit abbreviations for meter, minute and mile? Check the supplemental readings.

Problem 3.2: I have two objects of identical mass. Object A experiences a net force of 10 N westward while object B experiences a net force of 20 N westward. During the same period of time, which object experiences the greater change in velocity?

Problem 3.3: It was mentioned on page 57 that 65 mph equals 29.05 m/s. Show that this is indeed the case, using the fact that there are 1609 meters in a mile and 3600 seconds in an hour.

Problem 3.4: An eight-fluid-ounce bottle or glass of water is about 240 ml (i.e., 240 milliliters). There are 1000 ml in a liter. What is the mass of the 240 ml of water?

4. Using the Force and Motion Equation

Puzzle #4: If it takes 10 N to keep a 1-kg box moving across the floor at a constant velocity of 0.5 m/s, how quickly will the box come to a stop after the force is removed?

Introduction

To solve the puzzle, we need to relate the forces acting on the box with the box's change in motion. To do this, we'll use the law of force and motion and, in particular, the force and motion equation.

According to the law of force and motion, an object's velocity changes while it experiences a force imbalance. In the mathematical representation of the law, provided in equation 3.1, the change in motion is represented by $\Delta\vec{v}$ and the net force is represented by $\vec{F}_{\text{net}}$:

$$\Delta\vec{v} = \frac{\vec{F}_{\text{net}}\Delta t}{m}$$

The expression uses $\vec{F}_{\text{net}}$ because there can be more than one force acting on an object, and in that case it is the *net* force (or force imbalance) that is related to the change in motion. The greater the net force acting on an object, the greater the change in its velocity during the time the net force is present.

In this chapter, we utilize the equation for different situations. It is not just a matter of plugging numbers into the equation. We must also interpret the situation in light of the law of force and motion. Remember that the equation is just a *representation* of the law of force and motion, it does not *replace* it. This chapter examines what this means and how to interpret the results when using the equation.

4.1 Units

A mathematical equation equates two quantities or groups of quantities. In particular, the quantities on the left side of the equals sign, taken together, must be equal to the quantities on the right side of the equals sign, taken together.

If each quantity represented just a number, this would mean that the numerical values (numbers) must be equal on each side. However, physical equations involve quantities that have units as well as numerical values.

❢ The equals sign in a physical relationship means that the two sides are equal, not only in terms of the numerical value but also in terms of the units and the direction.

As long as we use SI units throughout, however, it turns out that the units will necessarily be equal so we don't have to check that each time. In fact, that is why have chosen to use SI units – so we don't have to worry about the units working out in the equation.

WHAT DOES IT MEAN FOR THE UNITS TO "WORK OUT"?

It just means that the units on one side equal the units on the other, just like the numerical values have to be the same on each side and the directions have to be the same on each side.

At first glance, this may not appear to be the case in equation 3.1. Using SI units only, the force and motion equation is as follows:

$$\begin{aligned}
\Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m} \\
\frac{\text{m}}{\text{s}} &= \frac{(\text{N})(\text{s})}{(\text{kg})} \\
\frac{\text{m}}{\text{s}} &= \frac{\text{N}\cdot\text{s}}{\text{kg}}
\end{aligned}$$

Based on this, it doesn't look like the units on the left side (m/s) equal the units on the right side (N·s/kg).

❢ The newton is a unit of force and is equivalent to a kg·m/s^2.

However, it turns out that 1 N is equivalent to 1 kg·m/s^2. Making that replacement and doing a little algebra[i], one finds that the units on the right

[i] Units obey the same algebraic rules as quantities like x and y.

do equal the units on the left.[ii]

The key point is that, as long as we use SI units throughout, we don't have to do any unit conversions. Without that consistency among the units, we would need to memorize and constantly use lots of **conversion factors**, such as the number of feet per mile, and the number of fluid ounces per gallon.[iii] This is the main reason why we are using SI units instead of units that may be more familiar to you, like feet and pounds.[iv]

❢ As we use SI units for all of the quantities in the force and motion equation, the units will be the same on each side.

✓ *Checkpoint 4.1: If the force is given in newtons, the time is in hours and the mass is in pounds, and we plugged the values into the force and motion equation, would the result give us the change in motion in meters per second (without any further unit conversions)? Why or why not?*

4.2 Numerical values

If we know the units will work out, do we need to include them?

Even if using SI units throughout, units are still important because they give meaning to the numbers. Without the units the numbers don't have meaning!

For example, you asked someone how far they had to drive to campus. If they answered "three", you'd be left wondering if they meant three miles, three blocks, three exits or perhaps they meant that it took them three minutes to get to campus.

❢ Units are important, as they give meaning to the numbers!

[ii]Replacing N by kg·m/s^2, I get that the units on the right-hand side end up with "kg" in the numerator (from the force) and "kg" in the denominator (from the mass). Those cancel. Similarly, we have "s" in the numerator (from the time) and "s^2" in the denominator (from the force). That results in "s" in the denominator. The end result is "m/s".

[iii]For example, 1 inch divided by 1 cm is 2.54 in/cm, which can be used to convert from centimeters to inches or visa-versa. This is an example of a conversion factor.

[iv]Another reason is that science is an international activity, and few people outside the United States are familiar with non-SI units. Imagine if you were given a velocity measured in unfamiliar units, such as furlongs per fortnight. That is what it is like for many people outside the United States when encountering units like feet and pounds.

But if we are using SI units throughout, do we still need to include them?

Technically, you wouldn't need to include them if you knew that the units were SI units. However, it is very, very easy to use equations divorced from any meaning and, once you do that, you are no longer doing any physics – you are just plugging numbers blindly into equations. The equations *represent* the physics – they do not *replace* the physics. What this means is that you should include them because otherwise you may end up just interpreting them as numbers without any meaning, and then you wouldn't be doing any physics.

In any event, to ensure that you are interpreting the physics and not just doing a bunch of math that has no meaning to you, make sure you *interpret* the situation and apply the law of force and motion *before* using the force and motion equation (or any equation). In other words, ask yourself – is there a force imbalance acting on the object and, if so, is the object speeding up or is it slowing down? Then, and only then, do you use the equation – for the purpose of determining *how much* the object speeds up or slows down. Then, once you know how much the object speeds up or slows down, you can then find out how fast it is moving at the end (by adding or subtracting how much it sped up or slowed down).

✍ It is like first determining whether you need pay someone or they need to pay you, then using an equation to find out how much money is transferred, then using that to determine how much you will have after the transfer (by adding or subtracting from what you currently have).

For example, suppose we have a situation where the forces are balanced (i.e., the net force is zero). From the law of force and motion, we know that the object is neither speeding up or slowing down. We don't need the force and motion equation to tell us that.

✍ As with any equation, if one side is zero then the other side is also zero. For the force and motion equation, that means that if the net force is zero then the velocity change must also be zero. That *doesn't* mean that the velocity is zero. It just means that it isn't changing.

In comparison, suppose the net force is *not* zero. In that case, the law of force and motion tells us that the velocity must be changing. Even a tiny amount of imbalance will make the velocity change. There is no "minimum" imbalance that is needed. Even if the net force itself *is* constant (i.e., maintains the same non-zero value throughout the time period), the velocity will *not* be constant.

Furthermore, we know that the object would be speeding up if the net force is in the direction of motion, and the object would be slowing down if the net force was opposite the direction of motion. We would only need the force and motion equation to tell us *by how much* the object was speeding up or slowing down.

❢ An object slows down when the change in motion is opposite the direction of motion and speeds up when the change in motion is in the same direction as the motion.

For example, suppose we have a situation where a constant net force of 10 N rightward acts on a 2-kg object for 3 seconds. If the object was initially at rest, how fast is it moving at the end of the 3 seconds?

From the law of force and motion, we know that the object is speeding up (since it started at rest). To find out *by how much* the object speeds up, we use the force and motion equation.

Since all quantities were in SI units, we know that the change in velocity will also be in SI units (m/s). Consequently, we have:

$$\begin{aligned}\Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m}\\ &= \frac{(10\text{ N})(3\text{ s})}{(2\text{ kg})}\\ &= 15\text{ m/s}\end{aligned}$$

This means that the velocity must change by 15 m/s during the 3 seconds. And, since we know it started at rest and was speeding up, that means its speed is 15 m/s at the end of the 3 seconds.

What if the object didn't start at rest?

❢ The force and motion equation doesn't tell us what the velocity is – it only tells us how the velocity has changed.

The force and motion equation doesn't tell us what the velocity is. It only tells us how it has *changed*. For example, if we find that in 3 seconds the change in velocity is 15 m/s, that means the object could be 15 m/s faster or 15 m/s slower at the end of the 3 seconds, but we don't know how fast it is moving at the end of the 3 seconds unless we know how fast it was moving at the beginning of the 3 seconds.

Let's say the object was *already* moving with a velocity of 5 m/s rightward at the beginning of the 3-second interval. In that case, the law of force and motion tells us that the object must be speeding up (since the net force is rightward also), meaning the object would speed up by 15 m/s and so would be moving 20 m/s rightward at the end of the 3-second interval.

✍ I'll refer to the beginning and end of the time interval as the **initial time** and the **final time**, respectively. Consequently, in this example, the initial velocity is zero and the final velocity is 15 m/s rightward.

Conversely, if the object was initially moving at 25 m/s *leftward*, then law of force and motion tells us that the object must be slowing down (since the net force is opposite the motion). Again, the force and motion equation tells us that the change is 15 m/s, but in this case that means the object is slowing down by 15 m/s. Since it started at 25 m/s leftward, its final velocity would be 10 m/s leftward.

✓ *Checkpoint 4.2: Suppose a net force of 5 N northward acts for 3 seconds on an object of mass 2 kg. If we don't know the object's velocity at the beginning of the 3 seconds, can we use the force and motion equation to determine how fast the object is moving at the end of the three seconds? If so, how fast is it moving? If not, why not?*

4.3 Using proportions

As noted in chapter 3, $\Delta\vec{v}$ and $\vec{F}_{\text{net}}$ are directly proportional, which means that the object's velocity will experience a greater change when the force imbalance is greater. That chapter also noted that $\Delta\vec{v}$ is directly proportional to Δt, and inversely proportional to m.

The idea of proportions allows us to predict how the velocity will change even if we aren't given specific numbers, as long as we are comparing two situations where only two of the quantities are different in the two cases. The following example illustrates what I mean.

> Suppose an object speeds up by 5 m/s when a given net force is exerted in the direction of motion for a given amount of time. By how much does it speed up if the same net force is applied for twice the amount of time?

To use the force and motion equation to find the object's change in velocity, one might think we'd need to know the net force, the time, and the object's mass. Since we don't know any of those three, at first glance it may seem that it is not possible to solve for the object's change in velocity.

However, in this case we are provided with a very important piece of information, which is that the net force and the mass is the same in the two cases. Consistent with the force and motion equation, if you double the time, while leaving everything else the same, the change in velocity will be double what it was before. Consequently, the answer is that the new change in speed is 10 m/s.

HOW DO YOU KNOW THAT THE CHANGE IN VELOCITY WOULD BE DOUBLE IF THE TIME IS DOUBLED?

One way to show this is to simply make up values for the unknown force and mass, and then calculate the change in velocity. If you repeat the calculation, using the same values you used before for force and mass but twice the time you used before, you'll get a change in velocity that is twice what you obtained before. Similarly, if we repeat the process but instead use triple the force, we find the change of motion is also triple.

✍ Being directly proportional doesn't mean that the two quantities are equal. It only means that the change in the same way. For example, if one doubles then the other must double (if the other quantities remain unchanged), and if one triples, the other must triple, and if one is halved, the other must be halved, and so on.

In any event, we don't need to know the values of Δt and m to know how $\Delta\vec{v}$ is impacted by $\vec{F}_{\text{net}}$ doubling, tripling or whatever, since $\Delta\vec{v}$ will experience the same proportional change (assuming Δt and m remain unchanged).

HOW DO WE KNOW THAT THE MASS IS THE SAME AS BEFORE?

Because the object hasn't changed – it is same object as before.

✓ *Checkpoint 4.3: Suppose an object's motion changes by 5 m/s when the object experiences a net force of magnitude 10 N for 3 seconds. What would be the change of motion for the same object if the magnitude of the net force is half as much (5 N) for 3 seconds? Why don't you need to know the object's mass for this problem?*

What happens if two quantities are inversely proportional?

To illustrate what is meant by inversely proportional consider two objects, of masses 1 kg and 2 kg, respectively. In this illustration, the 2-kg object has a mass that is twice that of the 1-kg object.

Now let's consider the case where the same force is applied to each object and for the same amount of time. According to the force and motion equation, the 2-kg object (with *twice* the mass of the other) will experience a change in velocity that is *half* that of the other. This is consistent with what was discussed in section 3.1.2.

❢ A light object will experience a greater change in velocity than a heavy object (for the same applied force and length of time).

Just as the inverse of two is one-half, the object with twice the mass experiences a change in velocity that is one-half that of the other. So, an object with three times the mass experiences a change that is one-third that of the other. And so on.

✓ *Checkpoint 4.4: Suppose the same force is exerted upon both a heavy bowling ball and a light Ping-pong ball for the same length of time. As a result, the bowling ball experiences a change in motion of 1 m/s. If the mass of the bowling ball is 2000 times greater than the mass of the Ping-pong ball, what is the Ping-pong ball's change in motion?*

What happens if we have two cases and more than two quantities are different?

If a problem specifies that more than two quantities has changed, you can still solve the problem. The next example illustrates how.

> An object experiences a change in velocity of 5 m/s upward when a given net force is applied for a given amount of time. What is the same object's change in velocity if the net force is applied for half the amount of time but with triple the magnitude?

If we only tripled the magnitude of the net force, while keeping the time the same, then the magnitude of the change in velocity would also triple, to 15 m/s upward. On the other, hand, if we only halved the amount of time, while keeping the net force the same, then the magnitude of the change in velocity would halve.

Since both quantities are changing, both effects are occurring. In other words, to find the change in velocity, first triple it (to get 15 m/s upward) and then halve that (to get 7.5 m/s upward).

You can do this in any order. For example, you can first half the change in velocity (to get 2.5 m/s upward) and then triple it (to get 7.5 m/s upward).

❢ You can apply the method of direct and indirect proportions in any order.

✓ *Checkpoint 4.5: Suppose an object experiences a change in velocity of 7.5 m/s northward when a given net force acts upon it for a given amount of time. What will be the change in velocity if the object is replaced with one that is three times the mass* and *we triple the magnitude of the net force?*

4.4 Determining $\vec{F}_{\text{net}}$, m or Δt

It is important to recognize that equation 3.1 does not mean that the only way to determine $\Delta\vec{v}$ is by multiplying the values of $\vec{F}_{\text{net}}$ and Δt and then dividing that by m. Rather, it just says that these quantities are related in this particular way. In practice, we measure each of those things separately. For example, one can measure velocity with a speedometer, forces with a force probe, time with a stopwatch and the mass with a balance.

What equation 3.1 *really* tells us is that, despite the fact that we can measure these four quantities separately, their values are not completely independent.

❢ Even though the net force, mass, time and change in velocity are measured separately, their values are not independent.

For instance, we can measure any three of the four quantities (for the same time interval) and then use the relationship to predict what value we'd get if we had measured the fourth. Indeed, we can measure any three of the four quantities and then use the relationship to infer the value of the fourth, without bothering to measure it.

This is illustrated in the next example, where we are asked to find the net force, rather than the change in velocity.

> I find that my velocity changes by 4 m/s westward in 0.8 seconds when I push off of a wall. My mass is 70 kg. What is the net force exerted on me during this time?

This is easiest to solve if we first use algebra to rearrange the force and motion equation so that it gives the net force. Otherwise, you can plug the numbers in first and then use algebra to solve for the net force.

For the example, let's do it the second way and plug the numbers in first.

$$\begin{aligned} \Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m} \\ (4\ \text{m/s}) &= \frac{\vec{F}_{\text{net}}(0.8\ \text{s})}{(70\ \text{kg})} \\ 4\ \text{m/s} &= \vec{F}_{\text{net}}\frac{0.8\ \text{s}}{70\ \text{kg}} \end{aligned}$$

At this point, it is just a matter of finding the value of $\vec{F}_{\text{net}}$ that makes the two sides equal. Since everything is in SI units, we know that the force will end up being in newtons. Numerically, then, we can focus just on the numerical values. To solve for $\vec{F}_{\text{net}}$, we can leave $\vec{F}_{\text{net}}$ on the right side of the equation and simply multiply both sides by (70/0.8). The left side becomes $4 \times 70/0.8$ while the right side becomes $\vec{F}_{\text{net}}$. Solve to get a net force of 350 N westward.

How do you know the net force is westward?

As discussed in section 4.1, the direction of the velocity change must be the same as the direction of the net force.

❢ Always consider the physics first, before using an equation.

At this point, you might start to think that physics is just a matter of plugging numbers into the force and motion equation. To illustrate how important it is for you to always consider the physics *before* using an equation, consider the following question.

> Suppose it takes 10 N to keep a 1-kg box moving across the floor at a constant velocity of 0.5 m/s. What is the net force acting on the box during this time?

Before you go plug numbers into the force and motion equation or think that the answer is 10 N (since that is the force provided in the problem), think about the physics here. It is asking for the net force on the box. Is the 10-N force the only force acting? If so, the box should be either speeding up or slowing down. In this case, it is doing neither – it is moving at a constant speed. According to the law of force and motion that means that the net force on the box is zero.

Notice how we didn't need to use the force and motion equation at all. We did need to use the law of force and motion, but not the equation form of it.

Let this be a lesson to you – always start with the idea of the law and only apply the equation form if needed.

What other forces are acting on the box?

In this case, friction is acting on the box, opposing the motion. The magnitude of the friction must be 10 N, in order to exactly counter the 10-N force that is being applied.

What would happen if we don't apply the 10 N force?

If moving at 0.5 m/s, as in the example, when the 10 N pushing force is removed, then the 10 N friction force (still present) will make it slow down.

This then brings us to the puzzle at the beginning of the chapter.

> Suppose we need to apply push with a force of magnitude 10 N to keep a 1-kg box moving across the floor at a constant speed of 0.5 m/s. How soon after we stop pushing on it will the box come to a stop?

We already know that the 10-N friction force is still acting on the box, opposing the motion. Since that is the only force acting, that is also the net force acting. That means we can apply the force and motion equation.

Since everything is in SI units, the units will work out and so we can focus only on the numerical values. We know the box slows from 0.5 m/s to zero, so its change in speed is 0.5 m/s. Plugging in, we have:

$$\begin{aligned}
\Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m} \\
0.5 \text{ m/s} &= \frac{(10 \text{ N})(\Delta t)}{(1 \text{ kg})} \\
0.5 \text{ m/s} &= (10 \text{ N/kg})\Delta t
\end{aligned}$$

One can now solve for Δt to get a time of 0.05 s (where seconds is the SI unit of time since all of the other quantities are in SI units).

✓ *Checkpoint 4.6: My 1000-kg car is rolling (without friction) with a velocity of 4 m/s westward. I find I can exert a force of 20 N eastward. How long does it take to stop the car?*

Now let's consider an example where the object doesn't start or end at rest.

Example 4.1: What net force is required to slow down a 5-kg object from 4 m/s westward to 1 m/s westward in 1.5 seconds?

Answer 4.1: Since everything is in SI units, the units will work out and so we can focus only on the numerical values. In this case, the object slows down by 3 m/s, so that is what we use in the equation:

$$\begin{aligned} \Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m} \\ 3 \text{ m/s} &= \frac{\vec{F}_{\text{net}}(1.5 \text{ s})}{(5 \text{ kg})} \\ 3 \text{ m/s} &= \vec{F}_{\text{net}}\frac{1.5 \text{ s}}{5 \text{ kg}} \end{aligned}$$

One can now solve for $\vec{F}_{\text{net}}$ to get a net force of 10 N eastward. It is eastward because the object is slowing down while moving westward.

✓ *Checkpoint 4.7: My 1000-kg car is rolling (without friction) with a velocity of 4 m/s westward. I find I can exert a force of 20 N eastward. How long does it take to slow the car down to 3 m/s westward?*

Now that we've shown how to solve for any quantity in the equation, we can combine this with the technique described in the previous section, where we used proportional reasoning to solve for a quantity. Consider, for example, the following question:

> Which requires less of a force imbalance, assuming the object is initially at the same speed: to stop the object in 30 seconds or to stop it in 3 seconds?

In this case, the two quantities, $\vec{F}_{\text{net}}$ and Δt, are both in the numerator and on the *same* side of the equation. Since the other quantities in the equation remain the same in the two cases, that means the *product* of $\vec{F}_{\text{net}}$ and Δt must remain the same in the two cases. For the product to remain the same, if one increases, the other must decrease. That means that stopping the object over a greater length of time requires less net force, so the answer is 30 seconds.

✍| Note that the magnitudes of $\vec{F}_{\text{net}}$ and Δt are inversely proportional.

The physics of this example is similar to the physics of a car air bag. In an accident, the air bag deploys and you hit the air bag instead of, say, the windshield. Both objects will stop you but the air bag stops you over a longer time period, using less force.

✓ *Checkpoint 4.8: A heavy bowling ball and a light Ping-pong ball are both at moving at 10 m/s. I exert a 10-N force on each object, opposite their motion. Which object stops in less time? Why?*

Summary

This chapter showed how to use the force and motion equation to make predictions about the motion, the mass, the net force and the length of time the net force is present.

The main points of this chapter are as follows:

- The equals sign in the force and motion equation means that the two sides are equal, not only in terms of the numerical value but also in terms of the units and the direction.
- The newton is a unit of force and is equivalent to a kg·m/s^2.
- As we use SI units for all of the quantities in the force and motion equation, the units will be the same on each side.
- The force and motion equation doesn't tell us what the velocity is – it only tells us how the velocity has changed.
- If the force and motion equation tells us the object slows down by more than the initial speed, it may be that the object changed directions.

- An object slows down when the change in motion is opposite the direction of motion and speeds up when the change in motion is in the *same direction* as the motion.
- You can apply the method of direct and indirect proportions in any order.
- Even though the net force, mass, time and change in velocity are measured separately, their values are not independent.
- Units are important, as they give meaning to the numbers!
- Always consider the physics first, before using an equation.

Frequently Asked Questions

DOES THE CHANGE IN VELOCITY TELLS US THE OBJECT'S MOTION?

No. The change in velocity, by itself, doesn't tell us if the object is speeding up or slowing down, nor does it tell us if the object is changing directions or going straight.

CAN A MOVING OBJECT HAVE A CHANGE IN VELOCITY THAT IS OPPOSITE TO THE WAY IT IS MOVING?

Yes. If the change is opposite the direction of motion, the object is slowing down.

Terminology introduced

Conversion factors
Final time
Initial time
Scalar
Vector

Additional problems

Problem 4.1: Suppose the net force on an object is zero. We find it is moving at 10 m/s northward. If the net force on the object remains zero, what is the object's velocity (speed and direction) two seconds later?

Problem 4.2: Suppose it takes 2 seconds to stop a particular object when applying a given net force. How long would it take to stop an object with triple the mass, assuming it was initially moving at the same speed, if the magnitude of the net force is doubled?

Problem 4.3: A 12-kg block is at rest on a horizontal frictionless surface. Three forces are acting on the block: (1) 10 N eastward, (2) 30 N westward and (3) 20 N eastward. Suddenly the 30 N force is switched from westward to eastward (still 30 N). This is then maintained for 2 s. How fast is the block moving at the end of the 2 s and in which direction?

Problem 4.4: Suppose a hockey puck is hit with a stick such that, after leaving contact with the stick, it slides across a surface made of ice. If the ice is extremely slippery (i.e., no friction) and we assume there is no air resistance, the puck will slide without slowing down (i.e., no change in speed or direction).
(a) What is the net force on the puck after leaving contact with the stick: zero or non-zero?
(b) Are there any horizontal forces acting on the puck as it slides? How do you know? Make sure your answer to (b) is consistent with your answer to (a).

Problem 4.5: My shoe is sliding on a frictionless surface with a velocity of 2 m/s westward. I then apply a force of 2 N westward for 0.8 s. At the end of the 0.8 s, my shoe has a velocity of 6 m/s westward. What is the mass of my shoe?

Problem 4.6: Show algebraically how the units of N·s/kg are equivalent to m/s, given that one newton is equivalent to one kg·m/s^2.

Problem 4.7: I observe a 10-kg object decelerate from 6 m/s eastward to 4 m/s eastward in 1 second.
(a) Given that the object had slowed by 2 m/s, what What is the object's instantaneous velocity at the initial time?
(b) What is the object's instantaneous velocity at the final time?
(c) What is the magnitude of the object's change in velocity?
(d) Using the answer in (c) and the force and motion equation, what is the magnitude of the net force acting on the object?
(e) Is the object speeding up or slowing down?
(f) Using the answer in (e), what is the direction of the net force acting on the object?

Problem 4.8: Suppose an object is moving upward and speeds up by 5 m/upward when a given net force is applied for a given amount of time. Suppose the object is replaced with one that has one-quarter the mass. By how much does the new object speed up if the same net force is applied for the same amount of time as before?

Problem 4.9: Suppose an object is moving northward and speeds up by 7.5 m/s when a given net force acts upon it for a given amount of time. Determine the change in speed for the following cases.
(a) If the magnitude of the net force is tripled and exerted for the same amount of time.
(b) If the object is replaced with one that is three times the mass but same force is applied for same amount of time.

Problem 4.10: Suppose it takes 2 seconds to stop a particular object when applying a given net force. How long would it take to stop an object with triple the mass, assuming it was initially moving at the same speed and the same net force is applied?

5. Forces as Interactions

Puzzle #5: From chapter 1, we know that each force on an object is due to its interaction with another object. How are the *other* objects impacted by those interactions?

Introduction

We are not alone.

What I mean by that is that we interact with objects around us. It is via this interaction that forces are exerted on us (due to those objects) and on those objects (due to us).

In this part of the book, we'll examine how objects interact and, from that, how we can identify the forces acting on an object. In this chapter, we'll examine a particularly important characteristic of interactions, namely that each object of the interacting pair experiences a force associated with that interaction, with equal but opposite forces on the two objects.

5.1 The law of interactions

As mentioned in the introduction, objects experience forces due to their interactions with other objects. What is important to recognize is that *both* objects experience a force. Indeed, when you push against something you can literally *feel* the force on you via the neurons in your tendons and skin.[i]

I'll refer to this idea as the **law of interactions**, which can be stated as follows:[ii]

[i]Technically, the neurons in your skin sense a closely-related property called pressure.

[ii]Newton stated the law as follows (translated from the original Latin): *To every action*

When two objects interact, they simultaneously experience a force due to that interaction, and those two forces have equal magnitude and opposite direction.

❢ An object cannot experience a force from another object without simultaneously exerting an equal but opposite force on that other object (law of interactions).

This means it is not possible for an object to experience a force from another object without simultaneously exerting an equal but opposite force on that other object.

✍ This relationship is sometimes called **Newton's third law** because it was the third law introduced by Newton in his Principia. Recall from chapter 1 that this is called a law because it *describes* a relationship. It does not explain why. We don't know *why* the two forces are equal in magnitude and opposite in direction. However, the law has been tested to very high precision, so we can be confident that it is correct.

What does it mean for two objects to interact?

In a little bit of circular reasoning, an interaction occurs when two objects exert a force on each other.[iii]

Of course, we aren't interacting with *all* of the objects around us. For example, at this moment I am not interacting with the painting hanging in my neighbor's living room. In addition, some objects may exert a force on us but the force may be too small to be of any consequence, like the force exerted on me due to an individual air molecule that bumps into me. On the other hand, if lots and lots of air molecules hit me, the force can be significant.

If we identify the interacting objects, does that mean we've identified the forces?

❢ If two objects aren't interacting, that means they are not exerting a force on each other.

Yes, in the sense that if two objects are *not* interacting, they *don't* exert a force on each other. Indeed, any individual force is associated with the *interaction* rather than with any individual object. For example, when I push on the wall, it is more proper to speak of the force on the wall "due to its interaction with me" and the force on me "due to my interaction with the wall", rather than "my force" and the "wall's force". To simplify the language, I'll refer to how there are two forces, a force on me due to the wall

there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

[iii] We can also describe an interaction as any event in which momentum is transferred, which is discussed in chapter 27.

and a force on the wall due to me, but don't fall into the trap of thinking that those forces are independent. They aren't. They are just two aspects of the same interaction.

✍ Because forces are associated with interactions, not the objects themselves, I'll tend to follow the naming convention where the name of the force is based on the type of interaction. For example, in chapter 12, we'll examine the gravitational force. The gravitational force exists because of a gravitational interaction between two objects. When two objects interact gravitationally, each experiences a gravitational force on it due to the other.

Example 5.1: I am standing on the floor and I throw a ball.
(a) There is a force on me, due to the floor, pushing upward on me and preventing me from sinking into the floor. Is there also a force on the floor due to me?
(b) While I am in the process of throwing the ball and pushing on it, is there a force on me due to the ball?

Answer 5.1: (a) Since the floor touches me, it is interacting with me and, therefore, there is a force the floor due to me, just as there is a force on me due to the floor.
(b) While I am pushing on the ball, the ball is interacting with me and, therefore, there is a force on me due to the ball.

✓ *Checkpoint 5.1: I hold a gun. While the bullet is in the gun, does the bullet exert a force on the gun? Does the gun exert a force on the bullet? Why or why not?*

5.2 Properties

Let's look at several important aspects of the law of interactions.

5.2.1 Forces are due to interactions between objects

As discussed earlier, forces occur when objects interact. You need at least two objects to interact in order to have an associated force. So, a force is not something that an object has, or doesn't have.

What may be potentially confusing about this is that outside of physics many people use the word *force* only for the initiating object (e.g., "to force somebody to do something"). That is not how it is used in physics.

Alternately, outside of physics, some people use the word force for something an object "has". For example, the phrase "may the force be with you," from the Star Wars movies, seems to suggest that each object "contains" a force within it. That is also not how it is used in physics.

To use the word force properly within the context of doing physics, we mustn't confuse the forces due to the objects with the objects themselves.

✍ An **intrinsic** property is something an object "has" and doesn't depend on what the object is interacting with. Mass and velocity[iv] are intrinsic properties. Force, on the other hand, is an **extrinsic** property, since it depends on the interactions between two or more objects.

✓ *Checkpoint 5.2: Why would it be improper, within physics class, to say that someone has a large force?*

5.2.2 Simultaneity

A crucial aspect of the law of interactions is that the two forces associated with the interaction (i.e., the force exerted on object B due to its interaction with object A, and the force exerted on object A due to its interaction with object B) occur at exactly the same time. There is no time delay.

❢ Forces are exerted *during* the interaction; not before, and not after.

In other words, the two forces occur **simultaneously**. This is consistent with the idea that the two forces really are part of the same interaction. There is not one "initiating" force followed by some other "responding" force.

[iv] Velocity is also dependent on the observer's reference frame but it is still considered to be an intrinsic property.

For example, suppose we each have a device that indicates the amount of force exerted on it. If we push on each other with the devices, each device will not only indicate the same magnitude of force but will indicate it at the same time. In other words, if I push harder on you, *both* devices will indicate an increased reading at exactly the same time.

✍ In the original Latin, Newton stated the law of interactions in terms of one force being the "action" and the other being the "reaction". I'm avoiding that terminology because it seems to imply that one force acts first and then the second force acts later in response. That is not the case. Both forces act at precisely the same time.

✓ *Checkpoint 5.3: Suppose I use my hand to push my brother on his shoulder. If I do that, a short while later he will push me back. Are the two pushes described by the law of interactions?*

5.2.3 Directions

As we know from the law of force and motion, you can change an object's motion by pushing on the object. How the motion changes depends on several factors, one of which is the direction of your push. Pushing in the direction of motion makes the object speed up, while pushing opposite the direction of motion makes the object slow down. We can use this idea to check an important aspect of the law of interactions, namely that the forces on each interacting object are opposite in direction.

❢ The forces on each interacting object (due to the other) are opposite in direction.

For example, a bowling ball is rolling down a bowling lane when it collides with a bowling pin, initially at rest. The ball slows down and the pin speeds up. If the ball is rolling northward, the collision exerts southward force on the bowling ball, slowing it down, while the *same* collision exerts a northward force on the bowling pin, speeding it up. Notice how the two forces, from the same collision, are exerted in opposite directions.

Similarly, if a tugboat is pushing on an ocean liner with a northward force then the direction of the force exerted on the tugboat due to the ocean liner would be *southward*, opposite the direction exerted on the ocean liner due to the tugboat.

✓ *Checkpoint 5.4: A person pushes on a car. If the force exerted on the car (due to the person) is eastward, what is the direction of the force exerted on the person (due to the car)? What physics principle did you use to answer this question?*

5.2.4 Magnitudes

So far, we've only compared the directions of the two forces (associated with the same interaction) and noted that they were opposite. However, the law of interactions also says the two forces have equal *magnitudes*.

❢ A "force" is not the same thing as the "effect"; the effect will depend on the object's mass as well as the force exerted upon it.

Keep in mind that this does not mean the *effect* of the interaction is the same on both objects. As we know from the law of force and motion, the change in motion depends not only upon the net force on it but also the length of time the force is exerted and the mass of the object. Consider, for example, the same force exerted on both a light object and a heavy object (like a tennis ball and a bowling ball). The light object will experience a much greater change in motion than the heavy object will.

So the law of interaction means that when I swat at a fly the magnitude of the force I exert on the fly is equal to the magnitude of the force the fly exerts on me as I swat it. However, the same magnitude force is associated with a much bigger change in the fly's motion than in my motion. On the other hand, if I swat at a truck with the same force, the impact is reversed, with a much bigger impact on me than one the truck. The same force results in vastly different effects.

So be sure to distinguish between the *force* due to the interaction (which is the same on both interacting objects) and the *effect* of that interaction (which can be different). As described by the law of force and motion, a force produces a larger effect (a greater change in motion) when exerted on a lighter object (compared to the same force on a heavier object).

✓ *Checkpoint 5.5: A heavy father and his light daughter are facing each other on ice skates. With their hands, they push off against one another. Which is larger in magnitude: the force exerted on the daughter due to the father or the force exerted on the father due to the daughter, or are they equal? What* *physics principle are you using to answer this question?*

5.2.5 Independence of motion

Does an object have to be moving in order for a force to exist?

No. For example, if I lean against the wall, the wall experiences a force due to me (and I experience a force due to the wall) even though I am not moving.

Certainly, if I hit the wall, the forces exerted on the wall (due to my hand) and on my hand (due to the wall) may be greater than when I am just leaning against the wall. However, the force can exist even if my hand isn't moving.

✍ A common misconception is to think that forces are exerted only when an object is moving. This conception may be rooted in the fact that one needs to move the two objects together in order for there to be an interaction. However, once in contact, there doesn't need to be any additional motion in order for the interaction to continue.

Are the magnitudes of the forces equal even if one object is speeding up and the other is slowing down?

Yes. Consider, for example, a bowling ball hitting a bowling pin. The pin makes the ball slow down, while the ball makes the pin speed up. Regardless, the forces on each (due to their interaction) have the same magnitude.

This is why the law of interactions doesn't mention anything about *how* the two objects are moving (e.g., speeding up, slowing down, changing directions, etc.). The law is independent of the objects' motion.

❢ The law of interactions holds even if the object is moving or changing speeds.

✓ *Checkpoint 5.6: (a) A football player throws a ball. During the act of throwing the ball, when the ball is speeding up, which is larger: the magnitude of the force exerted on the ball due to the player or the magnitude of the force exerted on the player due to the ball, or are they equal?*
(b) Another football player catches the ball and starts to run. As the player speeds up (with ball in hand), which is larger: the magnitude of the force exerted on the ball due to the player or the magnitude of the force exerted on the player due to the ball, or are they equal?
(c) What physics principle is used to answer these questions?

5.3 Evidence of the law of interactions

At this point, we have a firm idea of what the law of interactions says but how do we it is *actually* true?

As mentioned earlier, one piece of evidence for the law of interactions is that we can feel pressure when we push on objects and that pressure indicates that there must be a force back on us.

Further evidence is provided by how organisms move by pushing against something. For example, we wouldn't be able to walk unless there was a force on us (due to the floor) when we push against the floor. It is the force on us (due to the floor) that propels us forward, stops us, and allows us to change directions while walking. In a similar way, a fish wouldn't be able to swim if there wasn't a force on the fish (due to the water) when the fish pushes against the water.

Clearly, the "pushing" object is itself pushed. However, what evidence is there that the forces are *equal* in magnitude?

To answer that, we need to utilize the law of force and motion.

To illustrate how we can use the law of force and motion to test the law of interactions, consider two people, of equal masses, facing each other at rest on ice skates. With their hands, they push off against one another. According to the law of interactions, if they push off each other, the force on each must be equal in magnitude. Furthermore, according to the law of force and motion, those forces must result in the same change in motion of each, since they also have equal masses.

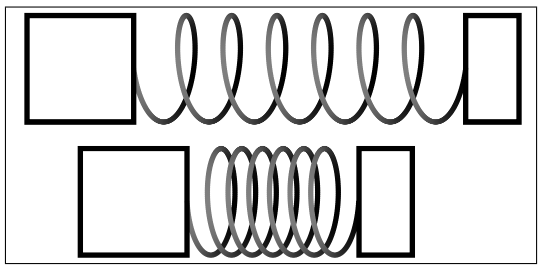
To test the law of interactions, then, we simply have to measure the speed of each person as they move away. If their speeds are equal then, since the two people have equal masses, that means the forces on each (due to the other) must likewise be equal in magnitude.

✍ We've assumed that there are no other forces acting on the objects. In this case, the two people would have to be on a frictionless surface, like ice.

WHAT IF THE TWO PEOPLE DID NOT HAVE THE SAME MASS?

Since the magnitude of the force is the same on each, the heavier person will experience a smaller change in velocity, and thus will move away more slowly. For example, if one object had *twice* the mass as the other, then the law of force and motion predicts that the heavier object will experience *half* the change in speed as the lighter object.

Consider, for example, two objects on a frictionless surface that are attached via a spring as shown in the figure below. Suppose the two objects are initially pulled away from each other, such that the spring was extended (see top illustration). When the two objects are released they will be pulled toward each other by the spring (see bottom illustration).



In the illustration, the left object is twice as big as the right object, representing an object that has twice the mass. Because it has more mass, the heavier object will experience a smaller change in velocity and thus won't accelerate as quickly as the lighter object. As a result, the left object doesn't move as far (see bottom illustration).

Indeed, when one of the objects is much, much more massive, like **Earth**[v], it won't move much at all compared to the lighter object. As a result, it may *appear* as though there isn't a force on it, but there is. The force just doesn't result in much of a change in velocity when applied to the more massive object.

✓ *Checkpoint 5.7: Suppose two people are facing each other at rest on ice skates, as described above, but one person has ten times the mass of the other. With their hands, they push off against one another. If the lighter person moves away at 3 m/s as a result of the push, how fast does the other person move?*

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[v] In keeping with the NASA Style Guide, "Earth" is capitalized when referring to the entire planet, with lower-case "earth" referring to the dirt that is part of the planet.

Summary

This chapter introduced the idea of force in the context of the law of interactions. The main points of this chapter are as follows:

- If two objects aren't interacting, that means they are not exerting a force on each other.
- Forces are exerted *during* the interaction; not before, and not after.
- An object cannot experience a force from another object without simultaneously exerting an equal but opposite force on that other object (law of interactions).
- The law of interactions holds even if the object is moving or changing speeds.
- A "force" is not the same thing as the "effect"; the effect will depend on the object's mass as well as the force exerted upon it.

By now you should be able to use the characteristics of forces (described by the law of interactions) to make predictions regarding the forces acting on one of the interacting objects given the force on the other object in the pair.

Frequently Asked Questions

Is identifying interacting objects the same as identifying forces?

See page 78.

Do two objects have to touch each other in order to interact?

No. The gravitational force is an example of a force that exists even when two objects aren't touching. This is discussed in chapter 12.

Why is the law of interactions called a law and not a theory?

It is called a **law**, not a **theory**, because the law of interactions does not attempt to explain why the two forces are the same. It only states that they are, indeed, equal.

If an object is in motion, is the force due to it necessarily in the direction of that motion?

No. While that may be true in some situations, it is not always true, and there is nothing in the law of interactions that requires it to be true. The law of interactions only states that the force exerted on object B due to its interaction with object A is equal (and opposite) to the force exerted on object A due to its interaction with object B.

What if there are more than two objects interacting?

The law of interactions applies to each *pair* of objects. Consider, for example, two people pushing on opposite ends of a car. There are three objects: the car and the two people. The force due to each person on the car can be different in that case because the two people are pushing on the car, not each other. The law of interactions only states that the force exerted on a particular person due to the *car* is equal in magnitude and opposite in direction to the force exerted on the *car* due to *that* person.

Terminology introduced

Effect	Inertia	Law of interactions	Theory
Extrinsic	Intrinsic	Newton's third law	
Force	Law	Simultaneously	

Additional problems

Problem 5.1: A tennis ball is hit with a tennis racket. At the moment of contact, the tennis racket is moving. Which is larger in magnitude: the force exerted on the ball due to the racket or the force exerted on the racket due to the ball, or are they equal in magnitude?

6. Collisions

Puzzle #6: How do accident investigators "reconstruct" an accident and determine the speed of the vehicles *prior* to the accident?

Introduction

In section 5.3, it was mentioned that we can test the law of interactions by observing how two objects change their motion as they push off one another (or get pulled in toward each other). In this chapter, we apply the same idea to collisions, as with accidents.

6.1 Collisions

Accidents often involve multiple cars. Reconstructing those accidents requires a knowledge of how cars interact with each other. It turns out that we can use the law of interactions and the law of force and motion to obtain information about each car's motion prior to the collision.

For our purposes, we'll consider interactions between two cars only. According to the law of interactions, each car participating in the interaction experiences a force due to that interaction, and the two forces are equal in magnitude and opposite in direction.

According to the law of force and motion, each force is associated with a change in the object's motion, and the less massive object will experience a greater change in motion, since the force on each is the same (law of interactions), the time is the same (same interaction) and the change in motion is inversely proportional to the mass (law of force and motion). So, for two cars in a collision, the less massive car will experience the greater change in motion.

✓ *Checkpoint 6.1: When a small car collides with a large truck, the small car undergoes a greater change in motion than the truck. Why? Is it because (a) a greater force is exerted on the car during the collision, (b) the force on the car is applied for a longer period of time, (c) the car's mass is less, or (d) all of the above?*

6.2 The basic process

When two objects collide, we know that each experiences a force due to the interaction, and the two forces have equal magnitudes, are applied for the same length of time and have opposite directions. However, we don't know the values of either the force or the time. It turns out we can still predict how the motion changes *even though we don't know the force or the time* by relating the motion changes of each object.

❢ When two objects collide, the forces on each, due to that collision, have equal magnitudes and opposite directions.

Consider, for example, the following situation:

> A 60-kg father and his 30-kg daughter are facing each other at rest on ice skates (father on the left, daughter on the right). With their hands, they push off against one another, with the daughter moving rightward and the father moving leftward. If the daughter is moving at 3 m/s after pushing off, how fast is the father moving after pushing off?

From the law of force and motion, we know that there must be a force on the daughter, to make sure speed up to 3 m/s. From the law of interactions, we know that there must also be a force on the father, equal in magnitude to the force on the daughter (since it is the same father-daughter interaction). Since the father is more massive, the same force results in a smaller change in the father's velocity, so the father must be moving at *less than* 3 m/s after pushing off.

The physics is similar to what happens when you shoot a gun, where the gun experiences something called **recoil**, which is a "kick" or thrust backward when the gun is fired. Basically, to fire the bullet, there must be a force on it due to the gun. And, by the law of interactions, there must be an equal and opposite force on the gun, pushing it backwards. Since the gun (and person firing the gun) is more massive than the bullet, the backwards motion of the gun/person is less than the forward motion of the bullet, even though the force is the same, in magnitude, on each.

CAN WE DETERMINE HOW FAST THE FATHER IS MOVING, OR JUST THAT THE FATHER IS MOVING SLOWER THAN THE DAUGHTER?

Yes, we can determine how fast the father is moving. In fact, you may have already guessed that the father is probably moving half as fast, since the father is twice as massive.

To show quantitatively that this is indeed the case, we'll repeat the logic we just went through but with the help of the force and motion equation. The process is easier if use the following version of the force and motion equation:[i]

$$m\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t$$

There are three steps to the process: apply the force and motion equation to one object to find $\vec{F}_{\text{net}}\Delta t$, use the law of interactions to find $\vec{F}_{\text{net}}\Delta t$ for the other object, then use the force and motion equation again to find Δv for that other object. Typically we know the directions so we usually just predict the change in speed, as illustrated below for the father and daughter example.

1. **Use the force and motion equation to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the daughter**
   Even though we don't know the value of the force on the daughter, we *can* find the value of $\vec{F}_{\text{net}}\Delta t$, where F is the magnitude of the force on the daughter and Δt is the time the force is applied on the daughter, because that is the left side of the force and motion equation, and the left side must be equal to the right side, which is the product of the daughter's mass (30 kg) and her change in speed (from zero to 3 m/s). The $m\Delta\vec{v}$ product this equals 90 kg·m/s rightward (notice that the units are the combination of the mass and velocity units), which means that $\vec{F}_{\text{net}}\Delta t$ on the daughter must be 90 kg·m/s rightward.
2. **Use the law of interactions to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the father**

[i]This version can be obtained from $\Delta\vec{v} = \frac{\vec{F}_{\text{net}}}{m}\Delta t$ by multiplying both sides by m.

The law of interactions states that the force on the father (due to the father-daughter interaction) must be equal in magnitude (and opposite in direction) to the force on the daughter (due to the same father-daughter interaction). In addition, the time must be the same for each force, since they both act only when the interaction is present. That means that $\vec{F}_{\text{net}}\Delta t$ on the father must be 90 kg·m/s leftward.[ii] Notice that the direction is leftward, not rightward, because the force on the father is opposite that on the daughter.

3. **Use the force and motion equation to determine the father's change in velocity**
   We know from the previous step that the right side is equal to 90 kg·m/s. From the force and motion equation, that means that the left side must likewise equal
   For the father, $\vec{F}_{\text{net}}\Delta t$ 90 kg·m/s, as determined in the previous step, so from the force and motion equation we know that $m\Delta\vec{v}$ for the father must likewise be 90 kg·m/s. Since $m\Delta\vec{v}$ for the father is 90 kg·m/s, we can divide that by the father's mass (60 kg) to get his change in speed (1.5 m/s). Since the father starts at rest, his speed must be 1.5 m/s.

❢ The 3-step process shows that the force and motion equation, along with the law of interactions, can be used to obtain an object's change in motion even when we don't know the force exerted on it.

Notice that the father did indeed end up moving at half the speed of the daughter, and this is because the father is twice as massive. Because of this, it may be tempting to skip the three steps described above and just figure out the father's final speed by just considering the ratio of the two masses. This is indeed possible. However, the purpose of this exercise is to illustrate how to apply use the law of force and motion and law of interactions. Consequently, I will continue to apply the three steps to situations involving collisions.

✓ *Checkpoint 6.2: Suppose the father was 90 kg instead of 60 kg. If the daughter is still 30 kg and they push off each other while at rest on ice skates (father on the left, daughter on the right), how fast would the father be moving after pushing off if the daughter is moving at 3 m/s after pushing off?*

[ii] We are assuming that there are no other forces contributing to the force imbalance on either person.

6.3 Situations where objects are not initially at rest

So far, we've only considered two objects initially at rest where they push off of each other. That is not really a collision, where at least one of the objects is moving and collides with the other object. The 3-step process is the same, though.

To illustrate this, consider the following situation:

> A red 4-kg cart rolls at 3 m/s eastward. It then collides with a blue 2-kg cart at rest. After the collision, the red 4-kg cart is observed to be rolling at 1 m/s eastward. How fast is the blue 2-kg cart moving after the collision?

Before doing any math, let's first get a sense of what is going on physically. The red cart collides with the blue cart and, due to that collision, slows down. In other words, there is a force on the red cart, due to its interaction with the blue cart, that makes it slow down. There must also be a force on the blue cart, due to its interaction with the red cart, and that force must be equal in magnitude to the force on the red cart. However, the blue cart is lighter, so it will experience a larger change in speed.

Let's now go through the three steps and repeat this analysis using some numbers.

1. **Use the force and motion equation to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the red cart**
   The product of the red cart's mass (4 kg) and change in speed (from 3 m/s to 1 m/s so the cart slowed by 2 m/s) equals 8 kg·m/s (multiply 4 kg by 2 m/s). Notice how we use the *change* in velocity (2 m/s), not the initial (3 m/s) or final velocity (1 m/s). From the force and motion equation, $\vec{F}_{\text{net}}\Delta t$ on the red cart must likewise be 8 kg·m/s.
2. **Use the law of interactions to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the blue cart**
   The law of interactions states that the force on the blue cart (due to the two-cart interaction) must be equal in magnitude (and opposite in direction) to the force on the red cart (due to the same two-cart interaction). In addition, the time must be the same for each force,

since they both act only when the interaction is present. That means that the magnitude of $\vec{F}_{\text{net}}\Delta t$ on the blue cart must be 8 kg·m/s (the same as the value on the red cart).

3. **Use the force and motion equation to determine the blue cart's change in speed**
   Now that we know $\vec{F}_{\text{net}}\Delta t$ on the blue cart, we can determine $m\Delta\vec{v}$ by again using the force and motion equation:

   $$m\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t$$

   We know from the previous step that the right side is equal to 8 kg·m/s. From the force and motion equation, that means that $m\Delta\vec{v}$ for the blue cart must likewise equal 8 kg·m/s. We can divide that by the blue cart's mass (2 kg) to get its change in speed (4 m/s). Since the blue cart starts at rest, its final speed must be 4 m/s (eastward).

Notice that the force and motion equation involves the *change* in velocity, so it is important that we determine that when going through the process.

✓ *Checkpoint 6.3: A 600-kg car traveling eastward at 30 m/s collides with a heavier 1200-kg truck that is at rest. Immediately after the collision, the 600-kg car is found to be moving 5 m/s eastward. What is the 1200-kg truck's velocity immediately after the collision? Assume no other forces are acting to speed up or slow down the car or truck during the collision.*

Summary

This chapter examined how objects interact in a collision.

The main points of this chapter are as follows:

- When two objects collide, the forces on each, due to that collision, have equal magnitudes and opposite directions.
- The 3-step process shows that the force and motion equation, along with the law of interactions, can be used to obtain an object's change in motion even when we don't know the force exerted on it.

By now you should be able to predict how the motion of an object changes during a collision by using the force and motion equation and the law of interactions.

Terminology

Recoil

Additional problems

Problem 6.1: (a) A 1000-kg car is traveling at a speed of 30 m/s eastward. It then stops in 10 seconds. What is the change in the car's speed during the 10 seconds?
(b) A 3000-kg truck is moving at 20 m/s northward. It then takes 20 seconds to slow to 15 m/s northward. What is the change in the truck's speed?

Problem 6.2:A 10-kg object (object 1) is moving at 10 m/s rightward on a frictionless surface. It then collides with a 5-kg object (object 2) that was at rest. After the collision, the 5-kg object moves rightward with a speed that is 3 m/s faster than the 10-kg object's speed (also moving rightward, so $v_{2,\mathrm{f}} = v_{1,\mathrm{f}} + 3$ m/s). What is the speed of the 5-kg object after the collision?

Part B

Definitions

7. Acceleration and Velocity

Puzzle #7: What is the difference between acceleration and velocity, and do we need both?

Introduction

What we know so far is that an object's motion changes while a net force acts upon it. This is the idea described by the law of force and motion.

The law of force and motion only tells us how the motion is *changing*. It doesn't tell us what the motion *is*. Stated another way, the law of force and motion tells us the *acceleration*, not the *velocity*. This chapter examines the difference between these two terms.

7.1 Zero vs. non-zero acceleration

The law of force and motion tells us that an object's velocity continues to change the entire time a non-zero net force acts upon it. In other words, the velocity doesn't just jump to the new value in an instant. Instead, it continually changes during the entire time the net force is present.

For that reason, it is useful to focus on the *rate* at which the velocity changes while the net force is acting. That rate is called the acceleration.

To distinguish between velocity and acceleration, let's compare what the object's motion looks like when it is speeding up (non-zero acceleration) compared to when it is moving at a constant velocity (zero acceleration).

7.1.1 Visualizing constant velocity

Suppose we have a toy car that moves at a constant velocity along a horizontal floor. Every second, I take a picture that shows the car's position at that

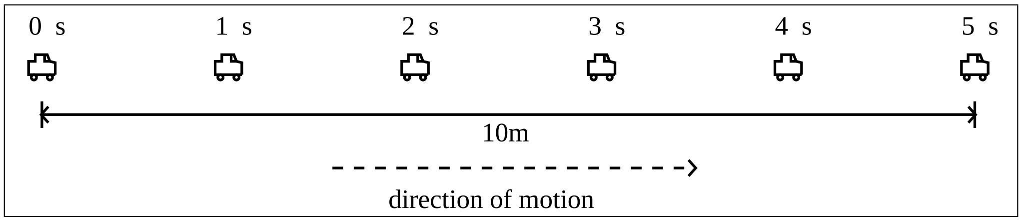


Figure 7.1: A time-lapse picture of a toy car.

moment (i.e., where the car is along the floor). If I then superimpose all of my pictures together, I get the multiple-snapshot image shown in Figure 7.1 (which I call a **time-lapse** picture).[i]

❢ In a time-lapse picture, the time interval between each "snapshot" needs to be the same in order to infer the velocity.

In the figure, the time interval between each "snapshot" is the same. In this case, the time interval is one second. For our purposes, the time interval doesn't have to be one second but the time interval does have to be the same between each snapshot.

Why does the time interval have to be the same?

The time interval has to be the same so that we can obtain information about the velocity. For example, in Figure 7.1, you can tell that the object is moving at a constant velocity because the position changes by the same amount during each time interval. If the time between each image was not the same, we would not be able to infer that.

Are we to assume that the car started at rest?

No. Just because the first image is taken at 0 s does not mean that the car was at rest at 0 s. Each image simply represents the position of the car at that instant. It is only through two successive pictures that we can get a sense of an object's motion.

It would be like if I stood on the side of a highway and took a picture. Just because it is the first picture I happen to take that day does not mean the cars were at rest when I took the picture.

So, unless specifically stated, we cannot assume an object started at rest during the time period of interest. Figure 7.1, in particular, is supposed to represent a car moving with a constant non-zero velocity, so the car would have the same velocity the entire time, even at the beginning.

[i]If the picture in Figure 7.1 does not make sense to you, ask the instructor.

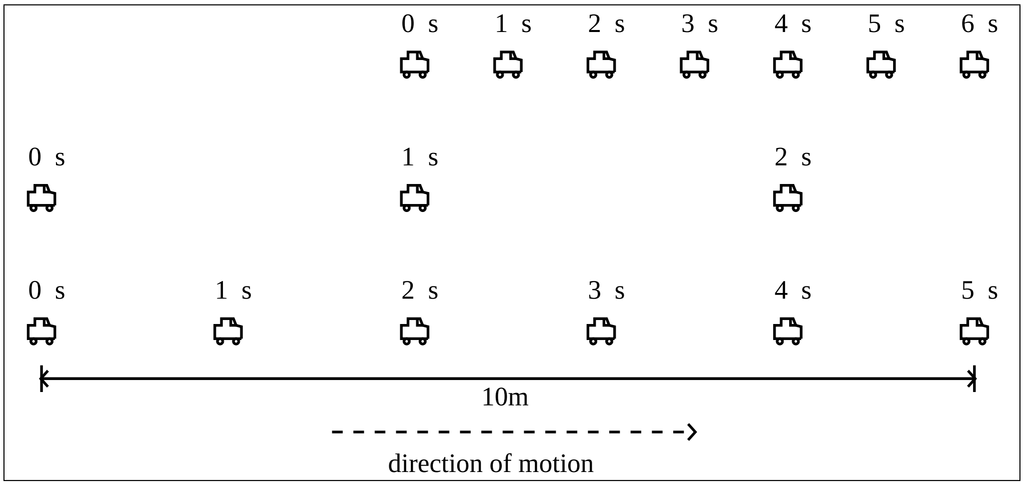


Figure 7.2: A time-lapse illustration of three toy cars.

WHAT IS THE CAR'S ACCELERATION IN FIGURE 7.1?

Since the car is neither speeding up or slowing down, we say that it isn't accelerating – its **acceleration** is zero. Notice that the car's acceleration is zero but its velocity is *non-zero*. Since the car is moving at a constant speed, the law of force and motion tells us that the forces must be balanced upon the car (zero net force).

✓ *Checkpoint 7.1: Figure 7.2 shows a time-lapse illustration of three different toy cars. At 0 s, the top car is to the right of the other two, yet it ends up to the left because it is moving slower.*
(a) How can you tell that the middle car in Figure 7.2 is moving the fastest?
(b) At 2 seconds, which car is in the lead?
(c) When does the bottom car catch up to the top car? How do you know?
(d) Do any of the three cars have a zero acceleration? If so, which one(s)?

7.1.2 Visualizing accelerating motion

Now that we can use time-lapse pictures to visualize constant velocity motion (zero acceleration), let's examine a time-lapse picture of an accelerating object. This is illustrated in Figure 7.3.

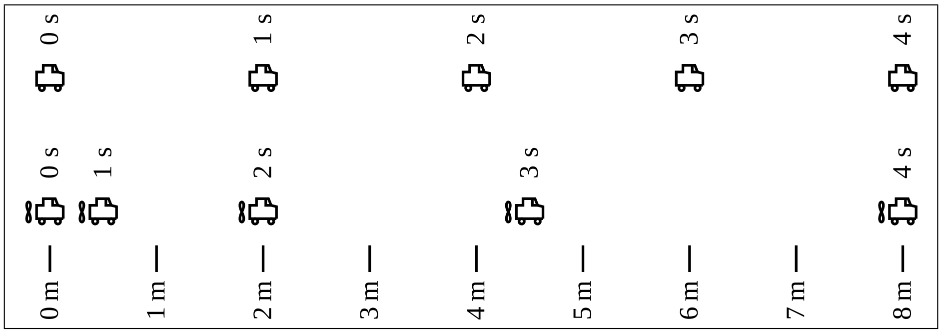


Figure 7.3: A time-lapse picture of a car moving at a constant speed and a fan cart that is initially at rest and accelerating.

The figure illustrations the motion of a car moving at a constant velocity (top set of images) and a fan cart[ii] that is speeding up (bottom set of images). Both are at the same position (0 m) at the initial time (0 s). However, the car is *already* moving while the fan cart is initially at rest. Consequently, car initially moves ahead of the fan cart (compare the two positions at 1 s). The fan cart, though, is accelerating, moving faster and faster, which we can tell from the increasing spacing between adjacent images, and by 4 s the fan cart has caught up to the car (both at 8 m).

Notice that the car's acceleration is zero whereas the fan cart's acceleration is *non-zero*. Since the fan cart is *not* moving at a constant speed, the law of force and motion tells us that there must be an imbalance of forces upon the fan cart (non-zero net force).

What is the fan cart's velocity and acceleration at 0 s?

As mentioned earlier, the fan cart is initially at rest, which means its velocity is zero at 0 s. The car, on the other hand, was *already* moving so its velocity is non-zero at 0 s.

On the other hand, the car maintained the same velocity the entire time (zero acceleration) while the fan cart was accelerating the entire time. That includes the time at 0 s. That means the fan cart's acceleration at 0 s is non-zero.

How can the fan cart's acceleration be non-zero at the moment

[ii] A fan cart is basically a cart upon which is attached a fan. When the fan is turned on, the cart slowly speeds up, going faster and faster as it moves in a straight line.

ITS VELOCITY IS ZERO?

The two terms refer to two different things. At the time equal to 0 s, the fan cart is at rest, meaning that its velocity is zero. However, it doesn't *remain* at rest – it is speeding up – and that means it has a non-zero acceleration.

✍ This is like having the air warm up (changing temperature) vs. having a temperature of 0°C. Both can be true, as long as the temperature doesn't *remain* at 0°C.

✓ *Checkpoint 7.2: (a) In Figure 7.3, which object is moving faster at time 0 s: the car or the fan cart? (b) Which object is moving faster at time 4 s? (c) Which object has a greater acceleration at time 0 s? (d) Which object has a greater acceleration at time 4 s?*

7.2 Definitions

We know that an object's velocity reflects how fast it moves from place to place (and the direction of that motion) whereas an object's acceleration reflects how quickly that motion is *changing* (for example, speeding up). Now that we have a sense of what it means for an object to have a velocity and/or acceleration, we can express these ideas in the form of two definitions. Defining the terms are important not just to help us distinguish between the two but also so that we can *quantify* their values.

For example, we can tell from Figure 7.3 that the car is moving faster than the fan cart at 0 s but the fan cart is moving faster than the car at 4 s. That means that at some time between 0 s and 4 s the two must have the *same* velocity! Having a specific definition of velocity will allow us to show that this is the case, and when this moment of equal velocity actually occurs.

7.2.1 Definition of velocity

From the time-lapse pictures shown earlier, we can see that an object's velocity depends on how far it moves in a given amount of time. A faster object moves farther in the same amount of time.

It is reasonable to find, then, that an object's velocity is defined to be equal to the change in its **position** divided by the time it takes for the position to change.

❢ Velocity is defined as the change in position divided by the time it took to undergo that change.

Using $\Delta\vec{s}$ for the change in position and Δt for the length of time it takes to undergo the change, we can write the definition as follows:

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{s}}{\Delta t} \tag{7.1}$$

Example 7.1: Figure 7.2 showed a time-lapse illustration of three different toy cars. What is the velocity of each car?

Answer 7.1: The bottom car travels 10 m rightward in 5 s, so its velocity is (10 m)/(5 s), which equals 2 m/s rightward. The middle car travels the same 10 m but in only 2.5 s, so its velocity is (10 m)/(2.5 s), which is 4 m/s rightward, twice that of the bottom car. During each second, the top car travels only half the distance as the bottom car, so its velocity should be half that of the bottom car, or 1 m/s rightward. It is a little hard to see, but the top car has only traveled 6 m rightward, and it took 6 s to do so, which is consistent with a velocity of 1 m/s rightward.

Why use $\vec{s}$ for the position? Wouldn't it make more sense to use $\vec{p}$?

We can use whatever letter (or group of letters) we want. I've chosen s because it is somewhat conventional to represent the position of an object as s.[iii] The abbreviation is likely derived from the Latin word *spatium*, meaning distance.[iv]. Galileo, for example, used the term *spazio*, which means distance in Italian, in his 1640 text *Discourses on Two New Sciences*.

✍ For information on typical abbreviations used for quantities, check out the supplemental readings.

Why use a Δ?

[iii] Actually, some physicists use x for position. I'm using s so as to not confuse position with the x direction, which we will be using later. You might think d could be used (for distance) but d is used in calculus to represent infinitesimal changes so it would be confusing to use it here.

[iv] I think of it as standing for "segment" but it doesn't matter as long as you recognize what I am using it to represent.

I am using the Δ along with $\vec{s}$ for the same reason I am using Δ along with t. It indicates that we are using the *change* in the position that occurs during the time interval Δt.

Why does $\vec{v}_{\text{avg}}$ have "avg" as a subscript?

Technically, this is the definition for the *average* velocity. For the time being, we don't need to worry about what that means. We'll explore what it means in chapter 8. I'm including it to be technically correct, and to avoid confusion later on.

Why is equation 7.1 called a definition?

A **definition** specifies what a particular quantity is in terms of other quantities. Definitions are relationships that are always true because, well, that is the way they are defined (see supplemental readings).

7.2.2 Definition of acceleration

Whereas the velocity is defined to be the rate at which the *position* changes ($\Delta\vec{s}/\Delta t$), the acceleration is defined to be the rate at which the *velocity* changes: In order to properly interpret the last sentence, we need a precise definition of acceleration. Mathematically, we can write the definition of acceleration as follows:[v]

❢ The acceleration is defined to be $\Delta\vec{v}/\Delta t$.

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \tag{7.2}$$

where $\vec{a}_{\text{avg}}$ is used to represent the acceleration.[vi] Note that the definition of velocity has the same form as the definition of acceleration. The difference is that the *velocity* is defined as the rate at which the *position* changes, whereas the *acceleration* is defined as the rate at which the *velocity* changes. In both cases, there is an "avg" subscript, the meaning of which we will explore later in this chapter.

Notice that the acceleration is not just the change in velocity $\Delta\vec{v}$ and it is not just the time Δt. By defining it as the ratio, the acceleration value is

[v] As with the definition of velocity, the definition of acceleration, being a definition, is always true, as discussed in the supplemental readings.

[vi] Technically, this is the definition of the *average* acceleration, but at the moment we don't need to worry about the difference.

larger when it takes a smaller amount of time to undergo the same change in velocity.

It is important to recognize that acceleration and velocity are not the same. The acceleration describes how quickly the velocity is *changing*.

To illustrate the difference, consider an object that is maintaining a constant non-zero velocity, as with the car in Figure 7.3. If the velocity is not changing (so $\Delta\vec{v}$ equals zero) then the object is not accelerating. In that situation, its acceleration is zero regardless of what the velocity value happens to be.

For example, a rocket could be moving through space at a constant speed of 400 m/s. If it maintains that speed[vii] then the rocket's acceleration will be zero. It doesn't matter how fast the rocket is moving. If it is not accelerating at that moment, the rocket's acceleration is zero.

In comparison, consider an object that is maintaining a constant non-zero *acceleration*, as with the fan cart in Figure 7.3. A non-zero acceleration means that the velocity is changing. A *constant* non-zero acceleration means that the velocity continues to change at the same rate. In other words, while the acceleration is constant, the velocity is not.

Example 7.2: Is it possible for an object's velocity to be zero while its acceleration is non-zero? If so, describe a "real" situation where that is the case. If not, explain why not.

Answer 7.2: It is possible, but only for an instant, since if the acceleration is non-zero that means the velocity has to be changing. For example, the moment a force is applied to an object initially at rest, the object accelerates, since the object can't stay at rest if there is a non-zero net force exerted on it. However, for an instant, the object's velocity is still zero, since that is what it was when the force was applied.

✓ *Checkpoint 7.3: Is it possible for an object's velocity to be non-zero while its acceleration is zero? If so, describe a "real" situation involving something* **you** *do where that is the case. If not, explain why it is not possible.*

[vii] And doesn't turn – since velocity includes direction. As we will see later, turning is also an acceleration.

7.3 Units

WHAT IS THE SI UNIT FOR VELOCITY?

An object's position indicates *where* the object is, and thus has units of length, which has the SI unit of meter (abbreviated as m). The SI unit of time is seconds (abbreviated as s). Since velocity is the change in position divided by the change in time, the SI unit for velocity is meters per second (abbreviated as m/s).

For example, suppose a car's position changes by 5 meters in a time of 10 seconds. According to the definition, the car's velocity during the 10-second interval would be 0.5 m/s (i.e., divide the change in position by the time).

✍ If the object's position isn't changing at all, then $\Delta\vec{s}$ is zero and the velocity is zero.

✓ *Checkpoint 7.4: Suppose a car's position changes by 10 meters in a time of 2 seconds. What is the car's velocity?*

WHAT IS THE SI UNIT FOR ACCELERATION?

Since acceleration is the change in velocity (with SI units m/s) divided by the change in time (with SI units s), the SI unit for acceleration is meters per second per second, or meters per second squared (abbreviated as m/s^2).

❢ The SI unit for acceleration is m/s^2.

Notice that the units are m/s2, not m/s. The presence of the "2" in the denominator is important – it indicates that the seconds are being squared. The squaring of the seconds corresponds to how the acceleration represents the rate that velocity is changing.

WHY ARE THE SECONDS SQUARED?

When we divide m/s by s, it is equivalent to dividing m by s and then dividing by s again. To see this, let's first rewrite the definition as the product of two ratios:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{v}}{1} \times \frac{1}{\Delta t}$$

Now we can consider that $\Delta\vec{v}$ has units of m/s while Δt has units of s. This means the acceleration has the following units:

$$\frac{\text{m/s}}{1} \times \frac{1}{\text{s}} = \frac{\text{m}}{\text{s}} \times \frac{1}{\text{s}} = \frac{\text{m} \times 1}{\text{s} \times \text{s}} = \frac{\text{m}}{\text{s}^2}$$

Are the units of acceleration the same as that for velocity?

The units are *similar* to those of velocity (m/s) but are *not* the same. The square of the seconds in the denominator is important because it indicates that we are looking at how quickly the velocity is changing, not the velocity value itself.

❢ The units of acceleration are different than the units of velocity because acceleration indicates how quickly the velocity is changing (rather than the velocity itself).

✍ Just as the units for the warming rate (in degrees per hour) is not the same as the temperature (in degrees), the units for acceleration (in m/s^2) is not the same as for velocity (in m/s).

✓ *Checkpoint 7.5: If an object has an acceleration of 5 m/s^2 does that mean it is moving at a speed of 5 m/s? If not, does it mean it is moving faster than 5 m/s, slower than 5 m/s or is it not possible say?*

7.4 Interpreting values

7.4.1 Speeding up

According to Tesla (tesla.com), the 2022 Tesla Model S Plaid has the quickest acceleration of any vehicle in production, going from zero to 60 mph in only 2.0 seconds. A more typical “zero-to-60” time is between 4 and 10 seconds. For example, the Toyota Camry takes 5.6 s and the Toyota Corolla takes 7.1 s, while the Toyota Pickup takes 10.0 s (source: http://0-60specs.com/0-60-times/). This leads us to several questions, like what are these speeds in m/s, what acceleration is this in m/s^2, and how do these accelerations compare to a typical acceleration when starting from a stop light at an intersection?

We'll start with the first question, and convert a speed of 60 mph into m/s.

It turns out that each mph is equal to 0.447 m/s or so. To get this conversion, one needs to know that each mile is 1609 m and each hour is 3600 s. Dividing a mile (1609 m) by an hour (3600 s), we get 1609 m divided by 3600 s, which equals 0.447 m/s. That means that one mile per hour is equivalent to 0.447 m/s. Multiplying the 60 mph by 0.447 m/s per mph, we get that 60 mph is equivalent to 26.8 m/s.

Since the numerical value is less, a speed of 26.8 m/s might seem slower than 60 mph but it is actually the *same* speed.

Now that we know that 60 mph is equivalent to 26.8 m/s, we can determine the acceleration of the Tesla Model S Plaid as it accelerates from zero to 60 mph. From the definition of acceleration, divide the change in velocity (26.8 m/s) by the time (2.0 s) to get an acceleration equal to 13.4 m/s^2.

How does this compare to a regular car?

As mentioned earlier, a Toyota Pickup takes 10.0 s to go from zero to 60 mph. From the definition of acceleration, divide the change in velocity (26.8 m/s) by the time (10.0 s) to get an acceleration equal to 2.68 m/s^2. A similar analysis shows that the Toyota Camry can accelerate at 4.8 m/s^2 and the Toyota Corolla can accelerate at 3.8 m/s^2. Notice how the smaller the time, the more quickly the vehicle can speed up and so the greater the acceleration (for the same change in speed).

Of course, these values are the *greatest* acceleration that can be achieved by these vehicles. A more typical acceleration, when starting at rest from a stop at an intersection for example, is around 2 m/s^2.[viii]

✓ *Checkpoint 7.6: When entering Interstate 80, you essentially start from rest and reach the highway speed of 50 mph within the length of the on-ramp. Do you expect your acceleration during that period to be greater than the 0-60 acceleration of the 2022 Tesla Model S Plaid or less than that acceleration?*

So far we have used the definition of acceleration with objects that start at rest. The definition needn't be restricted to that, however. For example, suppose a vehicle was traveling at 10 m/s (22.4 mph) and accelerates to 15 m/s (33.6 mph), taking 2 seconds to do so. To find the vehicle's acceleration during the 2 seconds, we take the *difference* in speed (5 m/s) divided by 2 seconds to get an acceleration of 2.5 m/s^2.

Notice that we do *not* simply take the final speed and divide by the time. Taking the final speed would not tell us whether the car was speeding up or not, which is what the acceleration indicates.

Furthermore, since the definition of acceleration contains three quantities (acceleration, change in velocity, and elapsed time), we can use any two of these quantities to determine the third. For example, suppose an object is

[viii] Source: Bogdanovic et al., 2013, The Research of Vehicle Acceleration at Signalized Intersections, *Promet – Traffic and Transportation*, vol 25 (1), 33-42.

speeding up at a rate of 3 m/s^2. This means is that the speed is increasing by 3 m/s every second. Over the course of 2 seconds, this would mean the object's speed would increase by 6 m/s.

Basically, when an object is speeding up with an acceleration of 3 m/s^2, it means that they are gaining speed such that their speed at any given time is 3 m/s faster than their speed a second before.

✓ *Checkpoint 7.7: An object speeds up at a rate of 5 m/s^2. At a certain time, it is measured to be moving at 20 m/s. How fast is it going 3 s later?*

7.4.2 Slowing down

Outside of physics, people usually use the word acceleration to mean speeding up and **deceleration** to mean slowing down. According to the definition of acceleration (equation 7.2), though, *any* change in velocity corresponds to a non-zero acceleration, just as *any* change in position corresponds to a non-zero velocity. That means that we can apply the definition of acceleration to both speeding up and slowing down.[ix]

❢ Any change in velocity is an acceleration.

For example, the U. S. Department of Transportation has determined that an average driver can decelerate at a rate of, at most, about 5.4 m/s^2 when driving on dry pavement (without slipping). Knowing this rate, one can determine how quickly one can come to a stop. For example, suppose you are driving at 55 mph (24.6 m/s) when you suddenly see something in the road and need to stop. From the moment you first brake, how long will it take you to come to a stop?

[x]We can solve this problem by using the definition of acceleration. From the definition, the acceleration (5.4 m/s6^2 in this problem) is equal to the change in velocity (24.6 m/s in this problem, since you are coming to a stop) divided by the time. To solve, use algebra[xi] (which leads to dividing 24.6 m/s by 5.4

[ix]Looking back, it may have been better to use a different word (like "alteration") so that students wouldn't get it confused with the ordinary usage of acceleration.

[x]Remember that the equation is just a representation of the idea, and doesn't replace the idea.

[xi]Multiply both sides by Δt and then divide both sides by the acceleration.

m/s^2) or simply continue to subtract 5.4 m/s from 24.6 m/s until you get to zero. The result is 4.6 s.[xii]

✓ *Checkpoint 7.8: An object slows down at a rate of 5 m/s^2. At a certain time, it is measured to be moving at 20 m/s. How fast is it going 3 s later?*

7.5 Direction

In the previous section, it was pointed out that velocity has a direction, and we need to pay attention to that direction when determining the acceleration.

In equations, we include a little arrow on top of the $\vec{v}$ to indicate that velocity has a direction. You may have noticed, though, that there is also a little arrow on top of the $\vec{s}$ (position) and there is a little arrow on top of the $\vec{a}$ (acceleration). This is because both position and acceleration have a direction as well. Let's take a moment to examine what that means.

What is meant by the direction of the position?

To adequately represent an object's position, we need to specify where the object is *relative to some reference position*. Usually, the reference position is obvious and so it goes without saying. In other cases, it is not.

For example, suppose you ask someone where they lived. If they respond "three miles", chances are they mean that they live three miles from a particular place, but what is that place? Without knowing the reference position (that particular place), the position is ambiguous. A more proper position would be something like three miles northward from East Stroudsburg. Notice that the direction of the position is *northward*.

Is the direction of an object's position the same as the direction of the object's velocity?

One cannot say. The velocity points in the direction the object *moves*. So, if the object moved from a position three miles northward (of some location)

[xii] According to the U. S. Department of Transportation, a typical reaction time is 1.1 s, which means it would take 5.7 s to stop from the time you first see the object in the road. That is not as short as some people may expect!

to another position that is five miles northward (of that location) then the velocity *would* be northward. On the other hand, if the object moved from a position three miles northward (of some location) to another position that is *one* mile northward (of that location) then the velocity would be *southward*.

Note that the velocity direction does tell us something about how the position is changing. If the position is initially northward, for example, then a northward velocity would mean the position is getting further and further northward, whereas a southward velocity would mean the position is getting less and less northward.

✓ *Checkpoint 7.9: Suppose a car's position is initially 3 miles eastward of an exit and then, 2 minutes later, its position is 1 mile eastward of that same exit. Which way was it moving and how fast?*

What is meant by the direction of the acceleration?

If the object is *speeding up*, then the acceleration is *in the direction* of motion. Conversely, if the object is *slowing down*, then the acceleration is *opposite* the motion.

For example, suppose an object is moving downward. If it is speeding up, it has a downward acceleration (i.e., in the same direction as the motion). If it is slowing down, it has an upward acceleration (i.e., opposite the direction of the motion).

✓ *Checkpoint 7.10: Answer the following based on how we use the word acceleration in physics:*
(a) A car faces northward at a stoplight. When the traffic light turns green (or, for some people, when the light turns yellow), the car speeds up. Is this an acceleration? If so, in what direction? If not, why not?
(b) A car is traveling northward when it encounters a stoplight. When the light turns yellow, cars are supposed to slow down. Is this an acceleration? If so, in what direction? If not, why not?

7.6 Force and motion equation

How do forces fit in with all of this?

The law of force and motion hasn't changed. To speed up, the net force on the object must be in the direction of the object's motion. To slow down, the net force on the object must be oopposite the direction of the object's motion.

However, notice the similarity with what we noted about acceleration and the direction of the object's motion, namely that an object speeding up is accelerating in the direction of motion, and an object slowing slowing down is accelerating opposite the direction of motion.

In other words, the *acceleration* direction is always in the direction of the net force, even though the object's *velocity* direction (i.e., the direction of motion) need notbe in the direction of the net force.

Indeed, as alluded to in the chapter introduction, the law of force and motion tells us how the motion is *changing* rather than what the motion *is*. Stated another way, the law of force and motion tells us the *acceleration* not the *velocity*. After all, as long as a force is acting on the object, the object's motion continues to change, meaning that the object is accelerating.

❢ As long as a non-zero net force acts on the object, the object continues to accelerate and it's motion continues to change

After all, according to the law of force and motion, the object's velocity continues to change as the net force acts on the object. If the net force is small, the object's velocity changes slowly (small acceleration). If the net force is large, the object's velocity changes quickly (large acceleration). Thus, we can get a sense of how great the net force is by the acceleration.

Mathematically, we can express this idea by combining the definition of acceleration with the force and motion equation. In particular, we can take the force and motion equation,

$$\Delta\vec{v} = \frac{\vec{F}_{\text{net}}\Delta t}{m}$$

and divide both sides by Δt to get the following:

$$\frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{F}_{\text{net}}}{m}$$

The left side is equivalent to the acceleration. Replacing the left side with the acceleration, the new expression is as follows:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} \tag{7.3}$$

This expression is called a **derived relationship** because it was obtained by rearranging, manipulating or combining one or more other relationships. Indeed, the new equation doesn't introduce any new ideas that aren't already embedded in the two relationships used to obtain it (i.e., the definition of acceleration and the force and motion equation).

Equation 7.3 doesn't need to be memorized, since we can always obtain it by combining the force and motion equation and the acceleration definition, but it can be very useful, which is why many people memorize it rather than deriving it each time they need it.[xiii]

WHAT IS THE ADVANTAGE OF USING THE COMBINED EXPRESSION?

The advantage is that it tells what the object is doing, in terms of the acceleration, *while* the net force is acting. These two things occur simultaneously – as the net force is applied, the object accelerates.

So, if the net force is zero ($\vec{F}_{\text{net}} = 0$) then the object is not accelerating ($\vec{a} = 0$). Unlike the acceleration, the velocity can have *any* value – it just isn't changing (if the net force is zero). And, if the net force is not zero ($\vec{F}_{\text{net}} \neq 0$) then the object must be accelerating ($\vec{a} \neq 0$).

❢ When the net force exerted upon an object is not zero, the object will undergo an acceleration with magnitude proportional to the magnitude of the net force exerted upon it.

Of course, we already know about objects in equilibrium (balanced forces) and that in such conditions an object at rest, will stay at rest, and an object in motion will remain with the same velocity.[xiv] Conversely, we already know that while the net force is *not* zero, the motion much be *changing*. What the combined expression allows us to do is relate the net force with the object's acceleration while the net force is acting.

In addition, and this turns out to be very useful, both the net force and the acceleration have a direction, and that direction must be the same for each.

WHY DO THE DIRECTIONS HAVE TO BE THE SAME FOR EACH?

[xiii] If you are struggling with distinguishing between acceleration and velocity, it is probably better to use the two separate expressions.

[xiv] In section 1.6 this idea was referred to as the law of inertia.

This is just another way of saying what we've already known to be true. For example, for objects that are speeding up, the law of force and motion tells us the force imbalance is in the direction of motion. Meanwhile, the definition of acceleration tells us that the acceleration must likewise be in the direction of motion if the object is speeding up.

Conversely, for objects that are slowing down, the law of force and motion tells us the force imbalance is opposite the direction of motion. And, by the definition of acceleration, the acceleration must likewise be opposite the direction of motion if the object is slowing down.

❢ The direction of an object's acceleration is the same as the direction of the net force exerted upon it.

✍ Although the acceleration and net force are always in the same direction, neither one needs to be in the same direction as the *velocity*. All three are in the same direction only when the object is speeding up.

✓ *Checkpoint 7.11: A car is moving at 10 m/s northward. Fifteen seconds later, the car is moving at 5 m/s northward.*
(a) What is the direction of the car's acceleration?
(b) In what direction is the net force acting on the object?

Summary

This chapter introduced the definition of acceleration and described what it means and how it differs from velocity. The main points of this chapter are as follows:

- In physics, the acceleration is defined as the rate the velocity is changing ($\Delta\vec{v}/\Delta t$) and thus *any* change in velocity means the object is accelerating.
  - An acceleration in the *same* direction as the object's motion means the object is speeding up.
  - An acceleration *opposite* the direction of motion means the object is slowing down.
- The units of acceleration are different than the units of velocity because acceleration indicates how quickly the velocity is changing (rather than the velocity itself).
- When the net force exerted upon an object is not zero, the object will undergo an acceleration that is in the same direction as the net force

exerted upon it and with a magnitude proportional to the magnitude of the net force exerted upon it.
- The SI unit for acceleration is m/s^2.
- As long as a non-zero net force acts on the object, the object continues to accelerate and it's motion continues to change

Frequently Asked Questions

Can an object's acceleration ever be in a direction that is different than the direction of the net force exerted on the object?

No. The direction of an object's acceleration is the direction that the velocity is changing, not the direction of the velocity itself.

According to the law of force and motion, an object's acceleration is always in the direction of the net force (i.e., force imbalance) but, as we know, the net force need not be in the direction of motion. For an object slowing down, the net force must be opposite the motion.

What does it mean for the acceleration to have a direction?

The direction of the acceleration, relative to the velocity, tells us if the object is speeding up or slowing down.

Shouldn't moving objects be accelerating in their direction of motion?

To *start* moving in a particular direction, the object needs to accelerate in that direction. However, once moving, the object isn't necessarily speeding up. Only if it is speeding up will it be accelerating in the direction of motion. If it is slowing down, we say it is accelerating in the opposite direction.

Can the magnitude of acceleration decrease while the speed increases?

Yes. This happens when an object continues to speed up but at a slower and slower rate.

Can the acceleration of an object have a direction that is perpendicular to its motion?

Yes. When the acceleration of an object is directed perpendicular to its motion the object is changing directions without speeding up or slowing down (e.g., going in a circle). We'll consider this situation in part E.

Terminology introduced

Acceleration	Definition	Position
Deceleration	Derived relationship	Time-lapse

Additional problems

Problem 7.1: If an object is moving with a constant acceleration, does that mean it is moving with a constant velocity? If so, why? If not, why not?

Problem 7.2: (a) If an object is "constantly speeding up" does that mean it has a "constant speed"? (b) If an object is "constantly accelerating" does that mean it has a "constant acceleration"?

Problem 7.3: A car is driving eastward and slows down. While it is slowing down, in which direction is the car's acceleration? Explain your reasoning.

Problem 7.4: Derive equation 7.3 ($\vec{a} = \vec{F}_{\rm net}/m$) from our previous version of the law of force and motion ($\Delta\vec{v} = \vec{F}_{\rm net}\Delta t/m$).

Problem 7.5: For each of the following, describe a situation, if possible, where the motion has the property given. If it is not possible, explain why.
(a) An acceleration in the same direction as the net force on the object.
(b) An acceleration opposite the direction of the net force on the object.
(c) An acceleration in the same direction as the object's velocity.
(d) An acceleration opposite the direction of the object's velocity.

Problem 7.6: An object is observed to accelerate from 10 m/s southward to 25 m/s southward in a time of 5 seconds, What is the object's acceleration during this time? Show how you obtained your units of acceleration as well as the magnitude and direction.

Problem 7.7: (a) Estimate your average acceleration while entering a highway (speeding up to highway speed) by assuming an initial speed of zero (at rest), a final speed of 50 mph (highway speed), and an approximate time for how

long it takes you to speed up to 50 mph (make a reasonable guess).
(b) In checkpoint 7.6, you predicted whether your acceleration was greater or less than the 0-60 acceleration of the 2022 Tesla Model S Plaid. Is your prediction supported by your answer in (a)? If not, which do you think is closer to the truth – your estimate in (a) or your prediction in checkpoint 7.6? Why?

Problem 7.8: My shoe is sliding on a frictionless surface. I then apply a force of 2 N westward for 0.8 s, during which time it accelerates with a magnitude of 5 m/s^2. What is the mass of my shoe? Can you say which way it is moving? If so, which? If not, why not?

Problem 7.9: A 12-kg block is at rest on a horizontal frictionless surface. Three forces are acting on the block: (1) 10 N eastward, (2) 30 N westward and (3) 20 N eastward. Suddenly the 30 N force is switched from west to east (still 30 N). This is then maintained for 2 s. What is the block's acceleration during the 2 s and what is the block's velocity at the end of the 2 s?

8. Distance

Puzzle #8: Suppose you are driving along the highway at the speed limit of 55 mph when you suddenly see something in the middle of the road. The U. S. Department of Transportation states[i] that the average driver will travel 275 ft before coming to a complete stop. How is that determined?

Introduction

Since velocity has SI units of meters per second, and meters represents a distance, it probably isn't surprising that we can use an object's velocity, along with the definition of velocity, to predict how far it travels. To do this, however, you must use the *average* velocity, not the *initial* velocity, *final* velocity or *change* in velocity. This chapter explains what is meant by the *average* velocity, how to figure out what the average velocity value is, and how to use that average velocity value to determine how far the object has traveled.

8.1 The meaning of average

Let's now explore why the definitions of velocity and acceleration have the "avg" subscript. The subscript stands for "average". The reason for that subscript is because these are definitions of the *average* velocity and *average* acceleration.[ii]

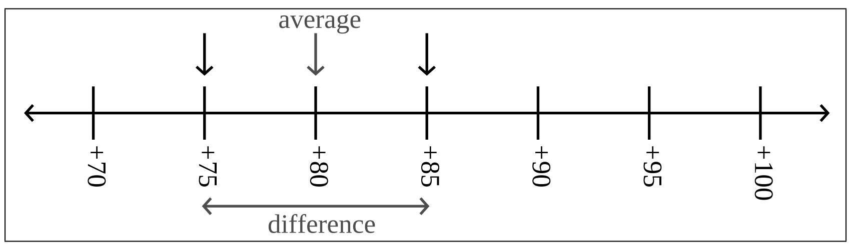
[i]Source: https://safety.fhwa.dot.gov/speedmgt/ref_mats/fhwasa10001/

[ii]For the same reason, the net force in the force and motion equation should also have the "avg" subscript, as it is actually the average net force.

What does the average mean?

The supplemental readings provide details about what an average is. However, there are some basics you should be comfortable with. For example, you probably know that your average grade is between your lowest grade and your highest grade. In particular, suppose you get an A in one course and a C in another course. Your average grade would be a B.

Similarly, if the temperature one day is 85°F and the temperature the next day is 75°F then the average temperature would be 80°F. Notice that the average in each case is equal to what we call the **midrange** value, exactly halfway between the two values. This is easier to see when we mark the three values on a number line:



With the number line, one can see that the 80 value is exactly midway between the 75 and 85 values.

Is the average value the same as the difference?

The average does *not* mean the difference. On the number line illustration, the difference between the two numbers is 10, whereas the average is 80.

Is the average always equal to the midrange value?

The average is always between the two numbers but it is only equal to the *midrange* value when the two values have equal weight. For example, most courses are three credits each. So, if the two courses each were three credits, the average grade would be right in the middle of the two grades. However, if one course is four credits and the other is three credits, the average would be closer to the grade in the four-credit class.

As with grades and temperatures, the average speed is always between the lowest speed and the highest speed (during a particular time period) but it is only equal to the midrange speed (right in the middle of the beginning and ending values) if the object is speeding up or slowing down *in a uniform way.*

This is what happens when the acceleration is constant, and we know from part A that the acceleration is constant whenever the net force is constant. Consequently, as long as the net force on the object is constant, we know that the average velocity is equal to the midrange velocity value.

For example, suppose we have an object that starts with a speed of 15 m/s and steadily speeds up, accelerating at a constant rate of 2 m/s^2 (gaining 2 m/s every second) for five seconds. In this case, we are told that the object starts at 15 m/s so that is its initial speed. Since it accelerates at 2 m/s^2, after five seconds it would be going 10 m/s faster than when it started. That means its final speed is 25 m/s (add 10 m/s to the initial 5 m/s).

The average speed in this case is midway between the 15 m/s and the 25 m/s. That would be 20 m/s.

How do we find the average mathematically?

Many times you can figure out the average simply by drawing a number line and indicating on the number line where the initial and final values are. You can then estimate what the midrange value is by selecting a position on the number line halfway between the initial and final values. However, mathematically, you can find the average by adding the initial and final values together and then dividing *that sum* by two.

✍ A common error is to ignore the initial or final value and only divide one of the values by two. That works when one of the values is zero (like when starting from rest). Otherwise, it won't work.

✓ *Checkpoint 8.1: Suppose an object is slowing down. Which would have the largest magnitude: the initial velocity, the final velocity or the average velocity? Give an example in support of your answer.*

Notice that the average *velocity* is obtained by taking the midrange between the initial and final *velocities*. Similarly, the average *acceleration* would be obtained by taking the midrange between the initial and final *accelerations*. However, in this case we don't have to do that for the acceleration – it is already provided as 2 m/s^2. In addition, the acceleration is constant, so its initial and final values, as well as its average value, are all the same – 2 m/s^2.

Be careful! By definition, the average acceleration is still $\Delta\vec{v}/\Delta t$, so whereas the average *velocity* is equal to the *average* of the initial and final velocities, the average *acceleration* is equal to the *difference* between the initial and final velocities divided by the elapsed time.

✓ *Checkpoint 8.2: Suppose an object starts with a velocity equal to 10 m/s northward and speeds up to 16 m/s in three seconds. If the acceleration is constant, what is the object's average velocity? What is the object's average acceleration?*

8.2 Constant velocity (zero acceleration)

Now that we know what is meant by the average value, and how it differs from the initial and final values, we'll apply this to various situations. In this section, we'll examine situations where the velocity is constant. This means that the velocity has the *same value* during the entire time period. And, since it has the same value during the entire time period, the *average* value likewise has that same value.

Furthermore, since the direction is constant for zero acceleration cases, we don't need to know the direction – and we can use the definition of average velocity with the speed (which is the magnitude of the velocity value).

For example, suppose you are traveling at 55 mph (24.6 m/s) and suddenly see something in the middle of the road (as in the puzzle). According to the U. S. Department of Transportation[iii], a typical **reaction time** is 1.1 seconds[iv], which is the time between when you recognize the need to stop and when you actually apply the brakes. How far do you travel in that time?

Since the speed is constant during this time, we can use 24.6 m/s as the value of the average speed. From the definition of velocity, a speed of 24.6 m/s means that it travels 24.6 meters every second. Since 1.1 seconds is a little more than one second, then, you must travel a little bit more than 24.6 meters. To solve, multiply the velocity by the time to get 27 meters.

[iii] *Speed Concepts: Information Guide*, Sep 2009, FHWA-SA-10-001.

[iv] This represents the expected reaction time for an average driver. It actually varies from driver to driver. A more conservative estimate is 2.5 s.

✍ Twenty-seven meters is almost 90 feet. That is the distance you travel before you even hit the brakes.

If you prefer, you can instead just plug the numerical values into the definition of velocity and solve for the unknown distance:

$$\begin{aligned} \vec{v}_{\text{avg}} &= \frac{\Delta\vec{s}}{\Delta t} \\ (24.6 \text{ m/s}) &= \frac{\Delta\vec{s}}{(1.1 \text{ s})} \end{aligned}$$

Multiplying both sides by 1.1 will get $\Delta\vec{s}$ by itself on the right side of the equation, allowing you to solve for it.

✓ *Checkpoint 8.3: A car maintains a constant speed of 12 m/s for 5 s. How far does the car travel during the 5 s?*

Now let's consider the second scenario, where we want to determine how long it takes for an object to travel a certain distance. For example, suppose a car travels at a speed of 40 mph. How long does it take for the car to travel 10 miles?[v]

Since it travels at 40 mph, that means it will take one hour to travel 40 miles. To travel 10 miles, it will take less than an hour. Indeed, since 10 miles is one-quarter of 40 miles (the distance traveled in one hour), the time needed is one-quarter of an hour, or 15 minutes.

CAN WE INSTEAD PLUG THE NUMBERS INTO THE DEFINITION OF VELOCITY EQUATION?

Yes, but you have to be careful. The units aren't SI, so you'll either have to convert everything into SI (in which case it will give you the time in seconds) or you have to keep track of the units.

For example, plugging the values into the definition of velocity (and ignoring the direction), we get the following (using mi/hr for mph):

$$\begin{aligned} \vec{v}_{\text{avg}} &= \frac{\Delta\vec{s}}{\Delta t} \\ (40 \text{ mi/hr}) &= \frac{(10 \text{ mi})}{\Delta t} \end{aligned}$$

[v]mph stands for miles per hour. See page 57.

Multiplying both sides by Δt and dividing both sides by 40 mi/hr leaves Δt by itself on the left side of the equation, allowing you to solve for the time. Numerically the answer is 0.25 (10 divided by 40), but what about the units?

If you ignore the units, you may be tempted to think the time is in seconds, since that is what time is usually expressed in. However, the unit is not seconds in this case because we didn't use SI units for the speed and the distance. By including the units when plugging in the values, you can probably tell from how it is written that the missing unit on the right side is "hr" (hours). That tells us that the time is 0.25 hours. This is why it is a good idea to includes the units along with the numerical values when using equations.

✓ *Checkpoint 8.4: (a) A car travels at an speed of 20 m/s. How long does it take for the car to travel 30 m?*
(b) A car travels at an speed of 20 feet per minute. How long does it take for the car to travel 30 feet?

8.3 Speeding up (constant acceleration)

When an object is speeding up, the velocity is not constant, so you can't use the initial velocity or final velocity. However, if the acceleration is *constant* then the average velocity is the midrange value (between the initial and final velocity values). The key is to first determine the average velocity *before using the equation*. Once you do that, the process is the same as before.

✍ As with zero acceleration motion, we'll assume the direction is constant for the "speeding up" case. That way, we don't need to know the direction and we can use the definition of average velocity with the speed.

Consider the case of an object that starts at rest and accelerates at a rate of 1 m/s^2, so that it is moving at 1 m/s one second after starting, 2 m/s at two seconds, and so on. By four seconds, it is moving at 4 m/s.

Suppose we want to know how far it moved during the four seconds. To do this, we need to know the *average* speed, not the initial speed (zero) or final speed (4 m/s). In this case, with it accelerating at a constant rate, the *average* speed is the midrange value, which is 2 m/s in this case.

As with constant velocity motion, we can now use the definition of average velocity, using 2 m/s. At that speed, the object would travel 8 m in 4 s (multiply the speed, 2 m/s, by the time). Algebraically, the process is as follows:

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{s}}{\Delta t}$$
$$(2\text{ m/s}) = \frac{\Delta\vec{s}}{(4\text{ s})}$$

Multiplying both sides by 4 will get $\Delta\vec{s}$ by itself on the right side of the equation, allowing you to solve for it.

Notice that it is not a one-step process. While we must use the definition of velocity relationship to find the distance, we can't start there – we must first figure out the average velocity. Furthermore, if we are given the initial velocity and the acceleration instead of the final velocity, we have to first use the definition of acceleration to get the final velocity. Only then can we get the average velocity and then, after that, get the distance. It is actually a three-step process.

✍ A common mistake is to skip the step where you determine the average velocity and instead just use the initial or final values. Usually, it is easy to notice the mistake because the answer won't make sense. Either the distance will be too small or it will be too large. Remember that if the object is speeding up or slowing down then that means the object is not traveling at the initial speed or the final speed the entire time.

✓ *Checkpoint 8.5: Suppose an object is accelerating at a rate of 2 m/s^2. If it starts with a speed of 10 m/s, how far does it travel in the next three seconds?*

8.4 Using the law of force and motion

If the net force is provided instead of the acceleration, the process actually requires four steps, since we must first figure out the acceleration.

For example, suppose we have a 2-kg object that experiences a net force of 10 N eastward. If the object starts with a velocity of 5 m/s eastward, how far does it travel in 4 seconds?

We know that the definition of velocity can give us the distance, but we need to now the *average* velocity, not the initial velocity. And to find the average velocity, we need to know the initial and final velocities. And to find the final velocity, we need to know the acceleration, which we first have to get from the force and motion equation.

So, from the force and motion equation (see page 7.3),

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$
$$\vec{a} = \frac{(10\ \text{N})}{(2\ \text{kg})}$$

This gives an acceleration of 5 m/s^2. Then, from the definition of acceleration (page 105),

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$
$$(5\ \text{m/s}^2) = \frac{\Delta\vec{v}}{(4\ \text{s})}$$

This gives a change in velocity of 20 m/s, and since the net force is in the direction of motion we know that the object must speed up by 20 m/s. Since it started at 5 m/s, it must end up at 25 m/s.

To figure out the distance, we need the average velocity, which is midway between the initial velocity of 5 m/s (eastward) and 25 m/s (eastward). The average is 15 m/s (eastward).

Finally we use the definition of velocity (page 104) to find out how far it traveled in the 4 seconds:

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{s}}{\Delta t}$$
$$(15\ \text{m/s}) = \frac{\Delta\vec{s}}{(4\ \text{s})}$$

This gives a distance of 60 m.

✍ Notice how this distance (60 m) is greater than what we'd get if we just used the object's initial speed of 5 m/s for 4 seconds (which gives 20 m), and it is less than what we'd get if we just used the object's final speed of 25 m/s for 4 seconds (which gives 100 m).

Can't we use the force and motion equation to get the velocity without getting the acceleration first?

Remember that the force and motion equation tells us how the velocity is *changing*. It does not tell us what the *average* velocity. Indeed, neither one of the following versions will give you the average velocity or the final velocity:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} \qquad\qquad \Delta\vec{v} = \frac{\vec{F}_{\text{net}}\Delta t}{m}$$

They only tell you how the velocity is *changing*.

✓ *Checkpoint 8.6: Suppose a 4-kg object is experiencing a net force of 20 N northward. If it starts with a velocity of 10 m/s northward, how far does it travel in two seconds?*

Notice that there are *two* ways of determining the average velocity. If we know $\Delta\vec{s}$ and Δt then we can get the average velocity by using the definition of velocity. However, we can also get the average velocity if the acceleration is constant and we know the initial and final velocities.

Both ways are valid. Which you use depends on the situation.

This is why you should always think about the physics before using any equations. When we only had the force and motion equation, it was easy to just reach for that equation for each problem. However, even then, it was important to keep in mind that the mathematical equations *represent* the laws and definitions – they don't *replace* them.

Also notice that sometimes you need to apply multiple ideas to a problem, and that may require multiple equations. For this reason, you need to avoid the temptation of focusing on just the letters and numbers in the equations. When you focus on just the letters and numbers, you leave yourself with only one path, which is to hunt for an equation that has the quantities presented in the problem. Not only does this avoid using any physics (and thus miss the point of doing the physics in the first place) but sometimes the solution isn't possible with just a single equation.

What if I find an equation that allows me to solve the problem in a single step?

❢ Avoid the use of "equation hunting."

Only if we were to encounter the same type of problem over and over again would it useful to use a single equation that combines all of the steps into one. Otherwise, doing so goes against the purpose of you learning all of this. After all, you are unlikely to ever be asked, outside of this course, to predict how far an object travels for a given acceleration or net force. That is not why you are taking the class. Instead, you are learning how to apply a small set of very powerful ideas to a variety of situations. To fully assess whether you understand these ideas, you need to use them, not find a shortcut through which you can avoid using them.

✓ *Checkpoint 8.7: (a) Suppose an object takes 3 hours to move from a position 2 mi north of the PA/NY border to a position 5 mi north of the border. To find the average velocity, do you take the difference between the two and five or do you take the average of two and five?*
(b) Suppose an object takes 3 hours to accelerate uniformly from 2 mph northward to 5 mph northward. To find the average velocity, do you take the difference between the two values or do you take the average of two values?

8.5 Slowing down (constant acceleration)

As we know, acceleration could be a speeding up or a slowing down. The way we approach a problem should be the same either way.

✍ As before, we'll assume the direction is constant so we don't need to know the direction and we can use the definition of average velocity with the speed.

For example, consider the scenario described in the puzzle. The U. S. Department of Transportation states that, when initially traveling at 55 mph (24.6 m/s) the average driver will travel 275 ft before coming to a complete stop. We already know that they assume the driver will take 1.1 s to react and that means the car will travel 90 feet before even hitting the brake.

At that point, the U. S. Department of Transportation assumes the car will decelerate at 5.4 m/s^2.[vi]

Because the car is slowing down, we don't want to use the initial speed (24.6 m/s) or the final speed (zero). The initial speed (24.6 m/s) would give us too far a distance, since we don't travel at that speed the entire time, and the final speed (zero) would give us too small a distance.

Instead, we need the *average* speed. Assuming the slowing is done in a uniform way (constant acceleration) then the average speed is midway between zero and 24.6 m/s, which is 12.3 m/s.

Now that we know the average speed, we can use the definition of velocity. At 12.3 meters every second, we'd travel (12.3×4.6) meters in 4.6 s.[vii] That gives us 56.58 m or about 185 feet. Along with the 90 feet traveled before the brakes were used, the car travels a total distance of 275 feet, which is how the U. S. Department of Transportation got their value.

✓ *Checkpoint 8.8: A 2-kg box is on a horizontal, frictionless surface with an initial velocity of 4 m/s leftward. A constant net force of 30 N rightward is applied on the box for 0.2 seconds. How far does the box travel during the 0.2 seconds?*

Summary

This chapter introduced the definition of average velocity and how that is different from the object's change in velocity.

The main points of this chapter are as follows:

- In a time-lapse picture, the time interval between each "snapshot" needs to be the same in order to infer the velocity.
- Velocity is defined as the change in position divided by the time it took to undergo that change.
- Avoid the use of "equation hunting."

[vi] This is equivalent to 17.71 ft/sec^2 and represents the expected deceleration for an average car and driver. The actual deceleration depends on several factors, including the tire condition and the mass of the car. A more conservative estimate is 11.2 ft/sec^2.

[vii] Multiplying 12.3 m/s by 4.6 s gives how far we move in 4.6 s when moving at 12.3 m/s.

Frequently Asked Questions

WOULDN'T THE AVERAGE VELOCITY ALWAYS BE EQUAL TO MIDWAY BETWEEN THE INITIAL AND FINAL VELOCITIES?

No. That is only true if the transition from initial to final velocity is done in a uniform manner (constant acceleration).

I'M STARTING TO GET OVERWHELMED WITH ALL OF THE EQUATIONS. HOW MANY EQUATIONS WILL WE NEED TO MEMORIZE FOR PHYSICS?

We've identified three equations[viii] so far. These three equations form the basis of the whole book, so while we might run into some more equations, most will just be versions of these three.

That may still seem like a lot, but you need to keep in mind a couple of things.

First of all, you can always look up the equation you need so there is no need to memorize them. Second, some equations can be obtained by combining others, so they don't represent new ideas. Third, all of the equations should "make sense". By that I mean that they represent ideas that make physical sense, so even if you didn't have the equation you could still make reasonable guesses as to what is likely to happen.

WHAT'S WRONG WITH EQUATION HUNTING?

The problem with equation hunting is that it rarely works in the real world, and if/when it does work, it is inefficient and risky. Often there are just too many possible equations or there are subtle differences between the quantities provided in the problem and the quantities represented in the equation.

Even if you've successfully used equation hunting in the past, there are two reasons not to do so here. First, you won't be able to assess your understanding of the underlying physical principles since the equation hunting technique doesn't require any knowledge of the underlying physical principles. Second, equation hunting generally only works for ivory-tower questions – those idealized problems only found in introductory textbooks.

[viii] The three equations are the force and motion equation and the definitions of acceleration and velocity.

Terminology introduced

Average	Position
Distance	Reaction time
Midrange	Time-lapse

Problems

Problem 8.1: In the men's 100m track and field event during the 2012 London Olympics, the difference between a medal and no medal was only 0.01 seconds.[ix] Nowadays they use speakers behind each runner to transmit the sound to start the race but in the past they used a starting gun place to one side of the track. The problem with the gun is that it takes time for the sound to reach each runner and runners farther away may be at a disadvantage. For eight runners on a track, the runner farthest from the gun is about 7.5 meters further from the gun than the runner closest to the gun. The speed of sound in air is about 344 m/s. Does the closest runner hear the gun more than 0.01 seconds before the furthest runner? If so, that runner will have an advantage.

Problem 8.2: The U. S. Department of Transportation states that the average driver will travel 320 feet (more than half a block) before stopping when driving 60 mph (88 ft/s) on dry pavement, assuming a deceleration rate of 17.71 ft/sec^2 (this is about 5.4 m/s^2). Using the definitions of average velocity and acceleration, show how this value was obtained. You can leave the units in feet, as long as all of the units agree.

[ix] A bronze medal was won by Justin Gatlin with a time of 9.79 s whereas Tyson Gay's time was 9.80 s.

9. Turning Around

Puzzle #9: A ball is thrown against a wall. It hits the wall and bounces back. When it is in contact with the wall, there is definitely a force on the ball due to the wall. Is it also accelerating?

Introduction

The puzzle looks at an object (the ball) that is turning around (changing directions). Investigating such motion forces us to dig deeper into the meaning of acceleration and its relationship with velocity and forces.

9.1 Law of force and motion

Chapter 1 mentioned how drag and friction act to slow down objects. With drag and friction, moving objects will eventually stop because the drag only acts if the object is moving and friction only acts if the object is sliding across a surface.[i] Thus, drag and friction disappear at the moment the object stops and the object remains at rest from that point on (unless some force acts on it).

However, what would happen if the force continued? In particular, suppose *we* apply an opposing force and we continue to apply the same force, even when the object comes to a stop. What happens then?

The answer is that the object then starts to move in the direction of the force, speeding up as it does so.

To illustrate this, consider a shopping cart rolling by itself in a store. To slow the cart, you could exert a force on it that is *opposite* the direction of

[i] We will expand our treatment of friction in chapter 17.

the cart's motion. You can do this either by pulling backwards on the cart (if you were behind it) or pushing it backward (if you were in front of it).

As long as you continue to apply that force, the cart continues to slow, eventually coming to a stop. If you stop pushing it at the moment the cart stops then the cart will remain at rest. However, if you *continue* to push on it, the cart will start moving again, this time in the direction of the force (which is opposite the cart's original direction of motion). And, if the force continues, the cart will speed up in the new direction. After all, at that point, you'd be pushing or pulling the cart forward, and an object speeds up when the force on it is in the *same* direction as the object's motion.

In this way, a force can not only *slow* an object, it can also change its direction.

Regardless, the law of force and motion still applies, since a change in direction is still a change in motion, even if the object ends up with the same speed it had originally but just in the opposite direction.

❢ As long as a force is acting on the object, the object's motion continues to change.

It is important to note that, at the instant the object is switching directions, the object is momentarily at rest but only for an instant. There is still a force acting on it and, as such, it doesn't stay at rest. After all, the object's motion continues to change the entire time a net force is exerted on it.

For example, in most ball games (like kickball, racquetball or baseball), the ball is hit such that it reverses directions. According to the law of force and motion, that happens because the force is applied for a long enough time period that the ball not only slows to a stop but also reverses direction and starts to move back in the other direction.

By the way, I'll refer to this as *turning around* but the object doesn't need to rotate and face the opposite direction. There is simply a change in the motion such that the object is moving opposite its original direction.

✓ *Checkpoint 9.1: Two students, Joe and Moe, are playing catch with a ball. At a particular moment, the ball is moving toward Joe. Joe then catches it and quickly returns it to Moe. For each of the following situations, what is the direction of the force on the ball due to Joe?*
(a) As Joe catches the ball in order to slow it down
(b) As the ball is reversing directions
(c) As Joe speeds up the ball during the act of throwing it toward Moe

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9.2 Acceleration and velocity

As mentioned in chapter 7, acceleration is defined as the rate at which the velocity changes. This means *any* change in velocity, both speeding up and slowing down, corresponds to an acceleration.

That means that turning around is also an acceleration, since velocity includes direction and the direction of motion is changing.

In other words, the acceleration is non-zero even though the velocity, for an instant, is zero. This is because the velocity is still changing – it doesn't remain zero.

This may seem strange but there are lots of situations where we look at the *change* and not the value itself. For example, suppose the temperature is falling rapidly from 2°C to −2°C. At some instant during this transition the temperature has to pass through 0°C, right? At that instant, the temperature is zero but the temperature is still falling. In a similar way, suppose you travel from a position two miles north of the state border and drive at a constant speed of 20 mph to a location two miles south of the state border. At some instant during this transition your position has to be right at the border and, at that time, your position would be zero miles north of the state border yet your velocity would still be 20 mph.

In a similar way to how the warming/cooling rate depends on the *change* in temperature, and velocity depends on the *change* in position, acceleration depends on the *change* in velocity. So, if you start with a velocity of 20 m/s northward then stop, turn around, and end up with a velocity of 20 m/s southward, at some instant you have to be at rest but that doesn't mean you aren't accelerating. As long as you were only at rest for an instant, your acceleration at that moment would not be zero.

HOW CAN YOU BE ACCELERATING IF YOUR FINAL VELOCITY AND INITIAL VELOCITY ARE BOTH THE SAME?

In the example I gave, the initial velocity was 20 m/s *northward* and the final velocity was 20 m/s *southward*. They were <u>not</u> the same – the directions were different! Indeed, the *difference* in those two velocity is 40 m/s, not zero.

Consider, again, the case where the temperature is falling rapidly from 2°C to −2°C. The change in temperature would be 4°C, not zero. Similarly, if you travel from a position two miles north of the state border to a location two miles south of the state border, the change in position would be 4 miles, not zero. In the same way, if your velocity changes from 2 mph northward to 2 mph southward, the change in velocity would be 4 mph, not zero.

What about at the moment in between, when I am stopped?

That depends on whether you stop for longer than an instant. If you are at rest then your acceleration is zero only if you *stay* stopped. If your change in velocity is due to a non-zero net force being exerted on you then, as long as that non-zero net force is present, your velocity must be changing and so you must have a non-zero acceleration, even if your velocity happens to be zero at a particular instant.

✓ *Checkpoint 9.2: Suppose an object starts with a velocity of* 5 m/s *eastward then slows down, turns around, and ends up with a velocity of* 5 m/s *westward. If it takes five seconds to do so, what is its (average) acceleration during the five seconds?*

9.3 Force and motion equation

In chapter 7, a version of the force and motion equation was provided that used acceleration, rather than the change in velocity. That version (equation 7.3) is not only equivalent to the force and motion equation used before but it is particularly useful for situations when an object is turning around.[ii] For example, consider the following problem:

> A 10-kg object is moving northward with an initial speed of 3 m/s when a net force of 20 N southward acts on it for 5 seconds. What is the object doing during the five seconds?

From the law of force and motion, we know that the object is slowing down, since the motion (northward) is opposite the net force (southward). Let's

[ii] Indeed, the version with acceleration is more popular than the version with the change in velocity.

see what equation 7.3 ($\vec{a} = \vec{F}_{\text{net}}/m$) can tell us about how quickly the object slowing down. Plugging in, we have:

$$\vec{a} = \frac{(20 \text{ N})}{(10 \text{ kg})}$$

This tells us that the acceleration is 2 m/s^2 southward (the same direction as the net force).

So far, so good, but what does that mean?

Based on the numerical value, we know that the object must be slowing down at a rate of 2 m/s^2, which corresponds to slowing 2 m/s every second. Over five seconds, it must slow by five times this, which is 10 m/s. Since it starts at 3 m/s, we subtract 10 m/s and end up with -7 m/s.

What does it mean for the object to be moving at -7 m/s?

The negative means that the object is now moving in the *opposite* direction. Since it was originally moving northward, it ends up moving southward by the end of the five seconds. The negative value means that the object slowed down so much it actually came to a stop for an instant and then moved in the opposite direction. In this case, that opposite direction is southward instead of northward.

✍ Notice how the acceleration was southward the entire time but the motion changed from northward to southward. It is similar to how someone can start north of the border and end up south of the border while traveling southward the entire time.

✓ *Checkpoint 9.3: Consider an object initially moving 3 m/s southward that then turns around and moves 2 m/s northward. In what direction is the object's acceleration while it is turning around (from southward to northward)?*

Can we also use the original version of the force and motion equation?

Certainly, but if the object turns around then you'll get a result that suggests the object slows by an amount greater than its initial speed. The following example illustrates what I mean.

> Suppose a net force of 10 N rightward acts for 2 s on an object of mass 5 kg. If the object is initially moving at 2 m/s leftward, what is the object's velocity at the end of the 2 s?

From the law of force and motion, we know that the object must be slowing down (since the net force, rightward, is opposite the leftward motion). To find out how much it slows down, we use the force and motion equation:

$$\begin{aligned}\Delta\vec{v} &= \frac{\vec{F}_{\text{net}}\Delta t}{m}\\ &= \frac{(10\text{ N})(2\text{ s})}{(5\text{ kg})}\\ &= 4\text{ m/s}\end{aligned}$$

This means that the velocity must slow down by 4 m/s during the 2 seconds.

Notice how the objects slows by an amount (4 m/s) greater than its initial speed (2 m/s). Subtracting 4 m/s from the initial speed of 2 m/s, we get −2 m/s, which means that it has turned around and is now moving rightward instead of leftward.

Also notice how the object's speed at the end is the same as the object's speed at the beginning, but the object's speed didn't stay the same *throughout* the motion. The speed was constantly undergoing a change. It just happened to end up with the same speed as it started with because it changed directions midway through the motion.

❢ If the force and motion equation indicates that an object slows down by more than its initial speed, that means it has turned around.

❢ If the force and motion equation tells us the object slows down by more than the initial speed, it may be that the object changed directions.

✓ *Checkpoint 9.4: Suppose a net force of 5 N northward acts for 3 seconds on an object of mass 2 kg. Determine the object's velocity at the end of the 3 seconds for each of the following cases.*
(a) If the object is initially moving at 5 m/s northward.
(b) If the object is initially moving at 10 m/s southward.
(c) If the object is initially moving at 5 m/s southward.

9.4 Collisions

The same approach needs to be used when dealing with collisions, as in chapter 6, as the equations can indicate that the object slows down by an amount greater than its initial speed. In such cases, the object is bouncing backwards off the other object. This is particularly likely when a lighter object is moving and collides with a heavier object. For example, consider the following scenario:

> A blue 2-kg cart rolls at 3 m/s *eastward*. It then collides with a red 4-kg cart at rest. After the collision, the blue 2-kg cart is observed to be rolling at 1 m/s *westward*. How fast is the red 4-kg cart moving after the collision?

Before doing any math, it helps to first get a sense of what is going on physically. The blue cart collides with the red cart and, due to that collision, experiences a force that sends it back the opposite way. There must also be a force on the red cart as well, due to the same interaction, and that force must be equal in magnitude to the force on the blue cart but, being heavier, the red cart will not experience as great a change in motion.

Let's now go through the three steps discussed in chapter 6 and repeat this analysis using some numbers.

1. **Use the force and motion equation to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the blue cart**
   The change in velocity in this case is from 3 m/s eastward to 1 m/s westward, which is a change of 4 m/s, not 2 m/s. The reason it is 4 m/s is because it didn't just slow down by 2 m/s. Instead, it slowed down by 3 m/s (to a stop) and then moved in the opposite direction at 1 m/s. That is a change of 4 m/s. Multiply 4 m/s westward by the mass (2 kg) to get that the magnitude of $m\Delta\vec{v}$ is 8 kg·m/s. From the force and motion equation, that must equal $\vec{F}_{\text{net}}\Delta t$ on the blue cart.
2. **Use the law of interactions to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the red cart**
   The law of interactions states that the force on the red cart (due to the two-cart interaction) must be equal in magnitude (and opposite in direction) to the force on the blue cart (due to the same two-cart interaction). In addition, the time must be the same for each force, since they both act only when the interaction is present. That means that the magnitude of $\vec{F}_{\text{net}}\Delta t$ on the red cart must also be 8 kg·m/s.
3. **Use the force and motion equation to determine the red cart's change in velocity**

Now that we know $\vec{F}_{\text{net}}\Delta t$ on the red cart, we can determine $m\Delta\vec{v}$ by again using the force and motion equation:

$$m\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t$$

We know from the previous step that the right side has a magnitude equal to 8 kg·m/s. From the force and motion equation, that means that the left side must likewise have a magnitude equal to 8 kg·m/s. Since $m\Delta\vec{v}$ for the red cart is 8 kg·m/s, we can divide that by the red cart's mass (4 kg) to get that it must have sped up by 2 m/s. Since the red cart starts at rest, its final velocity must be 2 m/s (eastward).

Can both objects experience a change in direction?

Yes. For example, consider the situation from before, with a 2-kg cart running into a red 4-kg cart but let's now have the red 4-kg cart initially rolling toward the blue 2-kg cart at 1.5 m/s westward. In this situation, let's suppose the blue 2-kg cart again bounces off the red 4-kg cart but, given the fact that the red 4-kg cart was originally rolling westward, the blue 2-kg cart bounces off with a greater speed this time and so its velocity after the collision is 3 m/s westward.

To figure out the red 4-kg cart's velocity *after* the collision, we use the same approach as before:

1. **Use the force and motion equation to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the red cart**
   The red cart has changed from 3 m/s eastward to 3 m/s westward, which is a change of 6 m/s (westward). The reason it is 6 m/s is because it slowed down by 3 m/s (to a stop) and then sped up to 3 m/s in the *opposite* direction. Multiply 6 m/s by the mass (2 kg) to get 12 kg·m/s. From the force and motion equation, the magnitude of $m\Delta\vec{v}$ must equal the magnitude of $\vec{F}_{\text{net}}\Delta t$, which means that $\vec{F}_{\text{net}}\Delta t$ on the blue cart must likewise have a magnitude of 12 kg·m/s.
2. **Use the law of interactions to determine the value of $\vec{F}_{\text{net}}\Delta t$ on the red cart**
   The law of interactions states that the force on the red cart (due to the two-cart interaction) must be equal in magnitude (and opposite in direction) to the force on the blue cart (due to the same two-cart interaction). In addition, the time must be the same for each force,

since they both act only when the interaction is present. That means that the magnitude of $\vec{F}_{\text{net}}\Delta t$ on the red cart must also be 12 kg·m/s.

3. **Use the force and motion equation to determine the red cart's change in velocity**
   Now that we know $\vec{F}_{\text{net}}\Delta t$ on the red cart, we can determine $m\Delta\vec{v}$ by again using the force and motion equation:

   $$m\Delta\vec{v} = \vec{F}_{\text{net}}\Delta t$$

   We know from the previous step that the right side has a magnitude equal to 12 kg·m/s. From the force and motion equation, that means that the left side must likewise have a magnitude equal to 12 kg·m/s. Since $m\Delta\vec{v}$ for the red cart is 12 kg·m/s, we can divide that by the red cart's mass (4 kg) to get that it must have slowed down by 3 m/s (we know it slowed down since it was a head-on collision). The red cart didn't start at rest, though, so its final velocity is *not* 3 m/s. That is just the *change*. Since it *started* with a velocity of 1.5 m/s westward, subtracting 3 m/s from 1.5 m/s that gives 1.5 m/s in the *opposite* direction (eastward). That means the red cart's final velocity is 1.5 m/s eastward.

Notice how everything is the same as before (when the red cart was initially at rest) except for at the very end, where we have to consider the red cart's initial velocity. The law of force and motion, after all, only tells us about *changes*. If an object is initially moving, we have to take that into account when figuring out the *final* velocity.

✓ *Checkpoint 9.5: A 1500-kg car is moving at 20 m/s eastward when it collides with a 500-kg car that was moving at 10 m/s westward. Determine the velocity of the 500-kg car after the collision for the following situations.*
(a) After the collision, the 1500-kg car is observed to be moving at 5 m/s eastward.
(b) After the collision, the 1500-kg car is observed to be moving at 12.5 m/s eastward.

9.5 Displacement

Once you know how fast the object is moving at the end of the time period, you can use that value along with the speed at the beginning of the time period to determine how far the object went, as in chapter 8. However, if the object turns around during the time period, you need to keep track of the initial and final velocity *directions*.

For example, suppose the object starts with a velocity of 5 m/s northward and experiences an acceleration of 1 m/s^2 southward for ten seconds. How far has it traveled in the ten seconds?

Since the acceleration is opposite the motion, this corresponds to a deceleration (at least initially), slowing down by 1 m/s every second. After ten seconds, it would slow down by 10 m/s (multiply acceleration by the time).

Since it started at 5 m/s, subtracting 10 m/s makes the speed $-5\ m/s$. The negative tells us that it turned around, so it is now moving southward, not northward.

Now that we have the initial velocity (5 m/s northward) and final velocity (5 m/s southward), we need the *average* velocity. But what is it?

It turns out that the average of 5 m/s northward and 5 m/s southward is zero!

Why is it zero?

The reason it is zero (and not, say, 5 m/s) is because the directions are opposite. It is equivalent to asking for the average of +5°F and −5°F, which is 0°F, or the average water level if it 5 m above sea-level one day and 5 m below sea-level the next day, in which case the average water level would be right at sea-level.

✍ Note that the average *velocity* is zero in this case but the average *speed* would not be zero, since speed doesn't take into account the direction of motion. The average velocity does not equal the average speed for objects that turn around during the time interval.

How can the average velocity be zero? Does that mean it didn't move?

The object certainly moved. However, the zero average velocity means it ended up at the same place it started. Just as an object at rest remains in the same place, an object with an average velocity of zero will end up in the same location where it started.

To see why, let's break up the problem into two 5-second intervals. Since it starts at 5 m/s, this means it slows to a stop in 5 s. During those five seconds, with an average speed of 2.5 m/s, it must have traveled 12.5 m (multiply 2.5 m/s by 5 s). This motion is northward, since the object is moving northward for the entire 5 seconds.

However, the problem states that the acceleration continues for five more seconds (for a total of ten seconds). That means the object then starts to speed up, in the southward direction now, gaining 1 m/s every second. After five additional seconds, the object would be going 5 m/s at the end of the ten seconds. During the last five seconds, with an average speed of 2.5 m/s, it must have traveled 12.5 m (multiply 2.5 m/s by 5 s). This motion is southward, since the object is now moving southward.

So how far has it traveled?

The answer actually depends on what you mean by "how far has it traveled".

If you mean "how far is it now, after the 10 seconds, from where it started", the answer is zero. After all, it has ended up in the same place it started. It traveled 12.5 m northward and then 12.5 m southward.

On the other hand, if you mean "how far would an odometer measure" then the answer is 25 m, since we'd include the distance both ways.

To distinguish between the odometer distance and how far the object ends up from the initial position, we call the former the "total distance traveled" and the latter the **displacement**.[iii] Displacement is similar to **distance**, except that displacement has a *direction*, which is the same direction as the object's average velocity.

For example, if I travel 20 m northward, the distance is "20 m" and the direction is "northward." The displacement is "20 m northward." For an object moving at a velocity of 20 meters every second *northward*, that means the object undergoes a displacement of 20 meters *northward* every second.

❢ An object's displacement is not only how far it moves but also the direction it has moved.

[iii] The word *displacement* has to do with "moving" stuff, just as "displacement of water" has to do with what happens to the water when you drop an object in a container of water.

✍ The difference between displacement and distance is just like the difference between velocity and speed. Velocity includes the direction of motion and speed does not. Displacement includes the direction of motion and distance does not. Velocity and displacement are vectors, while speed and distance are scalars (see page 54).

✓ *Checkpoint 9.6: An object is moving with a velocity of 20 miles per hour eastward. What is the object's displacement during two hours?*

To find the total odometer distance, we need to split the problem into two parts, one for when the object is slowing to a stop and one for when the object is speeding up. We then have to add up the distance traveled during each part.

To find the displacement, however, we don't need to do that. We can just use the definition of velocity equation once. This is because the $\Delta\vec{s}$ in the definition of velocity is precisely the displacement.

As another example, consider the following scenario.

> An object starts with a velocity of 5 m/s southward and experiences an acceleration of 3 m/s^2 northward for four seconds. What is the object's displacement during the four seconds?

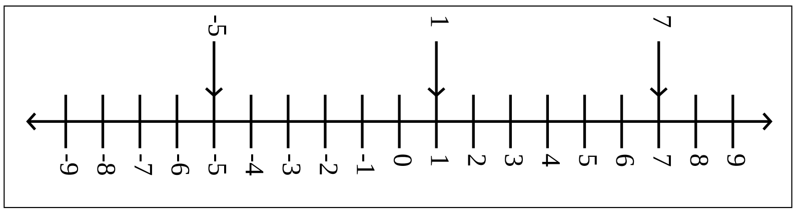
To find the displacement, we first need to figure out the average velocity, and to find the average velocity we first have to figure out the initial and final velocities.

In this case, the initial velocity is 5 m/s southward. It accelerates at 3 m/s^2 northward, which is in the opposite direction. That means it initially slows down, then turns around and speeds up in the opposite direction. In four seconds, the total change in velocity is 12 m/s northward (multiply 3 m/s^2 northward by four seconds). Since it starts at 5 m/s southward, a change of 12 m/s northward would make the final velocity 7 m/s northward.

Since the acceleration is constant, the average velocity is the midrange between 5 m/s southward and 7 m/s northward. The midrange value is 1 m/s northward.

How do you know the average velocity is 1 m/s northward?

The process is similar to how you'd get the average of two temperatures, +5°F and −7°F, or two water levels, one 5 m above sea-level and one 7 m below sea-level. Indeed, when dealing with changes in direction, it might be easier to treat one direction as positive and the other as negative. For example, if positive is northward, the initial velocity is −5 m/s and the final velocity is +7 m/s. If we then mark −5 and +7 on a number line,



one can more easily see that +1 is midway between −5 and +7.

To find the displacement, then, we treat the object as though its velocity is the midrange value for the entire four seconds. At 1 m/s northward, in four seconds the object would be 4 meters north of where it started. Consequently, the displacement is 4 meters northward.

✓ *Checkpoint 9.7: An 2-kg object starts with a velocity of 3 m/s eastward and experiences a net force of 8 N westward for three seconds. What is the object's displacement during the three seconds?*

Summary

This chapter examined situations where an object changes direction during the motion.

The main points of this chapter are as follows:

- As long as a force is acting on the object, the object's motion continues to change.
- If the force and motion equation indicates that an object slows down by more than its initial speed, that means it has turned around.
- An object's displacement is not only how far it moves but also the direction it has moved.

Frequently Asked Questions

What is accel dir when turning around? WHAT ABOUT WHEN THE OBJECT TURNS AROUND? AT THE MOMENT IT TURNS AROUND, IT IS MOMENTARILY AT REST. AT THAT MOMENT, ISN'T THE ACCELERATION ZERO?

No. Although motionless for an instant, it is still undergoing a change in direction and, as such, is accelerating (in a direction that is opposite its initial motion direction).

If the object *stays* at rest then its acceleration would be zero, but if the object turns around and speeds up from that point onward then its acceleration can't be zero.

HOW CAN THE ACCELERATION NOT BE ZERO IF THE VELOCITY IS ZERO?

The acceleration is zero only if the velocity remains *constant*. It has nothing to do with the *value* the velocity happens to have any particular moment. So, for example, the acceleration is zero only if the velocity *remains* zero. If the velocity doesn't remain zero, the acceleration is not zero. In the case of the object turning around, the object's velocity is momentarily zero but it cannot remain zero for otherwise it would just stay at that location. It doesn't. It is only there for an instant.

WHAT IS THE DIFFERENCE BETWEEN DISPLACEMENT AND DISTANCE?

The displacement is just the total change in position whereas the distance refers to the distance covered by the movement of the object.

Terminology introduced

Displacement   Distance

Problems

Problem 9.1: A fan cart moves one way and then returns to its initial position. Let's suppose it reaches a point 45 cm to the north of its starting location before returning to its initial position. What is the average velocity during the time between when it starts and when it returns to the initial position? Explain.

10. Graphs

Puzzle #10: So far, we've only examined situations where the net force on the object is constant. What about situations where the net force is not constant?

Introduction

So far we have only considered situations where the net force is constant and unchanging. However, there are a lot of interesting situations where the net force is not constant. Two examples of such a situation are the back-and-forth swinging of a pendulum and the up-and-down movement of an object on a spring.

The approach discussed in chapters 7 and 8 can still be used, but it is a little more difficult to figure out the average velocity when the net force is not constant. When the net force is constant, the average velocity is simply the midrange between the initial and final velocities. That won't be the case when the net force is not constant.

In addition, time-lapse illustrations, like the one in Figure 7.1 on page 100, are less useful when the net force is not constant. Time-lapse illustrations use "snapshots" of an object to show where an object is at various times. While they reveal the general nature of the motion, they don't show us what is happening *between* the snapshots.

We can solve this problem with graphs, as graphs can be used to show what is happening throughout the object's motion (as long as the motion is along a straight line). Consequently, in this chapter, we'll explore how graphs can be used to describe an object's motion. In chapter 11, then, we'll use graphs to help examine the back-and-forth motion of a swinging pendulum or object on a spring.

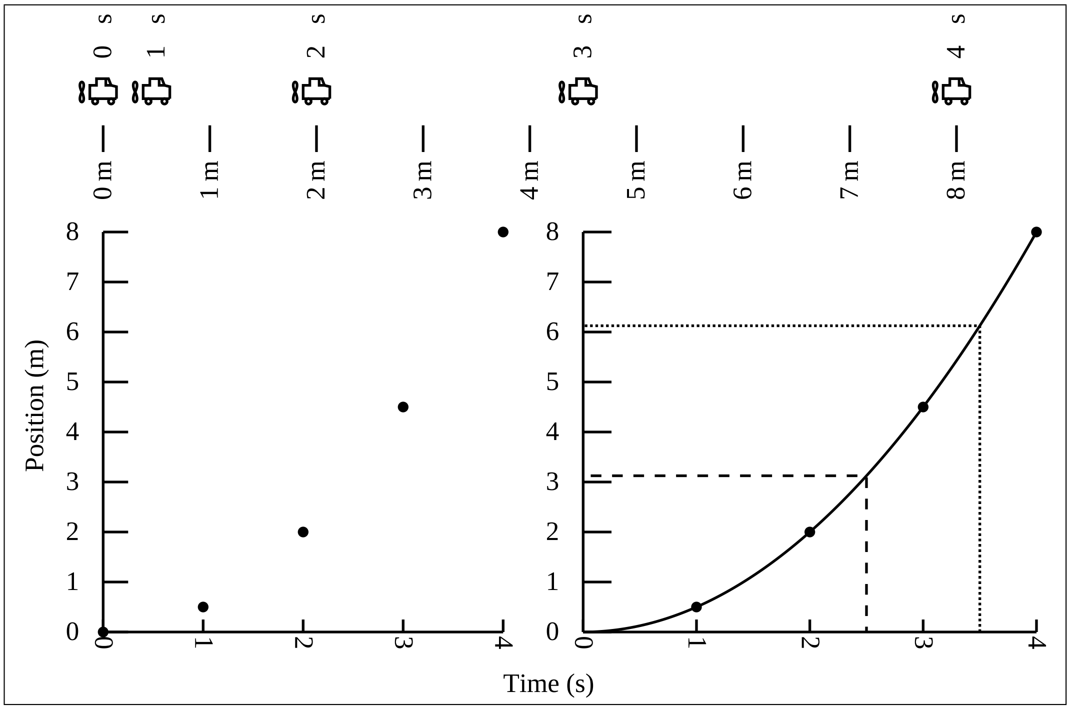


Figure 10.1: Two graphs, each showing the position of the object illustrated in the time-lapse figure (from zero to 4 seconds). The dotted and dashed lines are described in the text.

We'll examine two types of graphs. One type, the position vs. time graph, shows how an object's position is changing with time. The other type, the velocity vs. time graph, shows how an object's velocity is changing with time.

10.1 Position vs. time graphs

To illustrate what a position vs. time graph is, consider the two graphs shown in Figure 10.1. Each graph in Figure 10.1 is called a **position vs. time graph** since it shows how the position of an object varies during a certain time interval. Notice how one axis of the graph is labeled "Position (m)" and the other is labeled "Time (s)".

By convention, the two axes of a graph are called the **vertical axis** and the **horizontal axis**, based on how they are oriented on the page. It is important to recognize – and I want to emphasize this point – that these names do *not* refer to which way the object is moving. In other words, the vertical axis does not necessarily indicate an object moving vertically.

❢ The vertical and horizontal axes on a graph do not, in general, indicate the vertical and horizontal positions of an object.

Indeed, in this case, the horizontal axis does *not* indicate horizontal motion of an object, and the vertical axis does *not* indicate the vertical motion of an object. In other words, the curved line in the right graph in Figure 10.1 does *not* represent an object moving upward and rightward.

On the contrary, both graphs represent the motion of the cart in the time-lapse illustration shown at the top of the figure, which is the same illustration shown in Figure 7.3 on page 102, where a cart is moving rightward.

For every graph you encounter, take a moment to notice what each axis represents. The meaning of the graph depends on what the axes represent. In this case, the "horizontal" axis of each graph indicates *time*, not *position*, and the "vertical" axis indicates how far the object has moved *horizontally*, not vertically.[i]

Let's now examine the left graph in Figure 10.1 in more detail. The vertical axis is labeled "Position (m)" because it indicates the horizontal position of the fan cart. The horizontal axis is labeled "Time (s)" because it indicates the time corresponding to when the cart is at each of the positions indicated. Notice how the left graph, like the time-lapse picture, only contains information every second, indicating the position of the cart at each 1-second moment. Indeed, for each image of the fan cart in the time-lapse picture, there is a dot on the graph.

In comparison, the position vs. time graph on the right has a continuous line that connects all of the dots. The line not only tells us where the object is at each 1-second interval but also at any time in between. As mentioned in the introduction, the real benefit of a graph (over a time-lapse picture) is that it can be used to indicate what is happening throughout the motion.

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[i]There is no law that specifies what the two axes correspond to. In this particular case, the horizontal axis is time and the vertical axis is the horizontal position of the cart. However, we could make the axes be whatever we want them to be. Some axes assignments allow us to interpret the motion more easily and/or are more the convention. In this case, we have chosen to place time on the horizontal axis because it allows us to more easily determine the velocity, as will be shown later in this chapter.

✍ The dots, actually, aren't even necessary, and won't usually be plotted. They are plotted in the examples here only to show the correspondence with the time-lapse figure.

For example, whereas the left graph indicates that the object is at position 2 m at 2 s, and at position 4.5 m at 3 s, it doesn't tell us where the object is at times between 2 s and 3 s. The right graph, on the other hand, uses a curve to tell us the position throughout the time interval from 2 s to 3 s.

Example 10.1: Using the right graph in Figure 10.1, estimate the position of the object at 2.5 s.

Answer 10.1: The graph indicates a position that is a little more than 3 m. First identify the location along the time axis that corresponds to 2.5 s. That position would be half-way between 2 s and 3 s, and is indicated by the vertical dashed line. Follow that up to where it intersects the curve. That point on the position axis, shown by the horizontal dashed line, indicates the position of the object at that time.

✓ *Checkpoint 10.1: Using the right graph in Figure 10.1, estimate the position of the object at 3.5 s.*

A position vs. time graph can also be used to show the motion of multiple objects. For example, consider the time-lapse picture shown in Figure 7.2 on page 101 and reproduced here as Figure 10.2.

The time-lapse picture shows three toy cars. The top car starts at a position 4 m to the right of the other two cars. However, it travels slower, and the other cars pass it.

The middle car passes the top car sometime between 1 s and 2 s. This can be seen because at 1 s the middle car is behind the top car but at 2 s the middle car is in front of the top car.

The bottom car passes the top car sometime right at 4 s. This can be seen because the bottom car is behind the top car at times less than 4 s and is in front of the top car at times greater than 4 s. Right at 4 s the middle car is at the same position as the top car.

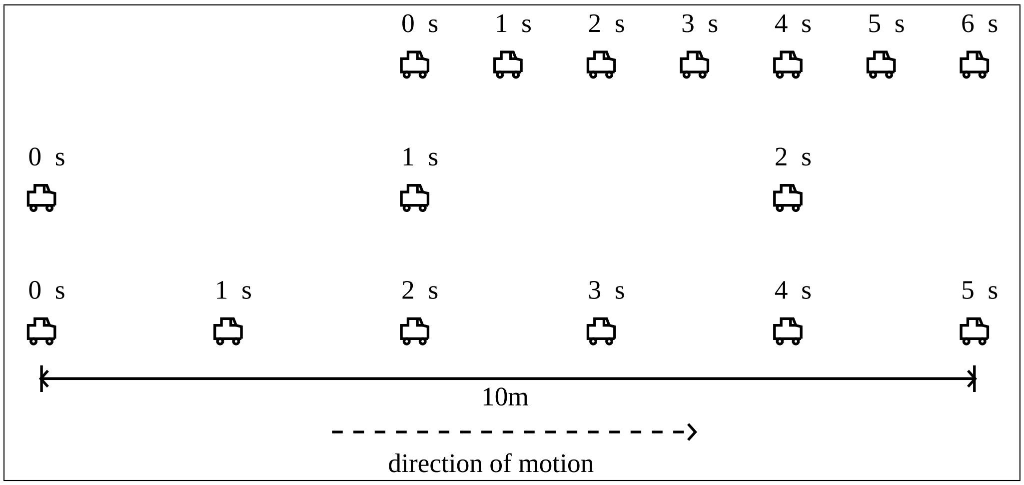

Figure 10.2: A time-lapse illustration of three toy cars as in Figure 7.2.

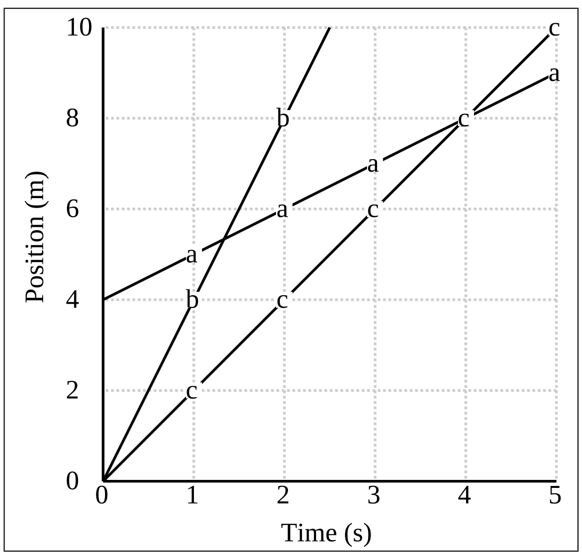

Figure 10.3: A position vs. time graph of the three cars shown in the time-lapse illustration of Figure 10.2.

Now let's look at the same situation but with the corresponding position vs. time graph, which is shown in Figure 10.3.

The line labeled "a" corresponds to the top car. Notice how it starts at position +4 m (where positive indicates rightward in the time-lapse picture) whereas the other two cars start at zero.

Line "b" corresponds to the middle car. Notice how the "b" line intersects the+ "a" line between 1 and 2 seconds. That intersection point indicates the

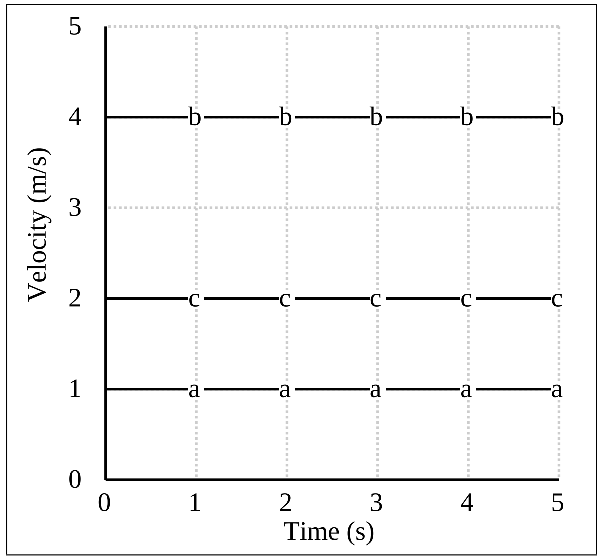

Figure 10.4: A graph showing the velocity for the three toy cars illustrated in Figure 10.2.

time when the middle car passes the top car.

Similarly, line "c" corresponds to the bottom car, and that line intersects line "a" at 4 seconds, which is when the bottom car passes the top car.

✓ *Checkpoint 10.2: According to the position vs. time graph in Figure 10.3, which car is furthest to the right at 2 seconds, and by how much?*

10.2 Velocity vs. time graphs

Just as we can use graphs to show how an object's *position* changes during a particular time period, we can also use graphs to illustrate how an object's *velocity* changes during the time period.

An example is shown in Figure 10.4.

The graph in Figure 10.4 is called a **velocity vs. time graph** because it shows how the *velocity* of one or more objects varies during a certain time interval. Notice how one axis of this graph is labeled "Velocity (m/s)", whereas the axis of the position vs. time graph is labeled "Position (m)".

Why do the curves in the velocity vs. time graph look different than the curves in the position vs. time graph?

The previous graph showed the *position* of the cars. This graph shows the *velocity* of the cars, so a flat horizontal line corresponds to a car that is moving with an unchanging velocity. When you are given a graph, it is crucial that you identify the *quantity* that is being plotted (e.g., velocity or position).

❢ When you are given a graph, it is crucial that you identify the *quantity* that is being plotted.

The position vs. time graph is like an odometer (which tells you how far you have traveled) whereas the velocity vs. time graph is like a speedometer (which tells you how fast you are traveling). If you are driving at a constant speed, the speedometer doesn't change but the odometer continues to increase. In the same way, the velocity graph would be flat but the position graph would have a non-zero slope.

For example, the top car in Figure 10.2 moves one meter to the right every second. In other words, its speed remains steady at 1 m/s during the entire time. Since it remains at 1 m/s the entire time, the corresponding line in the velocity vs. time graph will be a "flat" line with value 1 m/s. This is the line labeled "a".

The middle car moves four meters to the right every second, and maintains that velocity during the entire time (although the time-lapse picture doesn't show its position past 2.5 seconds). Consequently, its line on the velocity vs. time graph is a flat line with value 4 m/s.

✓ *Checkpoint 10.3: According to the velocity vs. time graph in Figure 10.4, which car is traveling the fastest? How can you tell?*

10.3 Positive vs. negative values

10.3.1 Positions

So far we have limited our analysis to situations where the values on the vertical axis are positive. It turns out there is no reason to restrict our attention to just positive values.

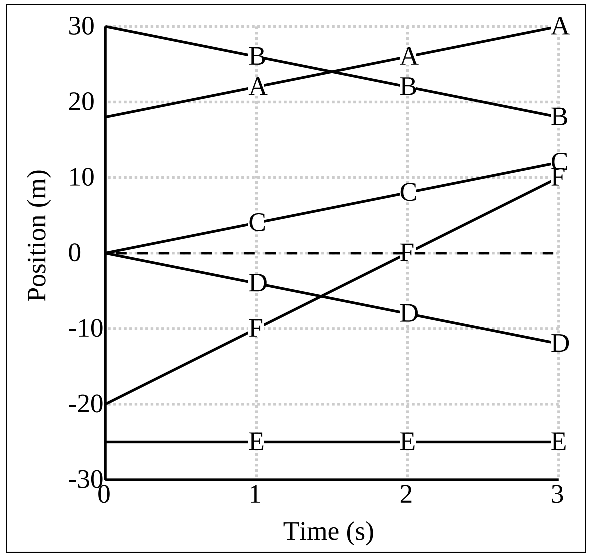

Figure 10.5: A position vs. time graph, representing the motion of six different objects.

What, then, does a negative value mean?

On a graph, positive and negative simply refer to opposite directions. For example, positive could mean northward, in which case negative means southward. Or, conversely, positive could mean southward, in which case negative means northward.

To see how this works, consider the positions of six different objects, as indicated by the six different lines in Figure 10.5, labeled A through F.

Let's first start with objects A and B, where their positions are indicated by lines A and B, respectively.

Whereas object A starts at the 18-m position and moves to the 30-m position in 3 s, object B does the reverse – starting at the 30-m position and moving to the 18-m position. Both travel 12 m in the 3 s but in opposite directions.

Object C exhibits a motion similar to object, in that it travels 12 m in the 3 s but it starts at different position. Rather than starting at the 18-m position, though, object C starts at the zero position. Because of their similar motions, object A and object C are always 18 m apart.

There is nothing special about the zero position. It just happens to be where we set zero. For example, consider object D. Object D starts at the same location as object C but it moves in the opposite direction, ending up at the -12-m position.

The negative position simply means it is on "the other side" of the zero position. Remember that there is nothing special about the zero position. It could represent a state boundary or the average sea level, for example. And, like a state boundary or the average sea level, it is not impossible for an object to move to the other side of it.

On a graph, we can't tell what the positive and negative values represent. For example, positive might mean a position north of a boundary and negative might mean a position south of a boundary. Or, conversely, positive might mean a position south of the boundary and negative might mean a position north of the boundary. We only know that positive and negative are opposite sides of the boundary.

On a position vs. time graph, the position only indicates where the object happens to be at a particular time. To obtain information about the motion, we need to consider if the position is different at different times.

Consider, for example, object E. It has a position at −25 m, and it remains there. Consequently, object E must be stationary. The actual value of its position has no bearing on it being stationary, as long as the position doesn't change.

✓ *Checkpoint 10.4: Line F in Figure 10.5 has a value of zero at 2 seconds. Does that mean that object F is stationary at that moment?*

10.3.2 Velocities

In Figure 10.5 we have objects moving in opposite directions. For example, object A and object B both both travel 12 m in the 3 s, for a speed of 4 m/s, but in opposite directions.

For example, if A's motion was northward then B's motion would be southward. Conversely, if A's motion was southward then B's motion would be northward. We can't tell from the graph which way the objects are moving – we only know that they are moving in opposite directions.

For lack of any other information, the best we can say is that object A is moving "in the positive direction" and object B is moving "in the negative direction." In other words, object A's velocity is positiveward and object B's velocity is negativeward.

Are "positiveward" and "negativeward" real words?

While not popular, these words are actually used sometimes to indicate what we are trying to do here – a direction much like we would use northward and southward.

However, it is actually much more popular to just say positive and negative, rather than positiveward and negativeward, so we are going to use that terminology instead. Consequently, we'll just say that object A has a positive velocity and object B has a negative velocity. This should be okay as long as you recognize that we are using positive and negative to indicate the opposite directions.

In the case indicated in Figure 10.5, then, object A's velocity is +4 m/s while object B's velocity is −4 m/s. Both have a speed of 4 m/s, though, since an object's speed does not include the direction of motion.

Similarly, objects C and D also have the same speed, but object C's velocity is +4 m/s while object D's velocity is −4 m/s. Object E, on the other hand, doesn't move, and so object E has zero velocity.

✓ *Checkpoint 10.5: (a) Which of the objects in Figure 10.5 have a negative velocity for the entire three seconds?*
(b) Is object F's velocity zero at 2 seconds? If not, is it positive or is it negative?

As we have seen, we can obtain information about the direction of an object's motion from a position vs. time graph. However, as noted in section 10.2, we can also plot the velocity directly, as in a velocity vs. time graph, and it turns out that we can get information about the direction of motion from that kind of graph as well.

Just as on a position vs. time graph, positive and negative on a velocity vs. time simply mean opposite directions. To see what this means, consider the graph in Figure 10.6.

Before examining the motion of the six objects plotted in Figure 10.6, I should first point out that these are six *new* objects, as reflected in the use

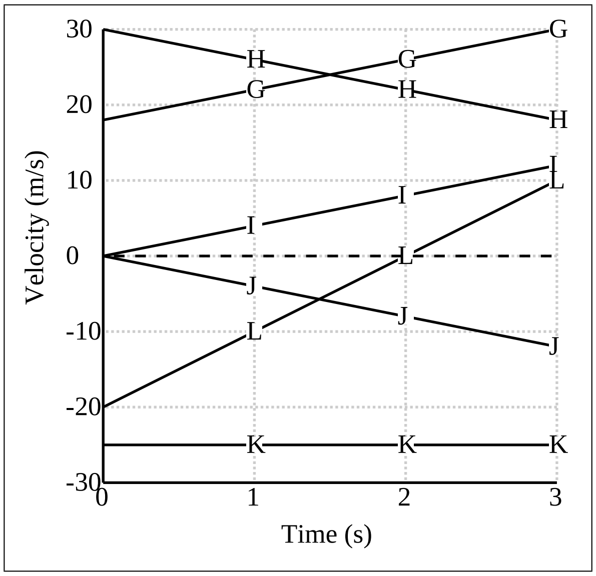

Figure 10.6: A velocity vs. time graph, representing the motion of six different objects.

of six *new* letters, and do not represent the same motion as that described by the graph in Figure 10.5.

How do we know the motion is not the same? The graphs look the same.

The *lines* look the same but this graph indicates the *velocity* whereas the previous graph indicated the *position*. There is a big difference between the two. Whereas the previous graph showed us *where* each object was at any time, this graph is shows us *how fast* each object is traveling at any time (and, as we will see, which direction it is traveling).

❢ Pay attention to what is plotted before interpreting a graph.

It is very important to identify what is being plotted *before* interpreting anything from a graph. For example, let's consider the line for object G. According to this line, object G starts with a *velocity* of +18 m/s and by 3 s its velocity is +30 m/s. This means the object has sped up by 12 m/s. In addition, the positive values mean it is traveling positiveward (i.e., in the positive direction).

Object H, on the other hand, starts with a *velocity* of +30 m/s and slows down to +18 m/s by 3 s. However, both objects are moving in the positive direction, since both have positive velocities.

✍ Notice how the motion of objects G and H differ from the motion of objects A and B (in Figure 10.5). While the lines themselves may look similar, the graphs are *different*. One is plotting velocity and the other is plotting position.

Now let's consider the lines for objects I and J. Both start at rest, with a velocity of zero. However, whereas object I speeds up to a velocity of +12 m/s by 3 s, object J speeds up to a velocity of −12 m/s. Both have the same *speed* at 3 s but object I has a *positive* velocity (moving in the positive direction) while object J has a *negative* velocity (moving in the negative direction).

Notice how the lines for objects G and I look similar, and the motion is similar as well, with the only difference being that object G starts off faster. Both are moving in the positive direction and both speed up by 12 m/s during the 3 s.

However, now let's compare objects H and J. Whereas the lines for those two objects also look very similar, objects H and J have very different motions. For one thing, object H is moving in the positive direction (as indicated by the positive velocity values) whereas object J is moving in the negative direction (as indicated by the negative velocity values). In addition, object H is slowing down (as indicated by it starting with a speed of 30 m/s and ending with a speed of 18 m/s) whereas object J is speeding up (as indicated by it starting with a speed of zero and ending with a speed of 12 m/s).

✍ Don't be tempted by what a graph "looks like". Read the actual values associated with the line on the graph then base on your interpretation on those values.

✓ *Checkpoint 10.6: (a) Which of the objects in Figure 10.6 have a negative velocity for the entire three seconds?*
(b) Is object L's velocity zero at 2 seconds? If not, is it positive or is it negative?

10.3.3 Accelerations

In Figure 10.6 we have objects that are speeding up (objects G, I and J), an object that is slowing down (object H), an object that is moving at a constant speed (object K), and an object that slows down for part of the time and speeds up for another part of the time (object L).

Let's review each of these and identify the acceleration for each one. As before, we need to identify the actual values associated with a line and then we'll base our interpretation on those.

Object G starts out with a velocity of +18 m/s and ends with a velocity of +30 m/s. This means it is speeding up. We know that speeding up is associated with an acceleration in the direction of motion. Since object G's velocity is positive and speeding up, that means object G's acceleration is *positive*.

Object H starts out with a velocity of +30 m/s and ends with a velocity of +18 m/s. This means it is slowing down. We know that slowing down is associated with an acceleration opposite the direction of motion. Since object H's velocity is positive and slowing down, that means object H's acceleration is *negative*.

Object I starts out with a velocity of zero and ends with a velocity of +12 m/s. This means it is speeding up. Since object I's velocity is positive and speeding up, that means object I's acceleration is *positive*.

Object J starts out with a velocity of zero and ends with a velocity of −12 m/s. This means it is speeding up. Since object J's velocity is negative and speeding up, that means object J's acceleration is *negative*.

Let's stop for a moment and think about the last case.

Object J's acceleration is *negative* yet it is *speeding up*. To some people that seems backward, since a negative acceleration seems to imply that the object is decelerating.

That would be incorrect.

Remember that in physics we use the word acceleration for both speeding up and slowing down. We are also using positive and negative to mean two opposite directions. To be consistent, if an object is slowing down while moving in the negative direction, its acceleration must be positive (opposite the direction of motion).

This is an important point. Negative does not mean "opposite the motion". It just means "opposite the positive direction". The motion itself could be in the negative direction, in which case a negative acceleration is in the *same* direction as the motion, meaning that the object is speeding up.

✍ If you have "negative means opposite motion" stuck in your head, you'll be inevitably be confused whenever we encounter a situation where an object speeds up while having a negative acceleration. Try to remember that negative just means opposite the positive direction.

❢ A negative acceleration simply means that the acceleration is in the negative direction. It does not necessarily mean the acceleration is opposite the direction of motion or slowing down.

Can't we always assign positive to be the direction of motion? That way a negative acceleration is always a deceleration.

While that sounds nice, it is actually much easier if we just keep the positive direction the same, regardless of which way the object is moving. After all, what happens if we have two objects moving in opposite directions? Which way do we indicate as positive?

As another example, consider the motion of object L in Figure 10.6.

Object L starts out with a velocity of −20 m/s and ends with a velocity of +10 m/s. This means it was initially moving in the negative direction but ends up moving in the positive direction.

Indeed, for the first two seconds, object L is moving in the negative direction but slowing down. At two seconds, object L's velocity is zero, which means it is momentarily at rest. This is when it turns around.

For the first two seconds, then, object L is slowing down and has a *positive* acceleration, since it is moving in the negative direction while slowing down. After two seconds, object L is speeding up. This is *still* a positive acceleration, since it is now moving in the positive direction while speeding up.

Notice how object L's acceleration remains positive for the entire three seconds, even at 2 seconds when it is momentarily at rest. At 2 s, object L is changing directions and so is momentarily at rest, but its acceleration is not zero since the velocity is still undergoing a change![ii]

✓ *Checkpoint 10.7: (a) Which of the objects in Figure 10.6 have a negative acceleration for the entire three seconds?*
(b) At 2 s, what is object L's acceleration: zero, positive or negative?

[ii]See the analogy with temperature on page 135.

10.3.4 Forces

Positive and negative can be used for any vector to indicate two opposite directions. Since force is a vector, just like position, velocity and acceleration, we can also use positive and negative to indicate the direction of a force.

Example 10.2: Suppose we have four forces acting on an object: +30 N, +40 N, −20 N and −50 N. What is the net force acting on the object?

Answer 10.2: We have a total of 70 N pulling in the positive direction (add together 30 N and 40 N) and we have a total of 70 N pulling in the negative direction (add together 20 N and 50 N). Since the magnitudes are equal and they are opposite in direction, the forces balance and so the net force is zero.

Notice that using positive and negative for opposite directions allows us to just add everything together: $+30\text{ N} + 40\text{ N} - 20\text{ N} - 50\text{ N} = 0\text{ N}$.

✓ *Checkpoint 10.8: An object has three forces acting on it: +5 N, +10 N and −20 N. What is the net force acting on the object?*

10.4 Slope

During our examination of the position vs. time graphs, you may have noticed that we not only get information about the position of the object but we can also get information about the velocity. You might have also noticed that the direction of the velocity corresponded to whether the line sloped up or down.

It turns out that the slope on a position vs. time graph is *equal* to the velocity.

To see why, recall that an object's average velocity during a time interval Δt is defined as follows (equation 7.1):

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{s}}{\Delta t}$$

where $\Delta\vec{s}$ is the displacement of the object during the time interval. Meanwhile, the definition of slope is as follows (see the supplemental readings):[iii]

$$m = \frac{\Delta y}{\Delta x}$$

Furthermore, if we are plotting the position vs. time, then our y coordinate is $\vec{s}$ and our x coordinate is t. Making those replacements in our slope equation, we get the same expression as the one for average velocity.

❢ The slope of a line on a position vs. time graph can be interpreted as the velocity.

✍ You can think of slope in much the same way a ski mountain has slopes. The greater the slope value, the steeper it is. A zero slope is horizontal. The floor of a building has a zero slope. For more information about calculating the slope of a line, check out the supplemental readings.

Let's apply this to object E in Figure 10.5. As seen in the position vs. time graph, the position of object A remains at −25 m and thus its displacement during the three seconds is zero. Based on the definition of velocity, that means its velocity is also zero.

Consistent with this, line E also has a zero slope, since the line is "flat" and doesn't rise or fall. The slope of line E is equal to the velocity of object E.

As another example, consider objects A and C. While the positions are different, both have a position that changes by +4 m every second, and thus have a velocity equal to +4 m/s. They also have identical slopes, equal to 4 m/s.

In comparison, the position of object B changes by −4 m every second, which corresponds to a velocity equal to −4 m/s and a corresponding negative slope.

✓ *Checkpoint 10.9: Answer the following questions based on the position vs. time graph of Figure 10.5.*
(a) Which of the objects in the have a negative velocity?
(b) Which of the lines have a negative slope?
(c) What is the velocity of object F?
(d) What is the slope of line F?

What about a velocity vs. time graph?

[iii] See the footnote in the supplemental readings for why we use m for slope.

For a velocity vs. time graph, the slope is *not* the velocity. After all, the curve itself is the velocity. To find out what the slope corresponds to in a velocity vs. time graph, let's apply the definition of slope.

❢ The slope of a line on a velocity vs. time graph can be interpreted as the acceleration.

On a velocity vs. time graph, the y quantity is the velocity $\vec{v}$ and the x quantity is the time t. That means that the slope of a line on a velocity vs. time graph is:

$$m = \frac{\Delta \vec{v}}{\Delta t} \text{ on velocity vs. time graph}$$

That ratio, $\Delta\vec{v}/\Delta t$, is exactly the definition of average acceleration.

For example, consider object K in Figure 10.6. As seen in the velocity vs. time graph, the velocity of object K remains at -25 m/s, without changing. Based on the definition of acceleration, that means it isn't accelerating and its acceleration is zero. Consistent with this, the slope of line K is likewise zero.

Now let's consider object G. As seen in the velocity vs. time graph, the velocity of object G changes by $+4$ m/s every second. In other words, it is speeding up at a rate of $+4$ m/s^2. Its acceleration is $+4$ m/s^2. This is also the slope of line G.

Line I likewise has a slope of $+4$ m/s^2. Like object G, then, object I undergoes a change in velocity of $+4$ m/s every second (i.e., an acceleration of $+4$ m/s^2). The only difference is that it starts off at rest.

In comparison, the line H has a negative slope, corresponding to its negative acceleration of -4 m/s^2, where its velocity changes by -4 m/s every second.

✓ *Checkpoint 10.10: What is the slope of the line corresponding to object L in the velocity vs. time graph of Figure 10.6? What is the acceleration of object L?*

Can we determine the acceleration from a position vs. time graph?

Since we can get the velocity from a position vs. time graph (by calculating the slope of the line), you might wonder if we can also get the acceleration.

We certainly can if the velocity is constant. If the velocity is constant, the acceleration is zero. When the position is plotted, we get a straight line,

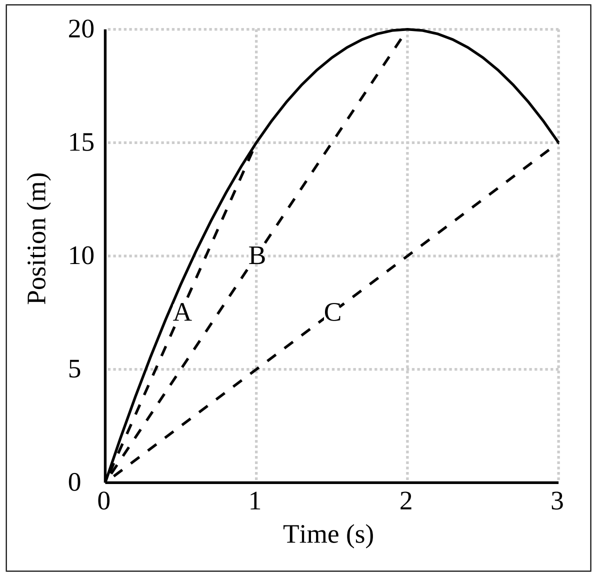

Figure 10.7: A position vs. time graph. Dashed lines are explained in the text.

with a single slope value. In those cases, it is straightforward to calculate the acceleration – it is zero.

However, what if the acceleration is not zero? In that case, the velocity is not constant, and when position is plotted, we'll get a curved line. For example, consider the position vs. time graph in Figure 10.7.

The solid curve indicates the position of the object, which starts at position 0 at time 0, and moves to position +20 m at 2 seconds before reversing directions, ending up at position +15 m at 3 seconds.

There is not a single value for the slope. The slope is positive between zero and 2 seconds, as the object moves in the positive direction, and the slope is negative between 2 and 3 seconds, as the object moves in the negative direction. At 2 seconds, as the object transitions from moving positiveward to negativeward, the slope is zero.

In a similar way, the object's velocity is positive between zero and 2 seconds, negative between 2 and 3 seconds, and zero right at 2 seconds.

If you apply the slope equation to the entire 3 s, you get a change in position equal to +15 m and a change in time equal to 3 s, which gives a slope (and velocity) equal to +5 m/s. However, that is the *average* slope and *average* velocity during the three seconds. It basically is the slope of a straight line drawn between the two points (see dashed line C in the figure).

The actual curve has portions where the slope is greater than +5 m/s and other portions where the slope is less than +5 m/s.[iv]

As an example, suppose we want to know the average slope between zero and 2 s. That gives a change in position equal to +20 m and a change in time equal to 2 s, which gives a slope (and velocity) equal to +10 m/s. Again, that is the *average* slope and *average* velocity during the two seconds. It is the slope of a straight line drawn between the two points (see dashed line B in the figure).

Notice how the average slope *between* zero and 2 seconds does not equal the slope *right at* 2 seconds. The slope (and velocity) right at 2 seconds is equal to zero. At that point, the object is in the transition between moving positiveward and negativeward.

The **instantaneous** velocity is the velocity *right at* a particular time. In this case, the *instantaneous* velocity at 2 s does not equal the *average* velocity between zero and 2 s.

✍ Note that the slope continually changes throughout the three seconds. That means it never experiences a zero acceleration, even at 2 seconds. At 2 seconds, the velocity is changing from positive to negative, and that corresponds to a negative acceleration, even though the *instantaneous* velocity at 2 seconds is zero.

Can we ever determine the instantaneous velocity from a position vs. time graph?

While we can tell when the slope is positive, negative or zero, based on whether the curve is upward, downward or flat (horizontal), it can be difficult to determine the exact numerical value unless it is straight (in which case the slope can be determined using any two points on the line). When the line is curved, all we can do is get an estimate of the slope.

A velocity vs. time graph, on the other hand, provides the instantaneous velocity values. That is why we have used a velocity vs. time graph to determine the acceleration.

How do we estimate the instantaneous velocity on a position vs. time graph?

[iv]In this case, it turns out that the slope is greater than +5 m/s for times less than 1.5 s and less than +5 m/s for times greater than 1.5 s.

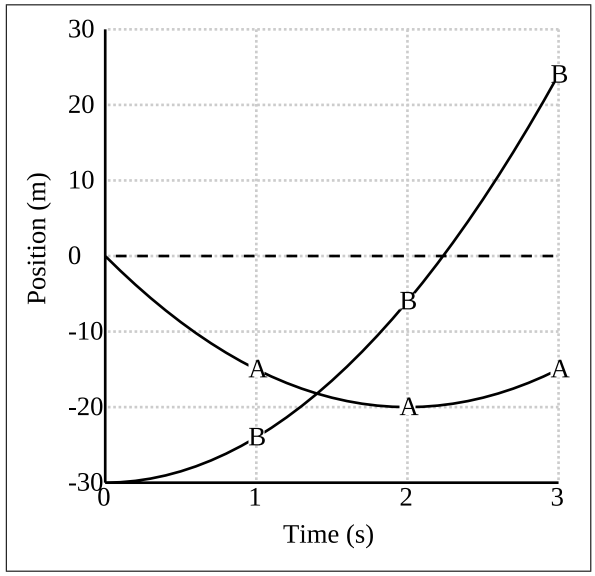

Figure 10.8: A position vs. time graph for two different objects.

Recall that the slope equation gives the *average* slope between two points. For example, in Figure 10.7, using the entire 3-s time period, we determine the average slope during the entire 3-s time period.

Also recall that the average slope during a particular time period is equal to the slope of the straight line drawn between the two points. For example, the average slope during the 3-s time period in Figure 10.7 is equal to the slope of the straight dashed line C.

So, the key to estimate the instantaneous velocity is to identify two points on the line for which a straight line between the two will have the same slope as the point of interest.

For example, suppose we want to estimate the velocity at 0.5 s (for the object whose position is graphed in Figure 10.7). Notice how the slope of dashed line A is similar to the slope of the solid curve right at 0.5 s. Dashed line A is the straight line between zero and 1 s. The slope of that dashed line is just the change in position during that time period divided by the time period. This gives a slope of +15 m/s. Since that slope is similar to the slope right at 0.5 s, we can estimate that the instantaneous velocity at 0.5 s is likely to be around +15 m/s.

✓ *Checkpoint 10.11: Based on the position vs. time graph in Figure 10.8,*

determine the following:
(a) The average velocity of object A between 1 and 3 seconds.
(b) The instantaneous velocity of object A right at 2 seconds.
(c) The average velocity of object B between 2 and 3 seconds.

Summary

This chapter introduced the definition of average velocity and how that is different from the object's change in velocity.

The main points of this chapter are as follows:

- The slope of a line can be interpreted as the velocity if position and time are chosen for the vertical and horizontal axes, respectively.
- The slope of a line can be interpreted as the acceleration if velocity and time are chosen for the vertical and horizontal axes, respectively.
- When you are given a graph, it is crucial that you identify the *quantity* that is being plotted.
- A negative acceleration does not mean the object is slowing down. It only means that the object is accelerating in the negative direction.

Frequently Asked Questions

WHY IS A STRAIGHT LINE ON A POSITION VS. TIME GRAPH SUGGESTIVE OF AN OBJECT TRAVELING WITH CONSTANT VELOCITY?

A straight line has a constant slope. When position and time are along the vertical and horizontal axes, respectively, then the slope is equivalent to the velocity (since the rise over run would be equal to the ratio of the change of position to time).

DOES POSITIVE ALWAYS MEAN UP, NORTH OR RIGHT?

No. Positive and negative simply indicate opposite directions. See section 10.3.[v]

[v] Some people may prefer to always assign positive to a particular direction, like north, in which case negative always means south. However, there is no rule that says north must *always* be positive.

Is positive always associated with the direction the object is moving?

No. While it *can* be, it does not *need* to be. Indeed, if an object is turning around, either the *initial* direction of motion is positive or the *final* direction of motion is positive. They can't *both* be positive.

In fact, the only rule that is set in stone is that, once you've chosen the positive direction, you need to maintain that convention for the entire problem. After all, once you've chosen the positive direction, the sign has *meaning* (i.e., a *particular* direction) and you can't change that midway through a problem without making things confusing.

For example, if you've set positive to be the object's initial direction of motion, you can't switch the direction of positive when the object turns around.

Since there is no hard and fast rule for which direction is positive, it is helpful to indicate which direction one has chosen to be positive for each problem, just so you don't forget midway through the problem.

If an object is moving in the negative direction, does that mean it is slowing down?

Not necessarily. The negative direction is simply opposite the positive direction, whichever direction that happens to be.

Consequently, an object slows down when the force on it is opposite its direction of motion. If the object is moving in the negative direction and the force on it is also in the direction then the object is speeding up.

Can we say that a deceleration is a negative acceleration?

While it seems tempting to do so, you shouldn't. In physics a negative acceleration doesn't always correspond to a slowing down.

If velocity is plotted versus time, can the slope of the line <u>ever</u> have a sign that is different than that of the net force exerted on the object?

No. When the velocity is plotted versus time, the slope is equivalent to the acceleration. The sign of the slope is thus the sign of the acceleration, and as explained in the previous question, an object's acceleration is always in the direction of the net force (i.e., force imbalance).

Why is a straight line on a velocity vs. time graph suggestive of an object upon which the net force is constant?

A straight line has a constant slope. When velocity and time are along the vertical and horizontal axes, respectively, then the slope is equivalent to the acceleration (since the rise over run would be equal to the ratio of the change of velocity to time). According to the law of force and motion, the only way the object's acceleration remains the same (i.e., the velocity will change at the same rate) is if the net force remains the same.

Terminology introduced

Horizontal axis	Velocity vs. time graph
Instantaneous	Vertical axis
Position vs. time graph	Y-intercept
Slope	

Problems

Problem 10.1: At 2 seconds, the object depicted in Figure 10.7 has a zero velocity, which means it is at rest for that instant. Is the net force exerted on it also zero at that moment?

Problem 10.2: At 2 seconds, which object in Figure 10.8 is going faster? How can you tell?

Problem 10.3: (a) Can an object speed up while undergoing a negative acceleration? If so, how? If not, why not?
(b) Can an object speed up while experiencing a force imbalance in the negative direction? If so, how? If not, why not?

Problem 10.4: Suppose an object is moving in the positive direction.
(a) If you want the object to slow down, in what direction do you exert a force on the object?
(b) Suppose the object stops and then moves in the opposite direction. Is it then moving in the positive direction?

11. Oscillations

Puzzle #11: An object is placed on a spring and then made to oscillate up and down. Why does it oscillate?

Introduction

So far we have only considered situations where the net force is constant and unchanging. However, as mentioned in chapter 10, there are a lot of interesting situations where the net force is not constant, like the back-and-forth swinging of a pendulum and the up-and-down movement of an object on a spring.

In this chapter, we'll investigate a class of phenomena called **oscillations** (pronounced with a silent "c"). To oscillate means to swing back and forth. The purpose of this investigation is to strengthen our understanding of the relationships learned so far: the law of force and motion, and the definitions of velocity and acceleration.

We'll also introduce the language for describing such motion, a language that will be used for any kind of repeating motion, like the beating of a heart and the motion we'll examine in part E.

11.1 An oscillation example

A simple situation involving an oscillation is shown in Figure 11.1. It consists of a block on a table, with the block attached to a fixed wall by a spring that is free to compress and expand, as the block moves toward or away from the wall. We'll assume there is no friction between the block and the table, just to simplify things.

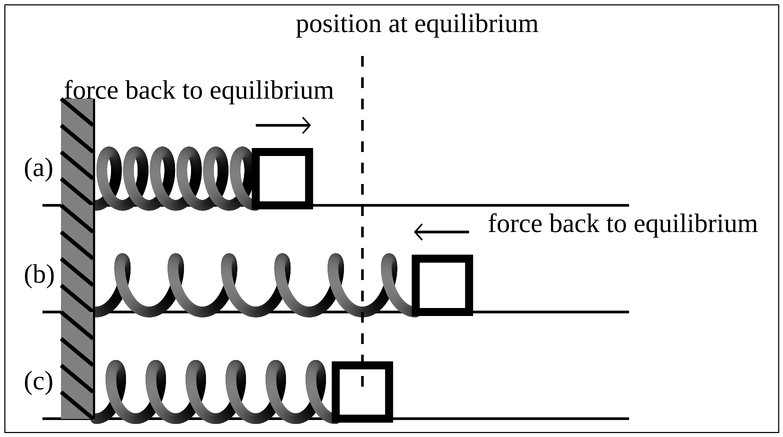

Figure 11.1: (a) A horizontally-oriented spring connected to a block at rest on a frictionless surface. (b) The position of the block after it is pulled to the right. (c) The position of the box at the spring's equilibrium position. In all three cases, the left side of the spring is held fixed to a wall.

In the figure, the dashed line indicates a reference location. We call that the equilibrium position, meaning that the spring is in its "relaxed" state when the block is there, as illustrated in part (c) of the figure.

In part (b) of the figure, the block is located to the right of the equilibrium position. Since it is still attached to the spring, the spring is stretched beyond its "relaxed" state, and the spring exerts a leftward force on the block. Conversely, in part (c) of the figure, the block is located to the left of the equilibrium position. Again, it is still attached to the spring so the spring is compressed and pushes against that, exerting a rightward force on the block. In other words, the spring always exerts a force pushing the block toward the equilibrium position.

What happens if the block is located right at the equilibrium position?

If the block is located right at the equilibrium position, as in part (c) of the figure, the spring doesn't exert any force on the block. That is why we call that location the **equilibrium position**, since that is where the system is in "equilibrium."

It is the existence of this equilibrium position that leads to the back-and-forth motion associated with an oscillation.

To see why, suppose we place[i] the block to the left of the equilibrium position, as in part (a) of Figure 11.1, and then let go. In this situation, the spring is compressed and pushes the block rightward (i.e., toward the equilibrium position). In accordance with the law of force and motion, the block will move back to the right and speed up.

❢ An object oscillates when the force acting on it is directed toward some equilibrium position.

As it moves rightward, the spring continues to push it rightward, making it continue to speed up. Eventually the block reaches the equilibrium position. At that point, the spring no longer exerts a force on the block and the block no longer speeds up. However, the block is already moving to the right at this point and so it continues moving to right and passes by the equilibrium position.

Once on the right side of the equilibrium position, the spring is stretched and, as such, pulls the block leftward (back toward the equilibrium position).

At first, that leftward force is opposite the motion (which is still rightward) and so the block slows down and eventually stops.

At that point, the block is on the right side of the equilibrium position, as in part (b) of Figure 11.1. Since the spring is exerting a leftward force on the block, the block doesn't stay at rest. It moves leftward, speeding up as it approaches the equilibrium position.

This leads to an oscillation of the block back and forth.

✓ *Checkpoint 11.1: Consider the block attached to a spring as shown in Figure 11.1 and no friction with the surface.*
(a) As the block moves back and forth, at what location is the net force on the block (due to the spring) zero?
(b) At the location you've indicated (where the net force on the block is zero), does the block stay at that location? If so, why? If not, why not?

As mentioned above, the back and forth motion associated with oscillations occurs when an object is displaced from some equilibrium position. A block attached to a spring is just one example of this. For another example, consider a small ball placed in a large bowl. The equilibrium position is the

[i]To place the block somewhere other than the equilibrium position, we'd have to apply a force on the block. However, we then assume that we *remove* our hand, so that the force due to the spring is only force acting on the block from that point onward.

bottom of the bowl. If the ball is moved slightly to one side, it will roll back toward the bottom, toward the equilibrium position.

A pendulum provides another example. A pendulum is just a ball hanging on a string. When the string is vertical, that is the equilibrium position. If the ball is moved slightly to one side, it will swing down toward the equilibrium position.

❢ The equilibrium position is the location where the object will remain stationary if released at rest there.

In all three cases, if the object is placed at the equilibrium position, it will stay there. It is for this reason that it is called the equilibrium position (i.e., the object is at equilibrium at that position). In many cases, the equilibrium position is in the middle of the back and forth motion.

Example 11.1: Consider a child swinging on a swing. Where is the child when at the equilibrium position?

Answer 11.1: At the bottom of the swing because if the child is placed there at rest, the child will remain there.

Is the object necessarily at rest if located at the equilibrium position?

No. At the equilibrium position, the net force on the object is zero. That means its acceleration is zero and so there is no change in its velocity. In other words, if the object is at rest at the equilibrium position then it will stay at rest. However, if the object is moving, it will continue moving (and move away from the equilibrium in the process).

By convention, we'll make the equilibrium position equal to zero, with positive positions on one side of the equilibrium position and negative positions on the other side. An example of a position vs. time graph is shown in Figure 11.2. Notice how the position varies between +2 m and −2 m, with the equilibrium position in the middle (zero).

✓ *Checkpoint 11.2: Suppose we have an object hanging from a spring. The object is initially at rest. If the object is then pulled down a small amount and released, it starts to oscillate up and down. Where is the equilibrium position in this case: (a) the position where the object was initially at rest or (b) the position where the object was released?*

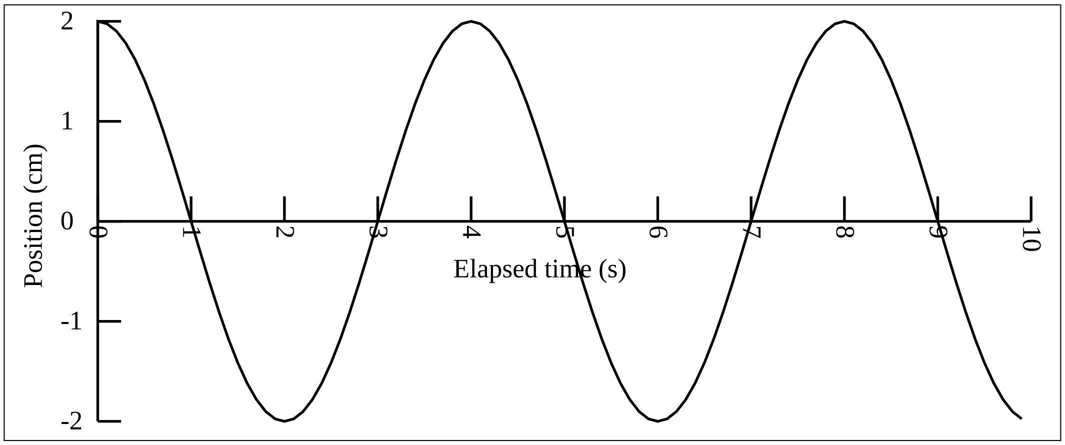

Figure 11.2: A position vs. time graph (see chapter 10) of an object undergoing an oscillation.

11.2 Velocity

So far we've described how to indicate the position of the object as it oscillates back and forth. We've set the equilibrium (middle) position as zero, with positive positions on one side and negative positions on the other side.

Let's now turn to the *velocity* of the object as it oscillates back and forth. Where is it when it is moving fastest? Where is it when it is moving slowest?

Let's address the second question first.

As the object moves back and forth, it must slow to a stop for an instant in order to turn around. Thus, the object's velocity is zero (if only for an instant) when it is farthest from the equilibrium position. This would be at zero, 2 s, 4 s and so on for the motion depicted in Figure 11.2.

❢ During oscillations, the velocity is zero at the moment the object is turning around.

Where is it fastest?

Since it slows down before coming to a stop, and it must speed up after turning around, it should make sense that the maximum speed comes when the object is passing through the equilibrium location. This would be at 1 s, 3 s, 5 s and so on for the motion depicted in Figure 11.2.

Is the velocity always positive?

The velocity, like the position, can be positive or negative. When the object is moving toward the maximum positive location, the velocity is positive.

When the object is moving back the other way, toward the maximum negative location, the velocity is negative.

Note that the velocity can be negative even though the position is positive (and visa-versa). For example, consider the situation where the object has just turned around and is moving back toward the equilibrium position.

✓ *Checkpoint 11.3: (a) For the motion depicted in Figure 11.2, suppose the object has passed the equilibrium point of an oscillation and is moving in the negative direction. Which way is the object moving: toward the* $+2$ *cm position or the* -2 *cm position?*
(b) Suppose the object has turned around at the -2 *cm position and is returning to the equilibrium position. What is the sign of the object's velocity as it returns to the equilibrium position?*

11.3 Acceleration

Now that we know where the velocity is positive, zero and negative, let's turn our attention to the acceleration.

We can do this two ways. One way is to consider the law of force and motion. The object's acceleration is in the same direction of the net force. Since the net force is always toward the equilibrium position, that means the acceleration is always toward the equilibrium position.

For example, if the object is oscillating up and down, then the net force is directed downward when the object is above the equilibrium position and the net force is directed upward when the object is below the equilibrium condition. Similarly, the object's acceleration is directed downward when the object is above the equilibrium position and the object's acceleration is directed upward when the object is below the equilibrium condition.

WHAT ABOUT WHEN THE OBJECT IS AT THE EQUILIBRIUM POSITION?

When the object is at the equilibrium position the net force on it is zero. Similarly, its acceleration is zero also.

WHAT ABOUT WHEN THE OBJECT IS AT ITS MAXIMUM DISPLACEMENT FROM THE EQUILIBRIUM POSITION?

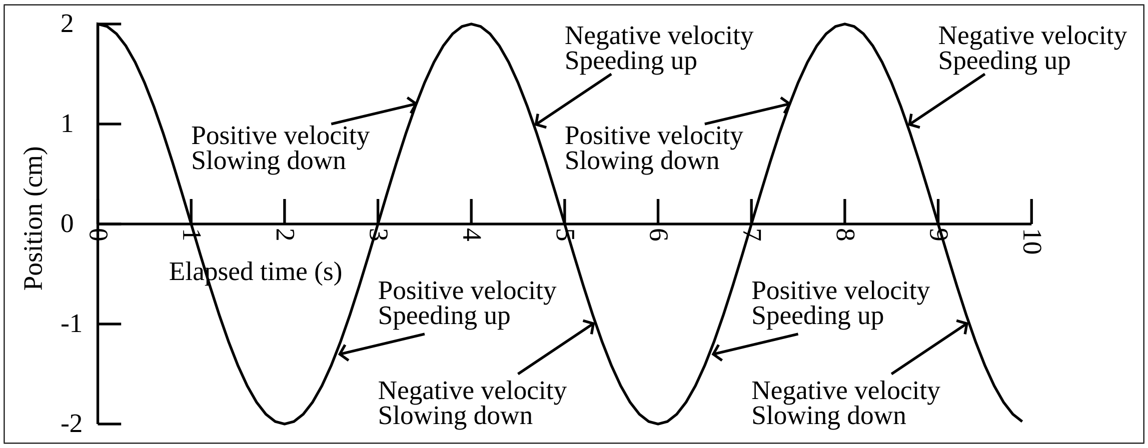

Figure 11.3: The same position vs. time graph from Figure 11.2 but with information about its motion.

When the object is at its maximum displacement from the equilibrium position, the net force on it is also at its maximum. Similarly, its acceleration is at its maximum, as the velocity switches directions.

What is the other way to figure out the acceleration direction?

The other way to determine the direction of the acceleration is to look at how the velocity is changing. This can be a little more illuminating that simply considering the law of force and motion.

Recall that if an object is speeding up, its acceleration is in the direction of motion, whereas if the object is slowing down, the acceleration is opposite the motion.

Thus, to figure out the direction of the acceleration, we simply need to analyze the motion and identify which way the object is moving and whether it is speeding up or slowing down.

To help with our analysis, in Figure 11.3 I've repeated the position vs. time graph from Figure 11.2 but have indicated which way the object is going and whether it is speeding up or slowing down.

The object speeds up as it moves toward the equilibrium position. In Figure 11.3, this would be the time period from 0 to 1 second, from 2 to 3 seconds, from 4 to 5 seconds, and so on.

The direction of the acceleration during these times is in the direction of motion. Again considering the oscillation depicted in Figure 11.3, this means the acceleration is negative from 0 to 1 seconds, positive from 2 to 3 seconds, negative from 4 to 5 seconds, and so on.

Now consider the times when the object is slowing down. In Figure 11.3, this would be the time period from 1 to 2 second, from 3 to 4 seconds, from 5 to 6 seconds, and so on. During these times, the direction of the acceleration is opposite the direction of motion.

For example, when the object is moving away from the equilibrium position and slowing down, as between 3 and 4 seconds, the velocity is positive and the acceleration is negative. In comparison, after the object reaches its maximum displacement from equilibrium (at 4 seconds), the object is moving toward the equilibrium position and speeding up, which means the velocity is negative and the acceleration is negative.

What happens right at 4 seconds?

Right at 4 s, the object's instantaneous velocity is zero as it turns around.

The velocity does not remain zero, however. Consequently, the acceleration is not zero at 4 seconds. Instead, it is in the negative direction, as the velocity switches from positive to negative.

Indeed, the object's acceleration is *negative* for the entire period from 3 to 5 seconds, when the object's position is *positive*. This is consistent with the law of force and motion, in that the net force (and thus the acceleration) is always toward equilibrium and thus opposite the displacement from equilibrium.

✍ Remember that velocity is not the same as acceleration. When the velocity is zero (at positions +2 cm and −2 cm in the figure), the velocity is still undergoing a change of sign (from positive to negative or negative to positive) and so the acceleration is not zero then (even though the velocity is zero).

Example 11.2: An object is undergoing an oscillation as depicted in Figure 11.2. When the object is at moving toward +2 cm, having passed the equilibrium position, what is the sign of the acceleration? Explain.

Answer 11.2: At that moment the acceleration is negative, since the object is slowing down while moving in the positive direction.

✓ *Checkpoint 11.4: An object is undergoing an oscillation as depicted in Figure 11.2. When the object is at moving toward −2 cm, having passed the equilibrium position, what is the sign of the acceleration? Explain.*

11.4 Terminology

As mentioned in the introduction, there are many physical situations that exhibit a back and forth motion. Besides an object oscillating on a spring, there is also the motion of a pendulum, the AC current in an electrical circuit, and many molecular vibrations. Even the up and down motion of a boat on the ocean and the back and forth motion of electrons in an antenna exhibit a back and forth motion that can be described using the approach described in this chapter. In this section, I'll look at the terminology we use to describe such back and forth motion.

11.4.1 Amplitude

For the oscillation plotted in Figure 11.2, the object starts out at a position +2 cm from the equilibrium position and moves toward the equilibrium position, passing it at 1 s at which point it moves to the other side of the equilibrium position (the negative side), reaching a position −2 cm at 2 s.

At 2 s, it turns around and returns to the equilibrium position, reaching the equilibrium position at 3 s. Since it is moving when it reaches the equilibrium position, it continues moving past the equilibrium position but slows down, reaching its original position, +2 cm, at 4 s, at which point it turns around and repeats the cycle.[ii]

Notice how the equilibrium position is midway between the two extremes on side. The farthest it gets, on either side, is 2 cm. The general term for the maximum distance from equilibrium is called the **amplitude**. The amplitude in this case is 2 cm (see Figure 11.4).

❢ For oscillations, the *amplitude* of the motion is equal to the *maximum* displacement from equilibrium.

[ii]This is similar to the back and forth motion of the pirate ship ride at an amusement park, if you happen to be familiar with that ride.

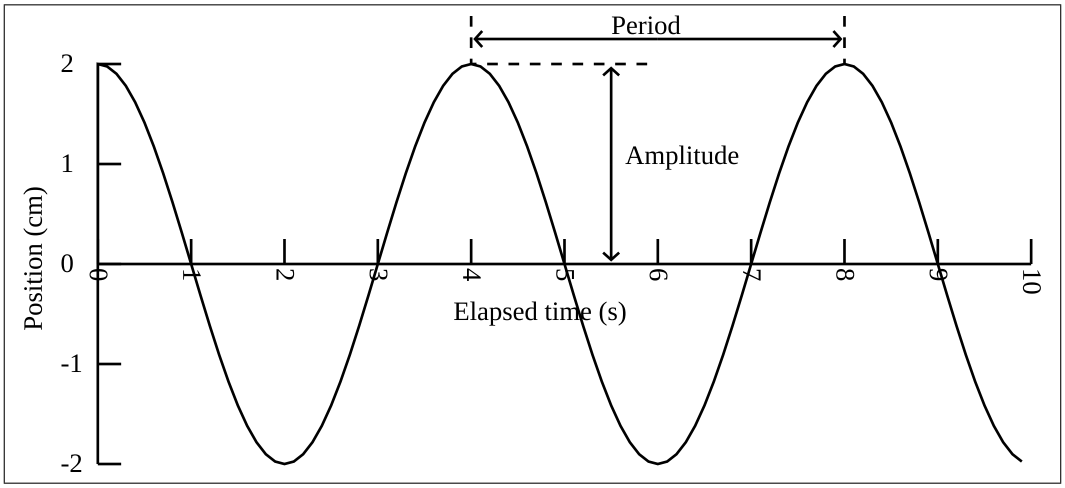

Figure 11.4: The same position vs. time graph from Figure 11.2 but with the period and amplitude of the oscillation indicated.

In mathematical expressions, I'll tend to use A to indicate the value of the amplitude. The object would oscillate, then, between the two maximum positions at $+A$ and $-A$.

Example 11.3: An object is released from rest 3 cm from the equilibrium position at which point it undergoes an oscillation, reaching a maximum of 3 cm on each side of the equilibrium position. Each extreme is 6 cm from each other. What is the amplitude of the motion?

Answer 11.3: The amplitude is defined as the maximum distance the object gets from the equilibrium position, which is 3 cm in this case.

✓ *Checkpoint 11.5: A block is released at rest from a point 4 cm from equilibrium. It then undergoes an oscillation with an amplitude of 4 cm and a period of 3 seconds. What is the maximum distance the object will ever be from the equilibrium position?*

11.4.2 Cycle

When describing oscillations, we say that an object completes one **cycle** when it returns to its initial position and velocity in the back-and-forth motion.

Consider, for example, the oscillation described by the position vs. time graph in Figure 11.4 completes one cycle every four seconds. We can tell because it starts with a position at +2 m and returns to that position 4 seconds later.

To identify one cycle, we could start at any point, as long as we are careful to determine when it returns to that position and velocity. For example, the object represented in Figure 11.4 is at −2 m at 2 s and returns to that position at 6 s, which is four seconds later.

COULD WE IDENTIFY A CYCLE STARTING IN THE MIDDLE OF THE OSCILLATION?

It doesn't matter where one starts, but it can be a little tricky identifying the cycle if you start in the middle of the oscillation.

For example, consider a child swinging on a swing. If we start a timer at the moment the child is at the bottom of the swing, as it is swinging leftward, one cycle is completed <u>not</u> when it *first* returns to its initial position but rather the *second* time. The first time it returns to its initial position, it is moving back rightward. The second time, it is again swinging leftward, as it did at the beginning of the cycle.

Remember, one cycle is completed when the object not only returns to its initial position but also when its velocity is in the same direction as its initial velocity.

✓ *Checkpoint 11.6: According to the position vs. time graph in Figure 11.4, the object is at the equilibrium position at 1 s and moving in the negative direction. When is the next time it is at that position and moving the same direction?*

11.4.3 Period

The **period** is the time it takes for an object to complete one cycle. For the situation depicted in Figure 11.4, the period is 4 seconds, as indicated on the

❢ The period is the time it takes for an object to complete one cycle.

graph.

Example 11.4: Suppose an object oscillates with an amplitude of 4 cm with a period of 2 s.
(a) After 0.5 s, what fraction of a cycle has the object experienced?
(b) If it starts at a position all the way on the right, where is it after 0.5 s?
(c) If it starts at a position $s = 4$ cm, where is the object 0.5 s after the start?

Answer 11.4: (a) In this instance, the period is 2 seconds. After 0.5 seconds, it has completed one-quarter of a cycle.
(b) Since it started all the way on the right, it must be in the middle of the oscillation.
(c) The time is the same as in part (b), so we already know that the object has completed one-quarter of its cycle. However, the starting position is given as $s = 4$ cm. Since it states that the amplitude is 4 cm and that is where it started, we know that the object must have started at one end of its back-and-forth motion (like at a position all the way on the right). Thus, this is exactly like part (b) and has the same answer: after 0.5 seconds it must be in the middle of the oscillation.

✓ *Checkpoint 11.7: An object undergoes an oscillation with a period of 3 s and an amplitude of 2 cm.*
(a) After 1.5 s, what fraction of a cycle has the object experienced?
(b) If it starts at a position all the way on the right, where is it after 1.5 s?
(c) If it starts at a position $s = -2$ cm, what is the value of s after 1.5 s?

Summary

This chapter examined the back and forth motion associated with oscillations (like the motion exhibited by an object oscillating back and forth on a spring).

The main points of this chapter are as follows:

- The equilibrium position is the location where the object will remain stationary if released at rest there.

- During oscillations, the velocity is zero at the moment the object is turning around whereas the acceleration is zero at the moment the object crosses the equilibrium position.
- When the force acting on an object is directed toward some equilibrium position (such as with a spring) then the object will undergo an oscillation about the equilibrium position.
- For oscillations, the *amplitude* of the motion is equal to the *maximum* displacement from equilibrium.
- The period is the time it takes for an object to complete one cycle.

Frequently Asked Questions

Other than springs, are there situations where an object oscillates because of a force directed toward some equilibrium position?

Yes. Examples include a pendulum (for small amplitudes) and a tight string (as in a stringed instrument like a violin).

What is the force due to the spring when the spring is neither stretched nor compressed?

The spring exerts no force at that position. That is why that position is called the equilibrium position.

What is meant by the spring's equilibrium position?

The spring is in its equilibrium position when it is relaxed and neither stretched or compressed from its "normal" length.

Is the velocity equal to the amplitude divided by the period?

No – the object does not travel a distance equal to the amplitude in the length of time equal to the period.

Is the object's maximum speed equal to the amplitude divided by the period?

No. To see why, consider the average speed of the object as it moves from one extreme, when its speed is zero, to the equilibrium position, when it achieves its maximum speed. The average speed would be the distance it moved, equal to the amplitude in this case, divided by how long it took, which is equal to *one-quarter* of the period in this case (since it has only completed one-quarter of the cycle).

Using A and T as the amplitude and period, respectively, this means that during this quarter of the cycle the object's *average* speed is A divided by $T/4$, which is the same thing as multiplying A by the inverse of $T/4$. That means the speed is equal to $4A/T$, or four times greater than just A/T.

So is $4A/T$ equal to the object's maximum speed?

No. The object's maximum speed would be *greater* than its average speed.

Would the object's maximum speed be twice as big as its average speed?

No. Only if the object was experiencing a constant acceleration then the maximum speed would be twice the average, since the minimum speed is zero. However, an oscillating object is not experiencing a constant acceleration.

Terminology introduced

Amplitude	Equilibrium position	Period
Cycle	Oscillations	Periodic

Additional problems

Problem 11.1: Suppose an object oscillates with an amplitude of 3 cm with a period of 4 seconds. The object is initially at a position equal to +3 cm.
(a) At what time is the position equal to −3 cm?
(b) At what time does the position become negative for the first time?
(c) Is there any time when the position is equal to +4 cm? If so, when? If not, why not?

Problem 11.2: A block is released at rest from a point 4 cm from equilibrium. It then undergoes an oscillation with an amplitude of 4 cm and a period of 3 seconds. One second after being released,
(a) Has the object passed the equilibrium position? Explain.
(b) Is it moving away from its initial position or toward? Is it moving away from the equilibrium position or toward? Explain.
(c) Is the object speeding up or slowing down? Explain.

Part C

Using Gravity

12. The Law of Gravity

Puzzle #12: When you release a rock, it falls to Earth. Why is it pulled toward Earth rather than toward the Sun, the moon, or anything else?

Introduction

In this part of the book we will take what we've learned in part B and use it along with the force and motion relationship learned in part A to examine how objects fall to Earth when they are released. This means there must be a force exerted on the object, forcing it downward, but what is that object interacting with? That interaction is the focus of this chapter.

12.1 Properties of gravity

The force responsible for pulling objects downward to Earth is the gravitational force. In this chapter, I'll introduce the **law of gravity**[i], which describes the properties of the gravitational force. And, like the law of force and motion, there is a corresponding equation, called the gravity equation that represents and quantifies the law.

First, though, we need to identify the characteristics of gravity that are described by the law of gravity and the gravity equation. I'll go through those characteristics in this section.

[i]The law of gravity is often called the **universal law of gravitation**. Some call it **Newton's law of gravity**, since it is one of several scientific laws identified by Isaac Newton during his lifetime.

12.1.1 Interactions

The first thing to recognize about gravity is that, like every other force, it is due to an *interaction* between two objects. You need two objects for the gravitational force to exist. It is not associated with a single object.

For example, when two objects, A and B, interact gravitationally then, due to that interaction, there is a "gravitational force on object A due to its interaction with object B" and a "gravitational force on object B due to its interaction with object A".

Unfortunately, saying "the gravitational force on object A due to its interaction with object B" is quite a mouthful. Consequently, I'll tend to just write "the gravitational force on object A due to object B."

Such phrasing is a bit risky, as the gravitational force on object A is not due to just one object – object B. Rather, it is due to the A-B interaction (i.e., both objects A and B). You just have to remember to mentally interpret "due to object B" as "due to its interaction with object B".

Some people will say "the gravitational force *of* object B on object A." I will not, as I think it is a bit too misleading. The force doesn't belong to object B. Rather, it belongs to the *interaction* between the two objects. Without *both* objects, the force wouldn't exist.

In the same way, it isn't appropriate to assign the force to just one of the two interacting objects or to say that an object "has" a force, just as it would be inappropriate to say that either object "has" a gravitational force, since the force is not just associated with the single object.[ii]

✍ For the same reason, I will avoid writing "the force of gravity," since it seems to imply that there is this thing called "gravity" that contains the force.

In all cases, then, the force depends on the properties of both interacting objects, not either one separately. And, in the same way that the magnetic force on the paper clip depends not only on the strength of the magnet, but also on the size and magnetic properties of the paper clip, the gravitational force associated with the two objects depends on the properties of both objects.

[ii]This is one of the many ways in which the word "force" is differently in physics than how you might use it in ordinary life.

To say the same thing the other way, you can't predict or understand the gravitational force until you look at *both* of the objects involved.

✓ *Checkpoint 12.1: Why is the phrase "the gravitational force due to the sun" preferable to "the gravitational force of the sun"?*

The law of interactions not only states that forces are due to interactions but it also states that the force, associated with that interaction, has the same magnitude on each object.

That means that when you interact gravitationally with Earth, the force pulls equally on both you and Earth, bringing you together. It is as though there is an invisible spring pulling you and Earth together.

If the forces have the same magnitude, why do you fall to Earth rather than Earth moving up to you?

As mentioned in chapter 5, it is important to recognize that "force" is not the same as "effect." When a truck and a fly collide, for example, the magnitude of the force on the fly (due to the truck) is the same as the magnitude of the force on the truck (due to the fly). However, the effect on the fly is much greater.

Earth is very massive. That means the impact of the gravitational force on Earth (due to its interaction with you) has little impact on Earth's motion, and so it appears as though there is no gravitational force on Earth at all. However, there is. And it is just as great on Earth (pulling it toward you) as it is on you, pulling it toward Earth. It is just that the effect is much, much different.

So, as you fall to Earth, the gravitational force (due to Earth) pulling you down toward Earth is equal in magnitude to the gravitational force (due to you) pulling Earth up toward you but the effects are quite different because Earth is so much more massive (consistent with the law of force and motion). In fact, since Earth is so much more massive, it does not experience a measurable change in velocity during the interaction.

What evidence is there that the forces are exactly the same in magnitude then?

Given our imperceptible effect on Earth, it is difficult to tell if there is, indeed, a force pulling Earth upward (and with the same magnitude as the force on us). However, there are two ways we can show that the law of interactions also applies to gravitational interactions.

One way is to examine how two objects interact via other non-contact forces, like the electric or magnetic forces. For example, we can observe how two magnets interact and show that the forces on each magnet are equal in magnitude and opposite in direction. We can then conclude that the same must be true for the gravitational force.

Another way is to observe the motions of two astronomical bodies, like the moon and Earth. The effect on each will be observable and allow us to infer the forces exerted on each.[iii]

✓ *Checkpoint 12.2: At this very moment, I am standing on Earth. According to the law of interactions, which is larger in magnitude: the gravitational force exerted on me due to Earth (pulling me downward toward Earth), the gravitational force exerted on Earth due to me (pulling Earth upward toward me), or are they equal in magnitude?*

12.1.2 Direction

As mentioned earlier, the gravitational force pulls objects together, like an invisible rubber band. As such, the direction of the gravitational force on one object is toward the other object. For example, the gravitational force on an object due to the sun is directed toward the center of the sun.

Does the gravitational force depend on how the object moves?

No. It doesn't matter if an object is in orbit[iv] around the sun or falling into the sun, the motion of the object does not impact the gravitational force. Regardless of an object's motion, the gravitational force on it due to the sun is always toward the sun.

✓ *Checkpoint 12.3: The moon is in orbit around Earth. In what direction is the gravitational force on the moon (due to Earth)?*

[iii] This is examined in section 22.6.
[iv] This is discussed in section 20.4.

12.1.3 Universality

One of Isaac Newton's great insights was that the gravitational force not only exists between, say, the sun and Earth, but between *any* two objects. Newton was able to see that the gravitational force only impacts both objects on Earth (like an apple) and astronomical objects (like the moon due to its interaction with Earth, and other planets due to their interaction with the sun).[v]

Not only that, but since gravity really is universal, an apple is not only attracted to Earth but also to everything else, including you, for instance. However, for most interactions the gravitational force is so weak that it has no impact on the interacting objects. To have a significant impact, the gravitational interaction must be with a massive object. That is why we can see the impact of the gravitational force on us due to Earth but not the gravitational force on us due to an apple.

This is discussed in more detail in section 12.2.

❢ The impact of the gravitational force is only evident when interacting with a very massive object.

The gravitational force also depends on how far apart the two interacting objects are. The nearer they are to each other, the greater the gravitational force. That is why there is a greater gravitational force on us due to Earth than, say, due to Neptune, which is much farther away from us.

✓ *Checkpoint 12.4: (a) Is there a gravitational force on you due to Earth?*
(b) A stapler sits on a desk across the room. Is there a gravitational force on you due to the stapler?
(c) Would the gravitational force on you due to the stapler be greater (in magnitude) if the stapler was closer to you?

12.1.4 Non-contact

Unlike the forces we've considered so far, gravity acts whether the two interacting objects are touching each other or not.

[v] The relationship between the gravitational force and orbits is examined further in sections 20.4 and 22.6.

In this sense, it is like the magnetic force. Suppose you brought a magnet close to a paper clip. At some point, the paper clip would get attracted to the magnet. In that case, the paper clip is interacting with the magnet even before they touch.

❢ The gravitational force acts even if the two objects are not in contact.

For the magnet and paper clip, the interaction is magnetic, so the attraction is due to the **magnetic force**[vi] not the gravitational force. However, the gravitational force is similar in that a rock is attracted to Earth even when the rock is not touching Earth's surface.[vii]

> ✍ Because gravity is a non-contact force, it can sometimes be tricky identifying when gravity is acting. Contact forces, on the other hand, are pretty straightforward to identify because we only need to identify which objects are actually touching. The contact forces can only exist when the interacting objects are touching.

✓ *Checkpoint 12.5: What is the name of the force associated with the following interactions?*
(a) The attraction of a magnet's north pole to another magnet's south pole
(b) The attraction of Earth to the sun

Since the gravitational force is a non-contact force, you can't "prevent" two objects from interacting gravitationally by, for example, placing another object between them. So, if a rock is being pulled toward Earth gravitationally, you can't remove that interaction by placing something between the rock and Earth. You can push the rock upward and thus *counter* the gravitational force that is pulling it downward but *both* forces are present – your upward force doesn't stop the gravitational force from acting.

✓ *Checkpoint 12.6: Suppose I am holding a rock at rest in my hand. Is the gravitational force (due to Earth) still acting on the rock?*

[vi]The magnetic force is discussed in more detail in volume II.

[vii]While only some objects experience a magnetic force, all objects attract each other via the gravitational force. It just may be very small.

12.2 Gravity equation

So far we have identified a couple of important characteristics about the gravitational force. The gravity equation expresses these ideas via a mathematical equation. I will first provide the gravity equation and then explain how the characteristics described in the previous section are reflected within the equation.

The **gravity equation** is as follows:

$$|\vec{F}_{\mathrm{g}}| = G\frac{m_1 m_2}{r^2} \tag{12.1}$$

In this expression, m_1 and m_2 represent the masses of the two interacting objects, r is their separation distance (measured from the center of one object to the center of the other) and G is a constant that is used for all gravitational interactions.[viii]

At first glance, this equation looks a little complicated. It is only complicated because it includes all of the properties described in the previous section. Let's go through them one by one.

1. It doesn't matter which mass one uses for m_1 and which for m_2.

   According to the expression, the two masses are multiplied together. Since the product of $m_1 m_2$ is the same as $m_2 m_1$, it doesn't matter which mass you use for m_1 and which you use for m_2. As discussed in section 12.1.1, this *has* to be the case, since the force on each object (of the interacting pair) is due to the interaction, not just one object or the other.

2. The expression works for *any* two objects.

   There is no restriction on which two masses we use for m_1 and m_2. This is because the gravitational force exists between *any* two objects.

3. Each mass represents the *entire* mass of each object.

[viii] As mentioned on page 341, I use capital R to indicate the radius of an object, whereas I use a lower-case r to indicate a general distance from the center (as in the gravity equation).

For the gravitational interaction between, say, a rock and Earth, the *entire* rock is interacting gravitationally with the *entire* mass of Earth. In other words, the rock is interacting with Earth's core, the mantle, India, Australia, and so on. Since the gravitational force is a non-contact force, the rock need not be *directly* in contact with every part of Earth in order to experience an attraction to it.

4. The gravitational force depends upon how far apart the objects are.

   In the gravity equation, r represents the distance from the center of object to the center of the other object. Since it is in the denominator of the gravity equation, a large value of r corresponds to a small value of $|\vec{F}_g|$. This is called an inverse proportion (see section 3.2.4).

   Note: r does <u>not</u> represent the radius of one of the objects.[ix] For example, for a rock on the surface of Earth, r represents the distance from the rock to Earth's *center*, not the distance from the rock to Earth's *surface*.

5. The gravitational force is insignificant unless one or both of the objects is very massive.

   The value of G, which is the same for all gravitational interactions, is $0.0000000000667430 \text{ N} \cdot \text{m}^2/\text{kg}^2$. This is very small. In fact, because the value of G is so small, we typically write the value using scientific notation (i.e., $6.67430 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$).

   The small value for G is consistent with the idea that the gravitational force is pretty small itself unless one, or both, of the interacting objects is very massive. In terms of the math, $|\vec{F}_g|$ is small unless the product Gm_1m_2 is large. And, since G is so small, the product Gm_1m_2 will likewise be small unless one or both of the m's is large.[x]

   Note: As discussed in section 12.1.1, the impact of the gravitational force is greater on the less-massive object in the pair.

[ix] As mentioned on page 341, I'm using r for distances and R for the radius of an object.

[x] Earth qualifies as something that is massive. Whereas my mass about 70 kg, Earth's mass is about 6×10^{24} kg (i.e., a six with 24 zeros after it).

Does the value of G depend on the situation?

No. It remains the same for every calculation regardless of what objects are involved or how far apart they are. In other words, the value of G is the same regardless of what objects you are considering (Earth, Sun, Moon, rock, person, rabbit, etc.), and regardless of their masses and their separation distances.

❢ The gravitational constant, G, always has the same value, regardless of what objects are involved or how far apart they are.

It is for this reason that the constant G is known as the **gravitational constant**.

Why does the value of G have units of $\mathrm{N \cdot m^2/kg^2}$?

The units of G appear to be complicated because the units must work with the SI units of $|\vec{F}_\mathrm{g}|$, r, m_1 and m_2.[xi]

How do we know that the value of 6.67430×10^{-11} is correct?

One way we've verified this value is by checking that it is consistent with the orbital characteristics of the planets.[xii] Scientists have also made very careful measurements of the gravitational attraction between small, heavy objects.

✓ *Checkpoint 12.7: In SI units, what is the value of G, the gravitational constant?*

The small number for G is a consequence of our choice to use SI units (i.e., newtons, meters and kilograms). SI was designed to be convenient for ordinary laboratory-sized masses, distances, and forces – such as a few kilograms, a few meters, and a few newtons – so the tiny number in the gravity equation (when used with SI units) tells you that the gravitational interaction of two ordinary objects acting on each other is quite tiny. This is why you are not likely to notice it between two ordinary objects (although this tiny force can be measured using special instruments).[xiii]

[xi] The SI unit of mass is kg. The SI unit of distance is m. When you calculate $m_1 m_2 / r^2$, you get SI units of $\mathrm{kg^2/m^2}$. Since the gravitational constant has SI units of $\mathrm{N \cdot m^2/kg^2}$, multiplying by the gravitational constant converts the units to newtons, which is a unit of force. If you wanted to convert it to another unit of force the number associated with G would be different.

[xii] This is discussed further in section 22.6. It is through the same process that we can show that the force is inversely proportional to r^2 instead of something like r.

[xiii] The idea that you can exert forces on objects without touching them might seem

So, for our purposes, we will ignore the gravitational force *except* in those situations where one or both of the objects are very massive (like Earth or the sun). In other words, we will assume that everyday objects (like you, me or ball) don't exert a measurable force on each other unless they are actually in contact, in which case the force is due to contact (to be discussed in chapter 15) rather than gravity.

✓ *Checkpoint 12.8: Suppose a 1-kg ball was sitting on a table two meters away from me. Is it reasonable to assume there is no gravitational force due to me on the ball? Explain.*

12.3 Using the gravity equation

To illustrate how to use the gravity equation, let's use it to determine the magnitude of the gravitational force on me due to Earth.

✍ As mentioned in a previous note, the direction of the gravitational force is always attractive and toward the center of the other object. Thus, the gravitational force on me due to Earth is directed downward, toward the center of Earth.

To calculate the magnitude of the gravitational force on me due to Earth, I need my mass. My mass happens to be 70 kg. I also need the mass of Earth (since it is the other object I am interacting with) and the distance between my center and the center of Earth. The gravitational constant is already given.

Earth's mass is 5.9723×10^{24} kg and its mean radius is 6.371×10^{6} m (both are given in the supplemental readings). The mean radius is the average distance from Earth's center to a point on Earth's surface. Since I am standing on Earth's surface, that distance is approximately equal to the distance from Earth's center to my center.

SHOULDN'T WE ADD HALF YOUR HEIGHT TO THE RADIUS OF EARTH?

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magical and conjure up images of Harry Potter and using wands. However, it is true – you *can* exert a force on an object without touching it. We call that the gravitational force. It is just that the force is too small to have any effect on that other object.

Perhaps, but my height is so small compared to the radius of Earth that adding a meter or so will not make the distance much different from the radius of Earth. Besides, I don't even know the radius of Earth within one meter. The number in the supplemental readings (6.371×10^6 m) is already rounded to the nearest kilometer.

Now that we have the mass of Earth and the distance from my center to Earth's center, we can calculate the magnitude of the gravitational force on me using the gravity equation:

$$|\vec{F}_{\mathrm{g}}| = G\frac{m_1 m_2}{r^2}$$

Plugging in what we know for each quantity,

$$|\vec{F}_{\mathrm{g}}| = 6.67430 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}\frac{(70\ \mathrm{kg})(5.9723 \times 10^{24}\ \mathrm{kg})}{(6.371 \times 10^6\ \mathrm{m})^2}$$

I get a gravitational force of magnitude 690 N exerted on me due to Earth.

How far can we go above Earth's surface before we can no longer use 6.371×10^6 m as the separation distance?

Earth is a really big object, so 6.371×10^6 m is quite a large distance. As long as one's height above Earth's surface remains small in comparison, one can use 6.371×10^6 m as the distance to the center of Earth.

A quick calculation (see the scaling section of the supplemental readings) reveals that at an altitude of 3.19×10^4 m the magnitude of the gravitational force on an object is only 1% smaller than it would be on the surface of Earth.

How far up is 3.19×10^4 m?

About 20 miles.[xiv]

This is pretty far up. As you can see, I can't jump high enough to make any significant difference in the value of r or Earth's gravitational force on me. Even if I go on top of a mountain or in an airplane (5 miles high), the gravitational force on me (due to Earth) is pretty much the same as when I am at sea-level.

[xiv] Since there are 1609.344 meters in each mile (see supplemental readings), we can divide 3.19×10^4 m by 1609.344 m/mi to get 20 mi.

✓ *Checkpoint 12.9: Suppose I stand on a ladder so that I am 2 meters above the floor (and so 2 meters further from the center of Earth).*
(a) Should the two meters affect the magnitude of the gravitational force? Why or why not?
(b) Test your answer to (a) by calculating the gravitational force on me in both cases (i.e., once using a distance equal to the radius of Earth and again using a distance equal to the radius of Earth plus the two meters). Do not round your answers.

At what height does the gravitational force due to Earth become zero?

Technically, the magnitude never becomes zero, it just gets smaller and smaller. The r is in the denominator (bottom part of the fraction), which means that the farther apart two objects are, the less the gravitational force exerted on each other.[xv]

The gravitational force on astronauts on board the International Space Station, in orbit about Earth, is actually pretty close to what it would be if they were on Earth. This is because they orbit not far from Earth's surface (about 200 miles).

One can use proportions to determine the gravitational force without going through the entire equation, but you need to keep in mind that the distance r is squared in the expression. For example, we'd need to double r in order to get a *quartering* of the gravitational force magnitude. Basically, we are applying the impact of the new r twice. See the scaling section of the supplemental readings for more details.

If the gravitational force on the astronauts is so strong, why do they appear to be weightless?

This will be explained in chapter 20. Basically, the gravitational force *appears* to be zero because the astronauts are in a spacecraft that is also in orbit. Since the astronauts are moving with the spacecraft, this leads to the perception that there are no forces acting on them (i.e., no gravitational force

[xv]As mentioned earlier, the inverse dependence on the separation distance explains why we are attracted to Earth more than to, say, Jupiter. Even though Jupiter is more massive than Earth, Earth is so much closer to us than Jupiter is to us.

acting on them).[xvi] This idea of relative motion is discussed in section 20.5 and the physics of orbits is discussed in section 22.6.

✓ *Checkpoint 12.10: According to the gravity equation, what happens to the magnitude of the gravitational force if the distance from one object to the other triples? Explain how you used the gravity equation to obtain your answer.*

Summary

This chapter examined the way objects interact gravitationally. The main points of this chapter focused on the gravitational force:

- The impact of the gravitational force is only evident when interacting with a very massive object.
- The gravitational force acts even if the two objects are not in contact.
- The gravitational constant, G, always has the same value, regardless of what objects are involved or how far apart they are.

By now you should be able to recognize that there is a gravitational force between any two objects, with the magnitude dependent on their masses and the distance between them, and use this relationship (called the law of gravity) to calculate the gravitational force on objects.

Frequently Asked Questions

Does the gravitational force depend on how the object is moving?

No. See page 190.

For gravity to exist, does one of the objects have to be Earth?

No. See page 193.

[xvi] NASA tends to call this situation an environment of "micro-gravity." This is because they are using a definition of the gravitational force that is frame dependent (see section 20.7), not what is given by the gravity equation.

Does every object exert a gravitational force on every other object?

Yes, but the gravitational force will be insignificant unless one of the objects is very massive (like the moon, Earth or sun).

Why don't we notice a gravitational force between us and the moon? Isn't the moon massive enough?

While the moon is massive enough to warrant a significant gravitational force, the gravitational force not only depends on the masses of the two objects but also how far apart they are. The moon is so far away that its gravitational force on the rock is not as great as Earth's.

So if we were on the moon instead of Earth, we'd notice a force between the rock and the moon?

Yes.

But there is no air on the moon. Don't we need air to have gravity?

No. The gravitational force is not due to the air pushing on you. This is discussed further in section 13.3.4.

Keep in mind that not only don't we need air in order to have objects attract gravitationally, but the two objects involved do not have to be touching at all.

Is the gravitational force the same thing as your inertia?

No. Inertia is the term we use to describe how an object wants to keep doing what it is doing (see chapter 1). An object's **inertia** has to do with itself, not its interaction with other objects. For example, you have inertia even if there are no other objects around.

Like other forces, the gravitational force is due to an *interaction* between two objects. For example, the gravitational force on you is due to your interaction with Earth (and is directed downward). The gravitational force was discussed in more detail in chapter 12.

Terminology introduced

Earth	Newton's law of gravity
Gravitational constant	Scaling
Gravitational force	Units
Gravity equation	Universal law of gravitation
Law of gravity	

Problems

Problem 12.1: At this very moment, I am standing on Earth. Which is larger in magnitude: the force exerted on me due to Earth, the force exerted on Earth due to me, or are they equal in magnitude?

Problem 12.2: Show that the units of the gravitational constant G, normally given as $\mathrm{N \cdot m^2/kg^2}$, can be expressed as $\mathrm{m^3/(s^2 \cdot kg)}$. In other words, show that $\mathrm{N \cdot m^2/kg^2}$ is equivalent to $\mathrm{m^3/(s^2 \cdot kg)}$.

Problem 12.3: According to the gravity equation, what does the separation distance r have to do in order for the magnitude of the gravitational force to decrease?

13. Gravitational Field Strength

Puzzle #13: If the gravitational force is something exerted upon objects, rather than something that objects "have", why do some people refer to Earth's gravity, as though that is something Earth "has"?

Introduction

To determine the gravitational force acting on the object, we can use the gravity equation introduced in chapter 12. If that equation seems complicated, don't worry. There is a simpler version that we'll use. However, with simplicity comes a major disadvantage – you need to be careful when you use it.

In this chapter, I'll introduce a simpler gravity equation and explain the conditions in which it can be used.

13.1 A special case

The gravity equation introduced in chapter 12 is universal, meaning that it can be used to find the gravitational force on *any* object due to any other object.

However, for most of situations we'll examine in this book, we'll need to find the gravitational force for a very specific situation, namely one where we want the gravitational force due to Earth on objects that are at or near Earth's surface.

For example, consider the following three situations:

1. The gravitational force due to Earth on a 0.2-kg bird that is flying 400 m above the surface of Earth.

2. The gravitational force due to Earth on a 1-kg rock that is on the surface of Earth.
3. The gravitational force due to Earth on a 40-kg child on a swing 1 m above the surface of Earth.

In each case, the object (bird, rock or child) is at or near Earth's surface and it is the gravitational force due to Earth that is acting on the object.

Given that we encounter such a situation over and over again, it can be tedious to use the universal version introduced in chapter 12:

$$|\vec{F}_{\rm g}| = G\frac{m_1 m_2}{r^2}$$

In its place, we'll use a simpler version. Rather than show you the simpler version right off the bat, I will instead show you where it comes from. So, be patient and it will soon become clear, hopefully, what the simpler expression is, why it works, and how to use it.

Basically, what makes the gravity equation seem complicated, perhaps, is the presence of the four quantities on the right side: G, m_1, m_2 and r. For our simpler version, we'll only have *one* quantity instead of four.

To see why how we can get by with only one, let's examine the values we'd use for each of the four quantities in the law of gravity for the three situations described above (i.e., for the bird, the rock and the child).

The quantity G is a value that is always the same. It doesn't change from situation to situation. Consequently, we'd use the same value of G for all three situations.

The quantities m_1 and m_2 correspond to the masses of the two objects that are interacting. In our three situations, one of those two objects is Earth. That means for all three situations one of the two masses is the mass of Earth. The other mass depends on the situation (i.e., 0.2 kg, 1 kg or 40 kg).

The quantity r corresponds to the distance from the center of one object to the center of the other object. Since all three of our situations involve Earth, that means the distance in each case is the distance to the center of Earth.

Technically, that distance is different for each situation, with the bird being 400 m farther from the center of Earth than the rock. However, even 400 m is tiny compared to the size of Earth, which has a radius of 6,371,000 m. As mentioned in section 12.3, that means we can use the radius of Earth as the center-to-center distance for all three situations.

So what does this all mean?

It means that G, r and one of the m's have values that are the *same* for all three situations. The only quantity that changes from case to case is the mass of the object upon which the gravitational force is being exerted.

To make it clear that those three quantities are the same for each case, let's rewrite the expression with those quantities separated from the single quantity that does vary. Below I write the right-hand side of the gravity expression for each of the three situations:

$$m_{\text{bird}}\frac{GM_{\text{earth}}}{R_{\text{earth}}^2} \qquad m_{\text{rock}}\frac{GM_{\text{earth}}}{R_{\text{earth}}^2} \qquad m_{\text{child}}\frac{GM_{\text{earth}}}{R_{\text{earth}}^2}$$

From left to right, we have the gravitational force on the bird due to Earth ($|\vec{F}_{\text{g}}|$ on bird), the gravitational force on the rock due to Earth ($|\vec{F}_{\text{g}}|$ on rock) and the gravitational force on the child due to Earth ($|\vec{F}_{\text{g}}|$ on child). Notice that the only thing that varies in each case is the mass of the particular object. Everything else is the same.

Rather than plug in all four quantities into the gravity equation all three times, we could instead just calculate the following quantity once and then use that for each situation:

$$\frac{GM_{\text{earth}}}{R_{\text{earth}}^2}$$

The three quantities in this expression are the gravitational constant G ($6.67430 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$), the mass of Earth M_{earth} (5.9723×10^{24} kg) and the radius of Earth R_{earth} (6.371×10^6 m). When we plug those values into the expression, we get 9.8 N/kg, and we can use that value for all three situations.[i] That means we could just use the following to determine the gravitational force due to Earth on our three objects:

$$m_{\text{bird}}(9.8\ \text{N/kg}) \qquad m_{\text{rock}}(9.8\ \text{N/kg}) \qquad m_{\text{child}}(9.8\ \text{N/kg})$$

Of course, this can be used for *any* time we need to find the gravitational force due to Earth on an object at or near Earth's surface. Simply multiply

[i]This value is only an approximation because of rounding. Some people use 9.81 N/kg. Others use 10 N/kg.

that object's mass by 9.8 N/kg. Mathematically, this can be written as follows:[ii]

$$|\vec{F}_{\mathrm{g}}| = m(9.8\ \mathrm{N/kg}) \tag{13.1}$$

To distinguish this equation from the one in chapter 12, from now on I'll refer to that one as the **universal gravity equation** and I'll refer to equation 13.1 as the **simplified gravity equation**. Both will give the same result when the object of mass m is interacting gravitationally with Earth. And, since that will almost always be the case from now on, we'll be using the simplified gravity equation almost exclusively from now on.

If it is so easy to figure out the gravitational force this way, why would anyone ever use the universal gravity equation?

The simplified gravity equation only works for calculating the gravitational force due to Earth on objects that are on or near Earth's surface. If you change either one (i.e., due to some other object besides Earth or exerted on an object that is not on or near Earth's surface), you cannot use this version.

How near to the surface of Earth does an object have to be in order to use this version?

As discussed in chapter 12, as long as the object is within 20 miles of the surface, we can use the radius of Earth as r and the force value will be correct within 1%. In other words, as long as the object is not very, very high (or in a very, very deep hole), the gravitational force is relatively independent of one's height above Earth's surface and we can use a value of 9.8 N/kg.

Example 13.1: Use the simplified gravity equation to determine the magnitude of the gravitational force due to Earth on an 8-kg bowling ball.

Answer 13.1: According to the simplified gravity equation, the magnitude of the gravitational force on the bowling ball due to Earth is the mass (8 kg) times (9.8 N/kg). This gives 78.4 N.

[ii] As will be discussed in section 20.7, the 9.8 N/kg value actually incorporates other factors besides just gravity, but those factors impact the value by only a small amount.

Example 13.2: According to the simplified gravity equation, how much greater is the magnitude of the gravitational force due to Earth on an 8-kg bowling ball than on a 0.01-kg Ping-pong ball?

Answer 13.2: According to the simplified gravity equation, the magnitude of the gravitational force due to Earth will be greater on the bowling ball since it is more massive. In fact, it will be 800 times bigger (since the mass is 800 times bigger).

✓ *Checkpoint 13.1: I throw a 0.3-kg ball in the air. When the ball is the air, what is the gravitational force on it (due to Earth)? Include units.*

13.2 The value of g

As discussed in the previous section, if you want to find the magnitude of the gravitational force due to Earth on an object that is on or near Earth's surface, you simply need to multiply that object's mass by 9.8 N/kg. You do not need to use the universal gravity equation provided in chapter 12.

WHAT IF WE WANT TO KNOW THE GRAVITATIONAL FORCE DUE TO SOMETHING OTHER THAN EARTH OR ON AN OBJECT THAT IS NOT ON OR NEAR EARTH'S SURFACE?

Then you have to use the universal gravity equation.

That being said, we can always go through the process described in the previous section to find a simple version of the gravity equation that happens to work for *any* situation.

For example, suppose we have a bunch of problems that involve determining the magnitude of the gravitational force due to the *moon* on objects that are on or near the moon's surface. Instead of multiplying the object's mass by

$$\frac{GM_{\text{earth}}}{R_{\text{earth}}^2}$$

we'd instead multiply it by

$$\frac{GM_{\text{moon}}}{R_{\text{moon}}^2}$$

That value turns out to be 1.6 N/kg. That means that if an object is on or near the moon's surface then we just need to multiply that object's mass by 1.6 N/kg to find the gravitational force on that object due to the moon.

Indeed, for *any* situation, to find the magnitude of the gravitational force on an object, we just need to multiply that object's mass by some factor. That factor will depend on what the other object is and how far away that other object is, but once we know the factor for that situation, we can then use it for any similar situation.

We call that factor, whether it is 9.8 N/kg or 1.6 N/kg, the **gravitational field strength** and we indicate it by the letter g. Mathematically, then, a simplified gravity equation can be written as follows:

$$|\vec{F}_{\text{g}}| = mg \tag{13.2}$$

where g is the gravitational field strength. The gravitational field strength can be calculated as follows:

$$g = G\frac{M}{r^2} \tag{13.3}$$

where M is the mass of the object exerting the gravitational force (e.g., Earth or moon) and r is how far one happens to be from that object.

IS g THE SAME THING AS G?

No. Be careful! It is common to confuse the two. The gravitational field strength (g) depends on the situation. The gravitational constant (G) never changes its value.

Since g and G refer to two different things, it can be tricky to tell which one is being referenced when you hear someone mention "gee". Unless the meaning is obvious from the context, the most common way to resolve the ambiguity is to refer to g as "little gee" and G as "big gee".

> ✍ Remember, the law of gravity applies to all situations (as far as we know), so the value of G does not depend upon the situation. The simplified version ($|\vec{F}_{\text{g}}| = mg$), on the other hand, has a value of g that depends on the situation. It is equal to 9.8 N/kg when dealing with the gravitational force due to Earth acting on an object near Earth's surface.

✓ *Checkpoint 13.2: When I am near the surface of Earth, the gravitational force on me due to Earth is around 700 N downward. If I was on the surface of Mars, the gravitational force on me due to Mars would be less. If I used mg to determine the gravitational force on me on Mars (due to Mars), would I use a value of g greater than 9.8 N/kg or less than 9.8 N/kg?*

13.3 Cautions

It is relatively simple to use mg to determine the gravitational force on an object. However, that simplicity also makes it easy to apply incorrectly. In this section, I go over some of the areas that can trip up students.

13.3.1 Force vs. field

Is g the gravitational force?

No.

First off, the gravitational force on an object isn't always the same (e.g., the gravitational force isn't always 9.8 N). Rather, the gravitational force on an object depends on the object's mass and will be larger on a more massive object. That is why you have to multiply g by the object's mass in order to find the gravitational force on it. For example, if an object's mass is 2 kg then the gravitational force on it (due to Earth) has a magnitude of $(2 \text{ kg}) \times (9.8 \text{ N/kg}) = 19.6 \text{ N}$ (assuming the object is on or near Earth's surface).

Second, the units are N/kg, not N. They may look similar but they are not the same thing.

It is for these reasons that g is called the gravitational field strength and not the gravitational force.

To illustrate the difference between the gravitational force and the gravitational field, consider the cost of gas at a local gas station. Usually the gas station advertises the cost of gas in terms of the price per gallon, like \$2.86/gallon. Certainly it does not cost just \$2.86 to fill up your entire tank with gas. To determine the cost to fill up your tank, you need to multiply the price per gallon by the number of gallons you put into your tank.[iii]

In a similar way, the gravitational force on a particular object has a magnitude with units of newtons, not newtons per kilogram. Only if you multiply the newtons per kilogram by the mass in kilograms will you get a value that has units of force. For the magnitude of the gravitational force on the object due to Earth, then, you need to multiply 9.8 N/kg by the object's mass in kilograms.

✍ Just as the cost for the gasoline is in dollars, not dollars per gallon, so is the gravitational force in newtons, not newtons per kilogram.

✓ *Checkpoint 13.3: Consider the three situations described on page 203 (0.2-kg bird, 1-kg rock and 40-kg child). All three are on or near the surface of Earth. On which object or objects is the magnitude of the gravitational force due to Earth equal to 9.8 N?*

13.3.2 Mass vs. gravitational force

Because the gravitational force is proportional to the object's mass, one may confuse the two or think they are the same thing (i.e., mass and the gravitational force). They are not. There are two important differences.

❢ Mass is not the same as gravitational force.

The first important difference between mass and the gravitational force is that the two are measured in different units. In SI units, that would be kilograms for mass and newtons for force.

The second important difference is that mass is something intrinsic to the object whereas a force is due to the object's interaction with another object.

An object always has a mass and it is always the same (unless you break part of it off, or add something to it). There is not necessarily a gravitational force on an object – it has to be interacting with another object.

[iii] While some people might call $2.86/gallon the "price of gas", it is actually the price *per gallon*. In a similar way, some people might call 9.8 N/kg the "force of gravity", but that is actually the force *per mass*.

✓ *Checkpoint 13.4: Consider the three situations described on page 203 (0.2-kg bird, 1-kg rock and 40-kg child). Each object has a value in kilograms. Is that the object's mass, the object's gravitational force or the object's gravitational field?*

13.3.3 Weight and pounds

I will avoid using the **pound** as a unit (typically used for weight).

Not only is the pound (abbreviated as lb) not a part of the International System of Units (see supplemental readings) but it is often used for *both* mass and force. The use of such a vague unit makes it hard to identify whether we are dealing with a mass or a force.

According to NIST (the National Institute of Standards and Technology), the pound is a unit of mass and is equal to 0.45359237 kg.[iv] That means that one kilogram is equal to about 2.2 pounds (obtained by taking the inverse of 0.45359237).

Example 13.3: My mass is 70 kg. What is my mass in pounds?

Answer 13.3: To determine my mass in pounds, I multiply 70 kg by 2.2 lb/kg to get 154 lb (or divide by 0.45359237 kg/lb). I would divide by 2.2 lb/kg (or multiply by 0.45359237 kg/lb) if I was given the reverse problem (i.e., given the weight in pounds and needing the mass in kilograms).

I will also avoid the use of the term **weight**, as the term suffers from the same ambiguity as pounds (i.e., it can be used for both force and mass).

On the one hand, most people in the United States give their weight in pounds, which is technically a unit of mass.[v] Indeed, a common usage is to

[iv] For more information, see the NIST guide to units at http://physics.nist.gov/Pubs/SP811/appenB8.html.

[v] NIST states that the poundal or pound-force is a unit of force, where one pound-force is equal to 4.448222 newtons. The 4.448222 value is not arbitrary. It is chosen so that an object of mass 1 pound will experience a gravitational force of magnitude 1 pound-force on Earth. Consequently, it is easy to convert between pound and pound-force. They have the same value (as long as you stay on Earth).

state the "weight of an object," which implies that the weight is a *property* of an object, like its mass.

On the other hand, many people also use the term to indicate how much something weighs or pushes down on a scale, which implies that it is a force. For example, one might say that an object is "weightless" in space, whereas the object's mass would be the same everywhere.[vi] This implies that weight is something that results from an object's interaction with its environment, like a force.

Because of the ambiguity associated with these two terms, I prefer to avoid them altogether.

✓ *Checkpoint 13.5: (a) Estimate your own mass in pounds.*
(b) From that estimate, determine your mass in kilograms by doing the conversion provided in the text.

13.3.4 Air

I know this chapter is about gravity but I'd like to take this opportunity to talk about air, just to make sure you realize that the gravitational force does not need air. For example, there is a gravitational force between the sun and Earth even though there is no air between the sun and Earth (except for the relatively thin layer of air on Earth that we call the atmosphere).

The incorrect idea that we need air to have gravity is due, perhaps, to the incorrect belief that forces *must* be the result of objects in contact. Air does exert a force but that force is not the gravitational force.

The force due to the air is called different things, depending on the situation, but regardless it is separate from the gravitational force. It can contribute to the *net* force (which is the sum of all the forces that are acting) but it does not impact the *gravitational* force, which is the attractive force between two objects due to their masses.

[vi] According to "Weight - An Official Definition" by Mario Iona in *The Physics Teacher*, Vol 37, p. 238 (April 1999), there is a standard definition of weight, given by the *International Organization for Standardization* (ISO) which says that the weight of a body in a specified reference system is that force which, when applied to the body, would give it an acceleration equal to the local acceleration of free fall in that reference system.

How do we know that gravity is not the result of air pushing down?

One can show that air is not necessary for gravity to act by removing the air in a tube and then dropping an object, like a rock, in the tube. You'd see that objects fall just the same inside the tube as objects outside the tube. You can also go to the moon, where there is no air. The astronauts who went there still experienced a gravitational force between them and the moon.

But doesn't the air push things down?

Yes and no. As mentioned in section 1.7, air is made up of lots of tiny little molecules that are bouncing around and hitting objects on all sides. Since there is less air the higher up in the atmosphere one goes, the force on the bottom of an object (pushing up) is slightly greater than the force on the top of the object (pushing down). This means that the air actually pushes things *up*, not down, and thus acts opposite to gravity.[vii]

❢ The gravitational force is not the result of air pushing down.

Not only is air not needed or responsible for the gravitational force pulling objects toward Earth, we will tend to ignore the air when identifying the forces acting on objects because the upward force of the air (on the bottom of the object) is very similar to the downward force of the air (on the top of the object).[viii]

When it is really windy, the wind can exert forces on things. How can we say that the force is insignificant?

❢ The force of the air is insignificant unless the object is very light or is moving very quickly.

It is insignificant for objects at rest (or moving slowly) with no wind. When the object is moving quickly through the air (or the air is flowing past an object), the force of the air in front, pushing backwards, can be significantly greater than the force of the air behind, pushing forward. This difference can cause a force called **air resistance** or **drag**, which opposes the motion (much like friction). For small speeds, typical of situations we'll be studying, the drag is small enough to be ignored.

[vii] This net upward force due to the air is called the **buoyancy** force, which is discussed in volume II. The buoyancy force tends to be small, but can become significant for very light objects, where the gravitational force on the object is also small.

[viii] If we wanted to know how much an object is "compressed," we'd need to know the contribution of the air. We just aren't interested in that, for the time being. For **thermodynamics**, such compression might become relevant.

✍ A crucial part of solving problems is being able to tell what is relevant and what isn't. It would be useful to try an experiment with and without air present to determine in which situations it is or is not relevant.

✓ *Checkpoint 13.6: Is it valid to ignore the force due to air on objects? Explain your reasoning.*

Summary

This chapter distinguished between (a) the gravitational force acting on an object and (b) the mass of the object. A simplified version of the law of gravity was introduced, which allows us to determine its magnitude when the gravitational field strength is known.

- For objects near Earth's surface, the magnitude of the gravitational force on them due to Earth is equal to the object's mass times 9.8 N/kg, which we call the gravitational field strength.
- For the gravitational force in other situations, where the object is not near Earth's surface of the two objects do not include Earth, we must use the universal law of gravity or a value of the gravitational field strength specific for that situation.
- An object's mass is intrinsic to the object, so it is appropriate to say an object has mass. Forces are due to how an object interacts with other objects, so it is not appropriate to say the object has a force.
- Mass is not the same as gravitational force.
- The gravitational force is not the result of air pushing down.
- The force of the air is insignificant unless the object is very light or is moving very quickly.

Frequently Asked Questions

Is there anything wrong with saying "the rock's gravity" or "Earth's gravity"?

It is highly recommended that you avoid this language as it is vague. If by "gravity" you mean the gravitational force then the phrase implies that the force is inherent to the object, which it is not. On the other hand, it is perfectly okay if you mean the gravitational field strength, since an individual does have a gravitational field strength, and that field strength depends only on that object's mass (and how far away one happens to be from the center of that object).

Is g always going to be equal to 9.8 N/kg?

No. The value happens to be 9.8 N/kg only because we are restricting ourselves to situations where the object is interacting with Earth and the object is on or near the surface of Earth.

Since the gravitational field strength is given in newtons per kilogram, does that mean an object has to be at least one kilogram in mass in order for there to be a gravitational force?

No. Consider, for example, the case where you are moving at 50 miles per hour. Just because the units are in miles per hour does not mean you need to be driving for an hour (or more) in order to have that speed. It simply means the *ratio* of how far you've traveled and how long it has taken is equal to 50 miles per hour. In a similar way, a gravitational field strength of 9.8 newtons per kilogram does not mean that the object has a mass of one kilogram (or more). It simply means the *ratio* of the gravitational force on the object (due to Earth) and the object's mass is equal to 9.8 newtons per kilogram.

When air is present, does it exert a force pushing things down?

The net effect of the air is an upward force, not downward. See page 213.

Do we need air to have gravity?

No. See page 200.

Terminology introduced

Air resistance	Gravitational field strength	Thermodynamics
Buoyancy	Pounds	Universal gravity equation
Drag	Simplified gravity equation	Weight

Problems

Problem 13.1: Each of the following situations involve an identical 200-g ball. Identify the gravitational force on the ball (due to Earth) in each situation.
(a) a ball that has been dropped and is falling downward
(b) a ball that has been thrown upward and is moving upward
(c) a ball that was thrown upward but has reached its highest point and has not yet started back down
(d) a ball hanging at rest from a string
(e) a ball that is being spun around in a circle with a string
(f) a ball that is in the process of being thrown

Problem 13.2: (a) A 2-kg block slides across some ice. While it is sliding across the ice, what is the gravitational force on the block (due to Earth)?
(b) While the block is sliding across the ice, what is the gravitational force on Earth due to the block?

Problem 13.3: The gravitational force (due to Earth) on a particular lump of clay is found to be 1 N. Suppose I take a second, identical lump of clay and combine it with first. What is the gravitational force (due to Earth) on the combined lump of clay?

Problem 13.4: To find the gravitational force due to the moon on an object near the moon's surface, we can use 1.6 N/kg for the value of g. Using values from the list of constants in the supplemental readings, show how equation 13.3) can be used to obtain this value of g.

Problem 13.5: A student says the simplified gravity equation states that the magnitude of the gravitational force on an object is 9.8 N, regardless of how massive the object is. Do you agree? Why or why not?

14. Free Fall

Puzzle #14: If you drop two objects, like two balls, you'll find that they will hit the ground at the same time, as long as they are released at the same time and from the same height. In addition, if you connect the two balls together, perhaps by a small rod or by gluing them together, the two balls together fall in the same way as the individual balls. In other words, the two connected balls, if released at the same time and same height as an individual ball, will hit the ground at the same time as the individual ball. Why is this?

Introduction

In this chapter, we look at how the gravitational force due to Earth impacts the motion of objects that are near the surface of Earth. By "near" I mean within 10 miles or so of the surface. While 10 miles might seem like a rather large distance, it is quite small compared to the radius of Earth. Indeed, 10 miles is only 0.25% of the radius.

As discussed in chapter 13, this means we can determine the magnitude of the gravitational force on the object due to Earth by using mg, where g is equal to 9.8 N/kg (rounded to the tenths place).

14.1 The definition of free fall

This chapter is called "free fall", so I first need to define what is meant by free fall. During **free fall**, the only force acting on the object is the gravitational force exerted on it due to Earth.

Since the gravitational force is a non-contact force (i.e., it acts even when the object is not in touching Earth) and since free fall involves only this one interaction, that means an object in free fall must not be touching anything. If you are holding the object, or the object is touching the ground, or there is significant aerodynamic drag, then the object is not experiencing free fall.

Notice that the *direction* of motion is not part of the definition. Consequently, an object could be moving up or down. As long as it isn't touching anything and the only force exerted on it is the gravitational force then it is technically in free fall. This may seem a bit strange, since one usually thinks of falling as meaning that the object is moving downward, but the definition of free fall doesn't require downward motion.

✍ In a similar way, the object could be moving horizontally and still be in free fall. The only requirement is that the gravitational force is the only force acting on it.

Consistent with free fall, for this chapter we'll restrict our analysis to those situations where the *only* force acting is the gravitational force due to the object's interaction with Earth. We'll add other forces in chapter 15.

✓ *Checkpoint 14.1: Suppose I throw a rock up in the air. After I let go and assuming no aerodynamic drag:*
(a) Is the rock experiencing free fall during its way up?
(b) Is the rock experiencing free fall after it hits the ground?

14.2 Direction of the gravitational force

❢ The gravitational force due to Earth on an object near Earth's surface is directed downward, toward the center of Earth.

The gravitational force pulls objects together, like an invisible rubber band. As such, the gravitational force on one object is directed toward the other object, which means that the gravitational force *due to Earth* is directed downward, toward the center of Earth.[i]

[i] Earth is not a perfect sphere. Consequently, straight downward is not necessarily along the line directed toward the center of Earth. The difference is inconsequential at this point in our studies, though.

Example 14.1: I am standing on the ground. In which direction is the gravitational force on me (due to Earth)?

Answer 14.1: The gravitational force on me (due to Earth) must be directed downward, toward the center of Earth.

As mentioned on page 190, it doesn't matter if the object is at rest on the ground, being thrown upward, moving horizontally or whatever. The actual motion of the object does not impact the gravitational force. So, regardless of its motion, the gravitational force on the object (due to Earth) is downward.

Example 14.2: Suppose the gravitational force (due to Earth) on a ball is 1 N and downward while it is at rest in my hand. If I throw the ball, what is the gravitational force (due to Earth) on the ball while it is in the air?

Answer 14.2: The gravitational force does not depend on whether the object is moving or not. It would still be 1 N downward.

✓ *Checkpoint 14.2: I throw a ball. When the ball is the air, in what direction is the gravitational force on it (due to Earth)?*

14.3 Applying the law of force and motion

We can approach free fall problems the same way we've approached all other problems since chapter 3. An object in free fall experiences a net force downward (since the only force acting is the gravitational force and that is directed downward). If the object is already moving downward, that downward net force will make the object speed up. If the object is moving upward, the downward net force will make the object slow down.

Is that what really happens?

Yes, as long as the gravitational force due to Earth is the only force acting.

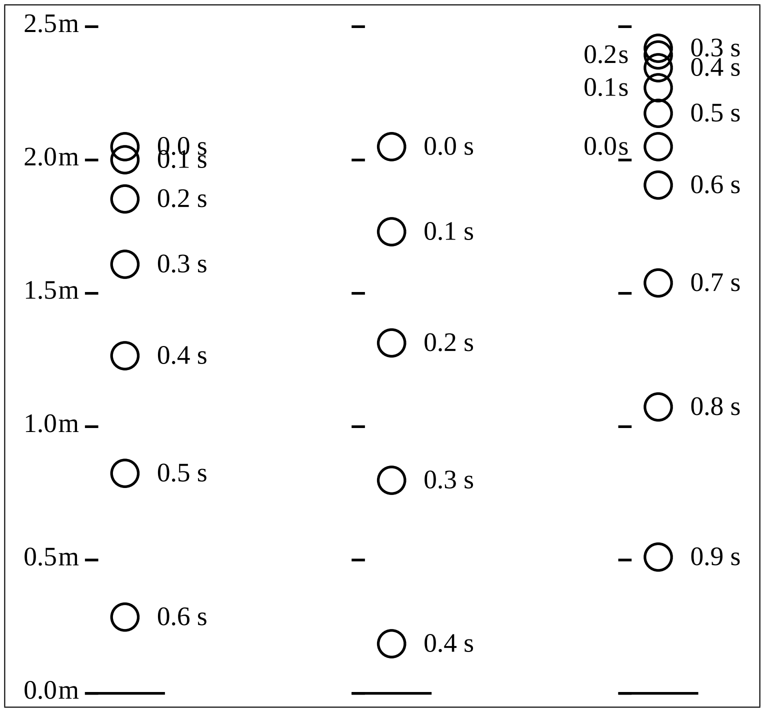


Figure 14.1: Time-lapse pictures of three balls:[left] ball dropped from rest, [center] ball thrown downward with initial speed of 2.7 m/s, [right] ball thrown upward with initial speed of 2.7 m/s.

Certainly, we can tell that an object thrown upward will slow down as it moves upward after being let go, consistent with the law of force and motion. However, it is a little harder to tell that a falling object actually speeds up.

For example, for a ball dropped from rest at a height 2 meters above the floor, the ball falls so quickly that it takes less than a second to hit the ground, much too fast to tell, with the naked eye, whether it is speeding up or not.

Fortunately, a high-speed camera can verify that the falling ball does actually speed up. The result is seen on the left side of Figure 14.1, where a time-lapse illustration of the falling ball is shown. The ball is released at a height of 2 meters above the floor, and the position of the ball is shown at 0.1-second intervals.

Notice how the ball speeds up as it fall, as seen by the increasing distance between each image. Recall that an object's average velocity during a time interval Δt is equal to $\Delta\vec{s}/\Delta t$, where $\Delta\vec{s}$ is the displacement of the object during the time interval. During the first 0.1 s, the ball hardly moves at all (about 0.05 m). During the time period from 0.5 s to 0.6 s, it moves a little more than 0.5 m, significantly larger than during the first 0.1 s.

14.3.1 Predicting the velocity

Not only can we use the law of force and motion to predict that an object in free fall slows down on its way up and speeds up on its way down, we can also use it to predict *how much* the speed changes.

Let's suppose we drop a 0.5-kg ball from rest and we want to know how fast it is going 3 seconds after being released (assuming it doesn't hit the floor). The only force acting is the gravitational force, which has a direction that is downward and a magnitude that is mg, where g is equal to 9.8 N/kg. Since the mass of the ball is 0.5 kg, that means the net force has a magnitude equal to 0.5 kg times 9.8 N/kg, which equals 4.9 N.

Plugging those values along with the ball's mass (0.5 kg) and the time (3 s) into the force and motion equation, we have:

$$\Delta\vec{v} = \frac{\vec{F}_{\text{net}}}{m}\Delta t = \frac{4.9 \text{ N downward}}{0.5 \text{ kg}}(3 \text{ s})$$

which equals 29.4 m/s downward.

That is the *change* in velocity. Since it started at rest, the final velocity three seconds later is also 29.4 m/s.

WHAT IF THE BALL DIDN'T START AT REST?

In the example, the object (ball) started its free fall at rest. However, when using the force and motion equation we didn't make any assumption about the object moving downward or starting at rest or anything like that. All we are assuming is that there aren't any other forces acting, like drag. So, during free fall, the *change* in velocity during a particular time (like three seconds) is the same regardless of the object's velocity.

✍ As mentioned before, just because we call this free *fall* does not mean the object is actually *falling*. It may be moving upward (after being thrown upward, for example). The only requirement is that the gravitational force is the only force acting.

So, during three seconds of free fall, the ball will experience a change of 29.4 m/s downward, regardless of its initial motion. If the ball started at rest then three seconds later the ball is moving downward at a speed of 29.4 m/s. On the other hand, if the ball was thrown downward and released with a speed of 10 m/s then three seconds later the ball would be moving downward at a speed of 39.4 m/s. This is because the ball was *already* moving downward at 10 m/s and we had to add an additional 29.4 m/s downward to that.

What if the ball was thrown upward instead of downward?

Let's suppose the ball was thrown *upward* with an initial speed of 29.4 m/s. The change in velocity during the first three seconds of free fall would still be 29.4 m/s downward, but the direction is opposite the motion, which means the ball is slowing down. In this case, a change of 29.4 m/s downward means it has reached its highest point and come to a stop (momentarily).

What if the ball was thrown upward with a speed less than 29.4 m/s?

If the change in velocity has a magnitude greater than the initial speed, the ball will reach its highest point and then return back down.

For example, consider a ball thrown *upward* with an initial speed of 10 m/s. As before, the change in velocity during the first three seconds of free fall is still 29.4 m/s downward but the ball first slows down by 10 m/s, turns around, and then speeds up on its way down, reaching a speed of 19.4 m/s at the end of the three seconds (for a total change of 29.4 m/s downward).

By the way, you don't need to split up the motion into two parts, as I did in this case, where I first considered how much it slowed down on its way up (10 m/s) and then how much it sped up on its way down (19.4 m/s), for a total change of 29.4 m/s downward. While it may be insightful to split up the motion into two parts, it is easier to just do it in one step.

To illustrate the one-step approach, let's repeat the exercise using positive for upward and negative for downward. That means the ball has an initial velocity of $+10$ m/s and experiences a change of -29.4 m/s. Basically, we are adding -29.4 to $+10$, which gives us -19.4, meaning that the final velocity is -19.4 m/s, where the negative value indicates a downward motion (since we set upwards as positive).

✓ *Checkpoint 14.3: Suppose I throw a 0.5-kg ball upward such that it has a speed equal to 14.7 m/s when it leaves my hand.*
(a) How long does it take for the ball to reach the top of its motion, where it is no longer moving upward but not yet started downward?
(b) What is its velocity 2 seconds after I release it?

14.3.2 Free fall acceleration

According to the force and motion equation,

$$\Delta\vec{v} = \frac{\vec{F}_{\text{net}}}{m}\Delta t$$

the change in velocity is proportional to the time. That means that the longer the time of free fall the more the velocity changes.

Put another way, the velocity continually changes the entire time the object is in free fall. There is no point where the object reaches a steady speed. As long as there is a non-zero net force acting on the object, the motion will continue to change.

In other words, the object continues to accelerate the entire time it is in free fall. As discussed in chapter 7, the acceleration represents the rate at which the velocity changes and is defined as follows:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

When we write the force and motion equation in terms of acceleration, we get:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

As shown before, for our 0.5-kg ball the net force during free fall is 4.9 N downward (since the only force acting is the gravitational force due to Earth). Plugging this into the expression we get:

$$\vec{a} = \frac{4.9 \text{ N downward}}{0.5 \text{ kg}}$$

which equals 9.8 m/s^2 downward.[ii]

Notice that this is the acceleration, not the speed. This value represents how quickly the ball's velocity changes during free fall, not how fast it is moving.

That means that when the ball is going upward, it is slowing down, losing 9.8 m/s in speed every second. And when the ball is going downward, it is speeding up, gaining 9.8 m/s in speed every second.

Notice also that the acceleration doesn't depend upon the time. In other words, it doesn't matter how long a time the ball is in free fall. As long as the ball is in free fall, the velocity will continue to change, at a rate of 9.8 m/s^2 downward.[iii]

✓ *Checkpoint 14.4: (a) For the 0.5-kg ball in free fall, does the ball experience a constant acceleration? If so, what is the magnitude and direction of the ball's acceleration? If not, why not?*
(b) For the 0.5-kg ball in free fall, does the ball experience a constant velocity? If so, what is the magnitude and direction of the ball's velocity? If not, why not?

So far we've only used a 0.5-kg ball as our object in free fall. Let's suppose we had a 0.1-kg ball instead.

Again, the force and motion equation in terms of acceleration is as follows:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

For our 0.2-kg ball the net force during free fall will not be 4.9 N downward as it was for the 0.5-kg ball. To find the net force acting on a 0.2-kg ball, we

[ii] Recall that a newton is equal to a kg·m/s^2 (see section 3.3.4). Consequently, a N/kg is equal to a m/s^2:

$$\frac{\text{N}}{\text{kg}} = \frac{\cancel{\text{kg}} \cdot \text{m/s}^2}{\cancel{\text{kg}}} = \text{m/s}^2$$

[iii] For really far drops, the object may reach a speed where the aerodynamic drag becomes significant compared to the gravitational force. At that point the object is no longer experiencing free fall and the acceleration won't remain 9.8 m/s^2 downward. For short distances, like the height of a typical classroom, the aerodynamic drag is small and the assumption of free fall is probably valid.

have go back to the mg calculation so we can first get the magnitude of the gravitational force acting on the ball.

Since the ball is near Earth's surface, the magnitude of the gravitational force due to Earth on the 0.2-kg ball is the ball's mass, 0.2 kg, times the gravitational field strength, 9.8 N/kg. That product is 1.96 N.

Plugging this into the force and motion expression we get:

$$\vec{a} = \frac{1.96 \text{ N downward}}{0.2 \text{ kg}}$$

which equals 9.8 m/s^2 downward.

Notice how that is the *same* acceleration experienced by the 0.5-kg ball!

How is it that the mass of the object changed yet the acceleration is exactly the same as before?

To see how that happens, let's go through the process a bit more carefully. The gravitational force is the only force acting. As discussed in chapter 13, the magnitude of the gravitational force on an object is proportional to the object's mass (i.e., $|\vec{F}_\text{g}| = mg$). The larger the mass, the larger the magnitude of the gravitational force on it (and thus the larger the net force on it, since the gravitational force is the only force acting during free fall).

On the other hand, in the force and motion equation,

$$\vec{a} = \frac{\vec{F}_\text{net}}{m}$$

the mass m is in the denominator, meaning that the acceleration is *inversely* proportional to the mass. The larger the mass, the *smaller* the acceleration (i.e., a heavier object is harder to accelerate).

❢ We can use the law of force and motion to show why an object's acceleration in free fall is independent of the mass of the object.

In other words, the magnitude of the net force is larger on the object of larger mass, but a larger magnitude is needed in order to produce the same acceleration with a larger mass object.

These two effects cancel out when the only force acting is the gravitational force.

✍ Whenever the net force is proportional to the object's mass (as in free fall), we find that the object's motion is independent of the mass. This is a useful piece of information, as sometimes a problem won't provide you with the mass because it isn't necessary.

Is this what really happens?

Yes. During free fall, the mass doesn't matter.[iv] In other words, it doesn't matter if the object is 1 kg or 100 kg – the acceleration is the same and does not depend on the mass![v]

✍ In the next section I describe a simple experiment that can demonstrate that the acceleration during free fall is indeed the same for both heavy and light objects.

Since all objects in free fall (near Earth's surface) experience the same acceleration of 9.8 m/s^2 downward, regardless of their mass, we call this value the **free fall acceleration**.

❢ Objects in free fall experience a constant acceleration, regardless of the object's mass or motion.

Is it a coincidence that the free fall acceleration and the gravitational field strength have the same value?

No, it is not a coincidence. They are necessarily the same for free fall. Indeed, as noted in the footnote on page 224, the units are also the same, which means you can write g as either 9.8 N/kg or 9.8 m/s^2. When written as 9.8 N/kg, I'll refer to it as the gravitational field strength, and when written as 9.8 m/s^2, I'll refer to it as the free fall acceleration.[vi]

✓ *Checkpoint 14.5: It was mentioned that an object in free fall experiences an acceleration of 9.8 m/s^2 downward. Suppose an object is dropped from rest. Once it reaches a speed of 9.8 m/s, does it remain at that speed for the rest of its fall?*

[iv] Scientists usually honor Galileo, an Italian scientist born in 1564, as the one who "discovered" this independence of mass, although he was not the first to formally document the relationship.

[v] For very light objects, the magnitude of the gravitational force on the object is small and thus it won't take long for the object to reach a speed where the aerodynamic drag becomes significant compared to the gravitational force. At that point the object is no longer experiencing free fall and there will be a dependence on mass (see section 2.6).

[vi] A word of caution: many people refer to g as the "acceleration of gravity." That is a bit misleading, since gravity isn't accelerating. Rather, it is the object, upon which the gravitational force is acting, that is accelerating.

14.3.3 Time to fall

In the previous section it was mentioned that there is a simple experiment one can do that demonstrates how the acceleration during free fall is indeed the same for all objects, regardless of the mass.

This experiment utilizes the fact that if the acceleration is the same, regardless of mass, then the change in velocity must likewise be the same, regardless of mass. Furthermore, if two objects start at rest and experience the same change in velocity, then their average velocity must be the same and thus take the same amount of time to fall a specific distance.

That means if you drop two objects of different masses from the *same* height, they will both take the same amount of time to fall to the floor. In other words, they will hit the floor at the *same* time.

❢ In free fall, all objects, when released from rest from the same height, will hit the ground at the same time.

You can demonstrate this yourself. Take a bunch of objects, of various sizes and masses, and drop them from rest all from the same height and at the same time. When you do this, you will find that each object takes the *same* amount of time to fall (<u>if</u> dropped from the same height).[vii]

But don't heavier objects, like a bowling ball, fall faster than something light, like a feather?

It is true that a bowling ball will fall faster than a feather, but that is because drag is significant for a feather (see section 2.6). For our analysis, we've assumed free fall, which means that drag is negligible. During free fall, objects will fall in the same way.

✓ *Checkpoint 14.6: When I drop a 20-gram ball from rest, I find it falls one meter in 0.45 seconds. How long do you expect a 40-gram ball will take to fall one meter if released from rest? Why?*

[vii] It may be tricky to actually drop them at the *same* height and at the *same* time. One way to do this is to place the objects on a ruler that is held horizontally and then quickly pull the ruler down and away.

14.4 Predicting the displacement

Not only can we predict the ball's change in velocity during free fall but we can also predict how far it falls.

The process is the same as that used in section 9.5. Basically, we first use the acceleration to figure out how the velocity changes over the time period, then figure out the average velocity given the range in velocity values, and finally use the average velocity to figure out how far the object has traveled during that time.

To illustrate the process, let's consider a ball that starts at rest and falls for three seconds. With an acceleration of 9.8 m/s^2 downward, it speeds up by 9.8 m/s each second:

0 s	1 s	2 s	3 s
0	9.8 m/s	19.6 m/s	29.4 m/s

Since the acceleration is constant, the average velocity is equal to the midrange value, which would be 14.7 m/s (downward). We can then use that average velocity value to determine how far it traveled in the five seconds. At an average rate of 14.7 m/s, the object must have traveled 44.1 m downward (multiply 14.7 m/s by 3 s).

What if the ball wasn't released at rest?

If the ball had an initial velocity that wasn't zero then we'd have to take that into account to determine the final velocity.

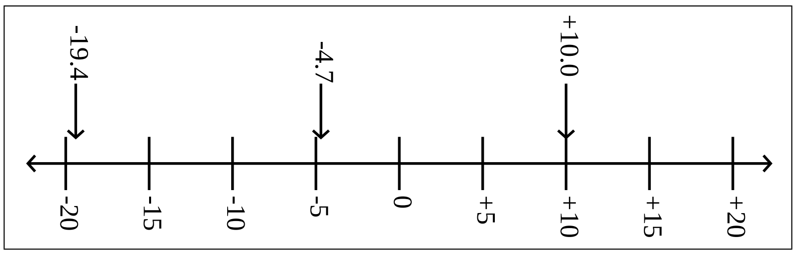
For example, let's suppose the ball has an initial velocity of 10 m/s upward. Using positive for upward, the acceleration would be -9.8 m/s^2, meaning the velocity will change by -9.8 m/s each second:

0 s	1 s	2 s	3 s
+10 m/s	+0.2 m/s	-9.6 m/s	-19.4 m/s

Notice how the ball initially slows down, stops momentarily then speeds up in the other direction. As in the previous example, when the ball started at rest, the acceleration is constant so the average velocity is equal to the midrange value, which would be -4.7 m/s in this case. At an average rate of

4.7 m/s, the object must have traveled 14.1 m downward (multiply 4.7 m/s by 3 s).

It is easier to see where the midrange value comes from when we mark the three values on a number line:



With the number line, one can see that the −4.7 value is exactly midway between the 10.0 and −19.4 values.

Mathematically, the midrange value of two numbers can be determined by adding the two numbers together and dividing the sum by two. An alternate method is to take the difference between the two (by subtracting one from the other) and then adding half of that to the smaller value. Use whichever you are more comfortable with.

✍ When dealing with changing directions, it is easy to overlook the difference in directions, which is why it is a good idea to use positive and negative to keep track of the directions.

✓ *Checkpoint 14.7: Suppose I throw a 0.5-kg ball upward such that it has a speed equal to 14.7 m/s when it leaves my hand. What is the ball's total displacement during the first three seconds of free fall?*

We can now apply all of this to the following scenario.

> There is a Winnie-the-Pooh video available where one of the characters, Rabbit, falls down a hole. You don't see Rabbit falling. All you hear is him falling and then hitting the bottom. It takes 12.5 seconds for Rabbit to fall. Assuming the gravitational force is the only force acting on Rabbit and that Rabbit started at rest, how deep is the hole?

Even though Rabbit's mass is not given, we know that he must be accelerating downward at 9.8 m/s^2 since we are assuming free fall.

Since he started at rest, this means he must gain speed at a rate of 9.8 m/s every second. Since it took 12.5 seconds to fall, the change in velocity can be obtained by multiplying 9.8 m/s^2 downward by the time 12.5 s, which gives 122.5 m/s downward. Since Rabbit started at rest, that means he is going 122.5 m/s just before he hits the bottom. This is really fast!

To find the displacement, we first have to determine Rabbit's *average* velocity during the 12.5 seconds, not Rabbit's *final* velocity.

Since Rabbit's velocity is increasing at a uniform rate, we know that Rabbit's average velocity during the 12.5 seconds is the midrange value between the initial velocity (zero) and final velocity (122.5 m/s downward). That midrange value, and thus the average value, is 61.25 m/s downward.

By the definition of average velocity, the displacement during the time period is the product of the average velocity and the time. Consequently, now that we have Rabbit's average velocity, we can multiply the average velocity by the time (12.5 s) to get the displacement: 765.6 m downward.

✍ A distance of 765.6 m is over 2500 ft. That is about twice as high as the Empire State Building (without the antenna)!

✓ *Checkpoint 14.8: In the example with Rabbit, why couldn't we just multiply 122.5 m/s (Rabbit's final velocity) and 12.5 s (the total time) to get the displacement?*

Summary

This chapter examined the application of the law of force and motion to free fall. It also introduced how one can use the definition of average velocity to predict an object's displacement during free fall (using the fact that the velocity changes in a uniform way).

The main points of this chapter are as follows:

- The gravitational force due to Earth on an object near Earth's surface is directed downward, toward the center of Earth.

- We can use the law of force and motion to show why an object's acceleration in free fall is independent of the mass of the object.
- Objects in free fall experience a constant acceleration, regardless of the object's mass or motion.
- In free fall, all objects, when released from rest from the same height, will hit the ground at the same time.

Frequently Asked Questions

IF AN OBJECT IS IN FREE FALL, DOES THAT MEAN IT FALLS AT 9.8 m/s?

No.

Be careful with units! If the units are m/s then the quantity is velocity. So, saying that "an object is falling at 9.8 m/s" means that the object is falling with a constant, unchanging velocity of 9.8 m/s downward.

This is certainly not the case during free fall, when the speed is increasing.

WILL THE OBJECT EVENTUALLY BE FALLING AT 9.8 m/s?

At some point, if it falls long enough, it may reach a speed of 9.8 m/s. However, with continued falling, it will not *remain* at 9.8 m/s, as the velocity continues to change at a rate of 9.8 m/s every second.

IS AN OBJECT'S ACCELERATION ALWAYS 9.8 m/s^2 DOWNWARD?

No. That is only true for free fall, where the gravitational force (due to Earth) is the only force acting on the object (and the object is near the surface of Earth).

WHAT ABOUT WHEN THE BALL IS THROWN UPWARD — IS THE ACCELERATION STILL DOWNWARD?

Yes.

Remember that what you may commonly call a deceleration (slowing down) is really "an acceleration opposite the direction of the motion." During the time that the ball is moving upward, it is slowing down. Since the velocity is upward, the acceleration must be downward (opposite the motion).

HOW ABOUT WHEN THE BALL STARTS TO COME BACK DOWN — IS THE ACCELERATION STILL DOWNWARD THEN?

Yes.

From that point on, the object speeds up, which means the acceleration and the velocity must be in the same direction.

Shouldn't objects that move upward be accelerating upward?

As someone throws an object upward, the acceleration may be upward. However, once the person lets go of the object, the object will slow down and that corresponds to a downward acceleration.

What about when the object reaches the top. At that moment, isn't the acceleration zero?

No.

The object is still undergoing a change in velocity (from moving upward to moving downward). An object at the top of its motion, although motionless for an instant, is still undergoing a change in direction and, as such, is accelerating. In this case, it is a downward acceleration, since the velocity is changing from upward to downward.

How can the acceleration not be zero if the velocity is zero?

The acceleration is zero only if the velocity remains *constant*. It has nothing to do with the *value* the velocity happens to have any particular moment.

So, for example, an object's acceleration is zero only if its velocity *remains* zero. If its velocity doesn't remain zero then the object's acceleration is not zero.

In the case of the object at the top of its motion, the object's velocity is zero. However, it cannot remain zero for otherwise it would just stay at that location. It doesn't. It is only there for an instant.

Can the acceleration decrease while the speed increases?

Yes. An example is examined in chapter 15 (section 2.6). With drag, a falling object will speed up as it falls but at a smaller and smaller rate.

Terminology introduced

Free fall
Free fall acceleration

Additional problems

Problem 14.1: I throw a ball upward. At release, the ball is moving upward at 2 m/s. How long does it take for the ball to reach the top of its motion?

Problem 14.2: In 2012, Felix Baumgartner jumped out of a balloon 128,100 feet above the ground. At that height, there is very little air and according to the Red Bull website he spent 4 minutes and 22 seconds in free fall, without drag. They say his maximum speed was 833.9 mph. Is that consistent with their assertion about free fall?

Problem 14.3: An object is dropped from rest, at which point it undergoes free fall. How fast is the object moving 1.5 seconds into its fall? Assume it hasn't hit the floor or anything.

Problem 14.4: A 1000-kg elevator is initially at rest. The elevator is hanging from a single cable. If the cable is cut, how fast is the elevator 3 seconds after the cable is cut? What if the elevator was initially moving upward at 3 m/s when the cable was cut?

Problem 14.5: (a) How much farther does an object in free fall travel in 2 s if it is initially moving at 3 m/s downward, compared to how far it falls if released at rest?
(b) How far does a object travel in 2 s if moving at a constant speed of 3 m/s?
(c) Any thoughts on why part (a) equals part (b)?

Problem 14.6: (a) Suppose I release a basketball by giving it an initial velocity of 2 m/s downward. How far above or below its starting height will the ball be half a second after I release it?
(b) Suppose I release a basketball by giving it an initial velocity of 4 m/s *upward*. How far above or below its starting height will the ball be half a second after I release it?

Problem 14.7: Suppose I throw a rock straight up into the air with an initial velocity of 2.0 m/s upward. If the rock hits the ground 0.7 seconds later, how high above the ground was the rock released?

Problem 14.8: A ball is thrown up in the air. For each case listed, indicate whether the acceleration is zero or non-zero. Do the same for the velocity. For non-zero values, indicate the direction (up or down) and explain your choice.
(a) On the way up

(b) On the way down
(c) When it is at its highest point

Problem 14.9: I throw a ball straight up into the air. At the moment the ball is at the top of its motion,
(a) What is the ball's velocity: downward, upward, or zero?
(b) What is the ball's acceleration: downward, upward, or zero?
(c) What is the direction of the net force on the ball: downward, upward, or zero?

Problem 14.10: Suppose we have an object in free fall with an initial velocity of 7 m/s upward.
(a) What is the object's average velocity between that moment and 1 s later?
(b) What is the object's displacement during that second?

Problem 14.11: I hold a ball at rest one meter above the ground. I then drop it, at which point it falls to the ground, eventually coming to rest upon the ground. The ball's velocity at the beginning (in my hand) was zero and the ball's velocity at the end (on the ground) is zero. Was the ball's average velocity zero? Why or why not?

15. Gravity with Other Forces

Puzzle #15: What force stops us from falling when we fall to the ground, and what is the magnitude and direction of that force?

Introduction

In this chapter, all of the situations will involve objects on or near Earth's surface. As in chapter 14, this means that there will be a downward gravitational force due to Earth exerted on the object, with a magnitude equal to mg, where m is the mass of the object and g is 9.8 N/kg.[i]

✍ From chapter 12, we know that there is a gravitational force between all objects, not just with Earth, but the gravitational force is insignificant unless one or both of the objects are very massive. Thus, for an object on or near Earth's surface, we will only concern ourselves with the gravitational force on the object due to Earth.

During free fall, the gravitational force due to Earth is the only force acting on the object, and the object continues to accelerate the entire time it is in free fall. The rate of the acceleration is 9.8 m/s^2 downward and does not change while it is in free fall, even as the object speeds up or slows down. This means an object's velocity will continually change during the entire time the object is in free fall (i.e., slowing down on the way up and speeding up on the way down).

But is it reasonable to expect the gravitational force to be the only force acting? What about situations where the object is clearly touching something else, like when the object is being held or the object is on the floor?

[i]If the object is not on or near the surface of Earth, we will need to use a different value of g. If we don't know the appropriate value of g, we would need to use the universal gravity equation (see chapter 12).

To answer those questions, we need a way of including contact forces in our analysis. Contact forces are forces that only exist when objects are touching. In a way, this makes them easy to identify, since to find out which contact forces are acting on a particular object, all we have to do is identify those things the object is touching.

In this chapter, we'll examine three types of contact forces: drag, strings and surfaces. In each case, the contact force will be upward, opposing the gravitational force, and we'll practice examining how that impacts the motion, as described by the law of force and motion.

15.1 Force diagrams

Since we'll be examining situations with more than one force acting, it is helpful to first review what the law of force and motion says about such situations.

Recall that the law of force and motion states that an object will speed up, slow down or change directions if there is a force imbalance acting on the object. We don't know which it is doing unless we are given some *additional* information about the motion itself.

For example, if the force imbalance results in a net force in the direction of motion, then the object is speeding up. Conversely, if the net force is opposite the direction of motion, the object is slowing down.

In this chapter, we will examine situations where there are two forces acting: the downward gravitational force and a separate, upward contact force that opposes the gravitational force. Sometimes the upward force will exactly counter the downward gravitational force, making the net force zero. Other times, the upward force will have a magnitude that is less than or greater than the magnitude of the gravitational force, making the net force downward or upward, respectively.

❢ It is a good idea to draw a force diagram, with labels indicating which force is which.

The various situations are illustrated in Figure 15.1 via what are called **force diagrams**.[ii] In a force diagram, a dot represents the object and arrows are

[ii] Some people call these **free body diagrams**, since the object (body) is drawn free from the other objects.

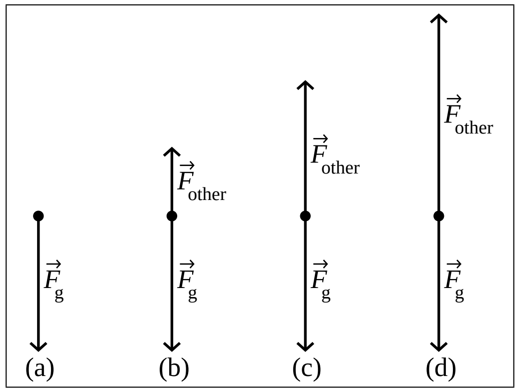


Figure 15.1: A force diagram showing (a) a downward gravitational force alone, (b) a downward gravitation force with a small upward force, (c) a downward gravitation force with an upward force of equal magnitude, (d) a downward gravitation force with a =b upward force of greater magnitude.

used to indicate the forces acting on the object. This is much simpler to do than to draw a realistic depiction of every situation we encounter.[iii]

In part (a) of Figure 15.1, I have drawn a force diagram illustrating the case where just the gravitational force is acting on the object. Notice that the arrow is directed downward. This is to represent the direction of the gravitational force, which is downward.

Also notice that I have drawn the "tail" of the arrow coincident with the dot. I will follow this convention for both pushes and pulls. Some people prefer to draw the "head" of the arrow coincident with the dot, and it isn't *wrong* to do so, but it is clearer to do it the way I'm doing, as it keeps the arrow heads separated.[iv]

Keep in mind that the downward-pointing arrow represents the force, *not* the motion. According to the law of force and motion, forces are related to the *change* in motion, not the motion itself.

So, in part (a) of Figure 15.1, we do not know which way the object is actually moving. It could be moving upward, in which case it is slowing down (since the net force on it is downward). Or, conversely, it could be

[iii]The actual object could be anything, like a hand, box, boat or chicken. In most cases, it is a lot easier to just draw a dot.
[iv]Also, most books do it the way I'm doing it.

moving downward, in which case it is speeding up. It could also be moving in some other direction, like rightward or leftward.

What if more than one force is acting on the object?

For each force, I will draw a separate arrow, each with its tail on the dot or circle that represents the object.

In parts (b), (c) and (d) of Figure 15.1, I have drawn the force diagrams that indicate an upward force in addition to the downward gravitational force. In (b), the upward force has a smaller magnitude than the gravitational force, meaning that the net force is still downward, like in part (a), but not as much. An upward moving object would still be slowing down and a downward moving object would still be speeding up, but less so than an object in part (a).

In (c), the upward force has the *same* magnitude as the gravitational force, meaning that the net force is zero. Regardless of which way the object is moving, the object wouldn't be accelerating. For example, for both upward and downward moving objects, the object would be neither speeding up nor slowing down.

In (d), the upward force has a *greater* magnitude than the gravitational force, meaning that the net force is now upward. An upward moving object would be speeding up and a downward moving object would be slowing down.

Should the force diagram include an arrow indicating the direction the object is moving?

No.

It is called a *force* diagram, not a *force and velocity* diagram. If we added an arrow indicating the motion, we might get confused and think the motion arrow was indicating an additional force. So, leave the motion out of the force diagram.[v]

> ✍ Remember, the force diagram does not include an arrow indicating the direction of motion. It only includes arrows for the forces.

[v] Some people like to indicate the motion via a dashed arrow. That isn't necessary, though.

✓ *Checkpoint 15.1: For the situation drawn in part (c) of Figure 15.1, can we tell if the object is moving or at rest? If so, which is it doing? If not, why not?*

15.2 Hard surfaces

With free fall, we considered the flight of a ball when you throw it straight up in the air. However, we only considered the portion of the flight *after* you let go and *before* you catch it because only during that time is the gravitational force the only force acting on the ball. Now let's consider what is happening *while* you are touching it.

While holding a ball at rest in your hand, the net force on the ball must be zero since, according to the law of force and motion, the net force must be zero if the object isn't accelerating. In this case, your hand exerts an upward force on the ball that exactly counters the downward gravitational force on the ball. This is illustrated in part (c) of Figure 15.1.[vi]

To throw the ball upward, we must accelerate the ball upward, To do this, the upward force exerted by our hand must be greater than the downward gravitational force on the ball. This is illustrated in part (d) of Figure 15.1.

Once we let go of the ball, the only force acting on the ball is the gravitational force, as illustrated in part (a) of Figure 15.1. That means the net force is downward. As the ball moves up, the downward net force makes the ball slow down. And, as the ball comes back down, the downward net force makes the ball speed up.

✍| We are assuming the force due to the air is negligible.

Finally, when we catch the ball, we want the downward traveling ball to slow down. That requires an upward net force, illustrated by part (d) of Figure

[vi] As mentioned in section 12.1.4, the presence of the hand (or any other object for that matter) does not "block" gravity from acting. As long as Earth is there, the gravitational force exists, regardless of whether there is anything "in the way" or not. In this sense, gravity is just like magnetism, in that you can't block either one. However, just because gravity always acts does not mean you can't counter it by applying opposing forces.

15.1, which means the upward force exerted by our hand must be greater than the downward gravitational force on the ball.

✓ *Checkpoint 15.2: A person throws a rock straight up into the air. Assuming drag is negligible, what forces are acting on the rock when (a) the rock is still in the person's hand and (b) when it has left the person's hand.*

15.2.1 Direction (normal direction)

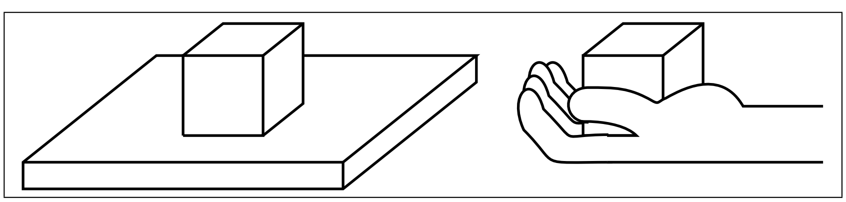
You may have noticed that the direction of the force due to the hand is always *away* from the hand. That is because the contact force essentially prevents the ball from sinking into the hand.[vii] This is true for all surfaces. The direction of the force is always away from the surface, which is *perpendicular* to the surface, a direction mathematicians (and physicists) refer to as the "normal direction."

❢ The surface force prevents objects from sinking into the surface and is always perpendicular (normal) to the surface.

WHY CALL IT THE NORMAL DIRECTION?

The word "normal" is from the Latin word for "carpenter's square" and means "perpendicular" in mathematics (and "regular" in common usage). As such, the normal direction is always perpendicular to the surface.

Compare, for example, a box at rest on the surface of Earth (left part of figure below) vs. a box at rest in a hand (right part of figure). What forces are acting on the box in each case?



[vii]Surfaces act in a way similar to that of a trampoline or bed mattress. However, the "compression" of hard surfaces is so small that one doesn't notice it. As we'll see in Volume II, the repulsion is due to the electric force. Interestingly, the electric force is also responsible for acting like the "glue" that keeps objects from falling apart and produces the tension in a string.

It turns out that the forces are the same in both cases. The ground holds up the box in the same way that the hand holds up a ball. Any surface, whether it be the ground, a table or a hand, applies a force that pushes the object away from the surface.

What is strange about the ground is that it is part of Earth. So, Earth pulls the box toward it (via the gravitational force) but, at the same time, it pushes the box away when the box touches the ground.

What is the difference between the surface force and the gravitational force?

The surface force, being a contact force, is associated with only those molecules on Earth's surface that are in contact with the box. The gravitational force, on the other hand, being a non-contact force, is associated with the interaction between the box and the *entire* Earth, not just the part that is closest to the object or just the part that the object is touching.

Are there any other forces acting on the box?

We are assuming the force due to the air is negligible, so only two forces are acting on the box: the gravitational force and the force due to the surface.

Now that we have identified the forces, what are their directions?

The gravitational force is directed downward, as always. The surface force, on the other hand, is in a direction that is normal (i.e., perpendicular) to and away from the surface. Since the surface is oriented horizontally and the box is on top of it, the surface force must be directed vertically upward (toward the top of the page in the figure).

The force diagram is the same as what we'd have for a hand holding a ball, as shown in part (c) of Figure 15.1.

✓ *Checkpoint 15.3: For a horizontal surface, what is the normal direction?*

15.2.2 Magnitude

The magnitude of the surface force depends on how much it needs to push to keep the object from sinking into the surface. Thus, to find the magnitude of the surface force, we typically need to apply the force and motion equation, since that equation relates the forces with the motion.

For example, consider again a box at rest on a frictionless horizontal floor. We've already established that there are two forces acting on the box: the gravitational force (due to Earth) directed downward, and the surface force (due to the floor) directed upward.

Suppose we knew that the gravitational force was 50 N. For the box to remain at rest, what would the surface force have to be?

The law of force and motion tells us that the forces have to balance in order for the box to remain at rest. Since the gravitational force in this case is 50 N, that means the surface force would likewise have to be 50 N (directed opposite the gravitational force).

✍ If we were given the mass instead of the gravitational force, we'd first have to multiply the mass by 9.8 N/kg to find the magnitude of the gravitational force.

✓ *Checkpoint 15.4: Suppose a 4-kg book was at rest on a table. What would be the magnitude and direction of the surface force (due to the table) on the book such that the book stays at rest? Describe your reasoning.*

Does the surface force and the gravitational force always have equal magnitudes?

No. It is equal only if the object isn't accelerating and the gravitational force is the only force pushing the object into the surface.

Keep in mind that surfaces prevent objects from "seeping" into the surface. The magnitude of the surface force automatically has whatever value is needed to keep the box from seeping into the surface.

How does the surface force know the value it must have?

The surface force only acts if it has something to act against. For example, when you step on a floor, the floor (hopefully) prevents you from sinking downward into the floor (i.e., perpendicular to the surface).

Because of this, we assume the surface force has a magnitude that is sufficient to prevent the two objects from meshing – no more, no less. In other words, it must increase or decrease depending on what other forces are acting to push the object into the surface. Sometimes that is equal to the gravitational force. Sometimes it is less. Sometimes it is more.

For example, for objects that are <u>not</u> in contact with the floor, the floor does <u>not</u> exert a force. So when you jump off the floor, the floor does not exert a force while you are in the air.

✓ *Checkpoint 15.5: If a box is not touching the floor does the floor exert a surface force on the box? Why or why not?*

As another example, suppose I push down on a 5-kg box with a force of 10 N. In that case, there is a 10-N force pushing down *in addition to* the gravitational force (49 N pushing down). The surface force would then have a magnitude of 59 N, enough to balance not only the gravitational force but also the 10-N force that I am applying.

CAN THE SURFACE FORCE HAVE A MAGNITUDE LESS THAN THAT OF THE GRAVITATIONAL FORCE?

Sure.

Consider, for example, if I pulled up on the box instead of pushed down on it. In that case, I would be countering the gravitational force and so the surface force wouldn't have to be as much.

CAN THE SURFACE FORCE EVER PULL THE OBJECT INTO THE SURFACE?

For our purposes, no.

So, if I pulled up with a magnitude greater than the gravitational force, the box would no longer stay on the surface. The surface force would be zero. It would not start pulling down on the box just to keep it at rest.

✓ *Checkpoint 15.6: A 2.0-kg box is sitting on a level, frictionless floor. If I push down on the box, is the magnitude of the surface force greater than, less than, or equal to what it would be without me pushing on the box? Describe your reasoning.*

Up to now, we've only considered surfaces that are stationary. Let's now consider surfaces that are moving.

For example, consider the following scenario:

> A 4-kg box is on the horizontal floor of an elevator. As the elevator starts to move upward, it speeds up at a rate of 0.4 m/s^2. What is the force exerted on the box due to the floor of the elevator?

From the law of force and motion, we know that there needs to be a force imbalance because the box is not maintaining a constant velocity. In addition, the net force on the box must be upward because it is moving upward and speeding up (i.e., the acceleration is directed upwards).

There are only two forces acting on the box: the gravitational force (due to Earth) acting downward, and the surface force (due to the floor of the elevator) acting upward. Since there is a force imbalance, with the net force directed upward (see part (d) of Figure 15.1), that means the surface force must have a greater magnitude than the gravitational force.

In particular, the surface force must have a magnitude that is greater by an amount provided by the force and motion equation, $\vec{a} = \vec{F}_{\text{net}}/m$. Rearranging to solve for $\vec{F}_{\text{net}}$, we get:

$$\vec{F}_{\text{net}} = m\vec{a}$$

Plugging in 0.4 m/s^2 upward for $\vec{a}$ and 4 kg for m, we find that the net force must be 1.6 N upward.

This is the force *imbalance*, remember, not the surface force. Basically, it tells us that the surface force must be 1.6 N greater in magnitude than the gravitational force. The gravitational force has a magnitude of 39.2 N (multiply mass by 9.8 N/kg), which means the surface force must have a magnitude of 40.8 N (add 1.6 N to 39.2 N).

✍ This is similar to throwing a ball upward, with your hand acting like the floor of the elevator. In order to accelerate the ball upwards, you have to exert an upward force on the ball that has a greater magnitude than the gravitational force pulling down on the ball.

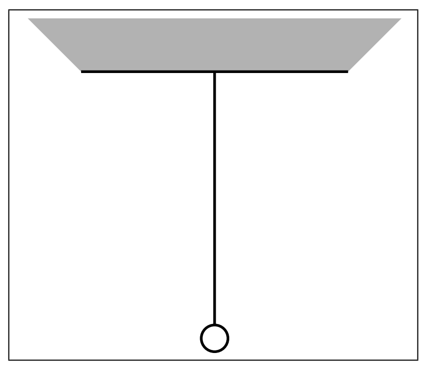
What if the elevator is moving upward but not speeding up?

If the box is not experiencing a change in velocity then the net force on it must be zero. Consequently, the force exerted upward on the box due to the elevator floor would have the same magnitude as the gravitational force pulling downward on the box.

Remember: it is not the direction of the motion that is important but rather the direction of the *change* in the motion.

✓ *Checkpoint 15.7: Suppose the elevator was slowing down as it approaches a higher floor.*
(a) Compare the magnitudes of the force due to the elevator floor on the 4-kg box and the gravitational force on the 4-kg box. Which is larger? Why?
(b) Suppose the elevator is slowing at a rate of 0.4 m/s^2 as it moves upward. What is the surface force exerted on the box?

15.3 Strings, ropes and cables (tension)



The purpose of this section is to identify the properties of strings, ropes and cables, and the forces due to them.

Consider, for example, a 0.5-kg ball hanging at rest from a string oriented straight up and down (see figure at right). What forces are acting on the ball?

Assuming the ball is near Earth's surface, there is the gravitational force due to Earth, pulling downward on the ball. And, unless stated otherwise, we can assume the object is not moving fast enough for the drag to be significant. The only other force acting would be the force due to the string.[viii]

HOW DO YOU KNOW THE STRING EXERTS A FORCE ON THE BALL?

Any object that touches the ball is interacting with it and thus there is a force associated with that interaction.[ix]

[viii]Remember, the ball does not exert a force on itself, even if it is moving.

[ix]Remember that we are assuming the only non-contact force is the gravitational force and that force is only significant if the interacting object is massive. The string's mass isn't significant enough to interact gravitationally in a significant way.

❢ Strings, ropes and cables can only pull, not push.

What are the properties of the force due to the string?

A string can only pull. It cannot push. Furthermore, a string can only pull in a direction parallel to the string. Since strings can only pull in a direction parallel to the string, that means the string in this case must exert a force that is directed upward.

✍| Ropes and cables have the same properties as strings.

✓ *Checkpoint 15.8: What is it about the string that tells us the force exerted on the ball due to the string is directed upward as opposed to any other direction?*

What is the magnitude of the force due to the string?

Much like we did with an object at rest on the ground (see section 15.2), the ball in this case is at rest and remaining at rest and so the net force on the ball must be zero (according to the law of force and motion). With the two forces acting, on upward (string) and one downward (gravity), the situation must be like that shown in part (c) of Figure 15.1, with the two forces having equal magnitudes.

We already know that the gravitational force due to Earth has a magnitude equal to mg, where g is 9.8 N/kg. Since the ball has a mass of 0.5 kg, that means the gravitational force has a magnitude of 4.9 N (multiply 0.5 kg by 9.8 N/kg). And, since the force due to the string has the magnitude, it must be 4.9 N upward.

Notice how the gravitational force on the ball was not mentioned in the statement of the exercise, even though it plays an important role. Don't assume that any phenomenon not mentioned explicitly in the question can be ignored.[x] At the other extreme, also don't assume that every little piece of information provided in a problem statement needs to be used.[xi]

[x]This is especially important in the real world, as your boss (or customer) will likely not ask you a question *and* give you all of the relevant factors needed to answer it. Maybe they feel the factors are so obvious that they aren't worth mentioning or, perhaps more likely, they aren't aware of the relevant factors (which is why they are asking the question in the first place).

[xi]For example, the color and size of the ball doesn't matter. If the information was given you'd have to use physics to know that they weren't relevant. Always use physics to interpret a problem before using an equation.

Is the force due to the string always equal and opposite to the gravitational force on the ball?

No. That is only the case for when the forces balance, as in when the ball is at rest and staying at rest. Consider the following scenario:

> A 1000-kg elevator is being held up by a cable. As the elevator starts to move upward, it speeds up at a rate of 2 m/s every second (i.e., it experiences an acceleration of 2 m/s^2 upward). What is the force exerted on the elevator due to the cable)?

Without plugging in any numbers, we know that there needs to be a force imbalance because the elevator is not maintaining a constant velocity, and the law of force and motion states that a force imbalance is needed to change the motion.

We also know that the net force on the elevator must be upward because it is moving upward and speeding up, so the net force must be in the direction of motion. This means the forces must be like that indicated in part (d) of Figure 15.1, with the downward-directed arrow representing the gravitational force and the upward-directed arrow representing the force due to the cable.

The net force is upward, consistent with the upward-moving elevator speeding up. That net force is related to the elevator's acceleration via the force and motion equation, $\vec{a} = \vec{F}_{\text{net}}/m$. In this case, we know that $\vec{a}$ equals 2 m/s^2 upward. Plugging that in, along with 1000 kg for m, we find that the net force must be 2000 N upward.

This is the force *imbalance*, remember, not the force due to the cable alone. Basically, it tells us that the force due to the cable must be 2000 N greater in magnitude than the gravitational force. The gravitational force has a magnitude of 9800 N (multiply mass by 9.8 N/kg), which means the force due to the cable must have a magnitude of 11,800 N (add 2000 N to 9800 N).

✍ A real elevator probably doesn't accelerate at 2 m/s^2. I just used that number so that there would be a significant difference between the two magnitudes.

What if the elevator is moving upward but not speeding up?

If the elevator is not experiencing a change in velocity then the net force on it must be zero. Consequently, if the elevator is not speeding up, the force due to the cable would have the same magnitude as the gravitational force pulling downward on the elevator, as illustrated in part (c) of Figure 15.1.

And if the elevator is moving upward and slowing down, then the net force must be down, which is illustrated in part (a) or part (b) of Figure 15.1.

✓ *Checkpoint 15.9: Suppose the 1000-kg elevator was slowing down as it approaches a higher floor.*
(a) Which has a greater magnitude: the force due to the cable or the gravitational force due to Earth? Why?
(b) Suppose the elevator is slowing at a rate of 0.4 m/s every second as it moves upward. What is the force due to the cable?

When a string, cable or rope pulls on something, **tension** occurs the string, cable or rope. The value of the tension is equal to the magnitude of the force exerted by the string, cable or rope. From now, I will tend to refer to the tension rather than "the magnitude of the force exerted by the string, cable or rope," as it is just a lot less to write.

✍ The greater the string, rope or cable pulls on something, the greater the tension within the string, rope or cable.

To illustrate this, let';s consider a situation where we are given the tension and asked to find acceleration or the change in velocity:

> A 1000-kg elevator is initially at rest. The elevator is hanging from a single cable. If the tension in the cable becomes 8000 N and maintains that tension for 3 seconds, how fast is the elevator moving at the end of the 3 seconds and in which direction?

To know which way the elevator is moving, we need to determine whether our situation corresponds to part (b) in Figure 15.1 (where the gravitational force has a magnitude greater than the upward force due to the cable), part (c) in the figure (where the forces balance) or part (d) in the figure (where the upward force is greater).

To determine which we have, we need to determine the magnitude of the two forces. We already know the magnitude of the force due to the cable – 8000 N (the tension in the cable). And we can determine the magnitude of the gravitational force via the product of the mass (1000 kg) and 9.8 N/kg. That means the gravitational force (due to Earth) is 9800 N downward.

Now that we have the two forces, we can see that we have a situation corresponding to that in part (b) of Figure 15.1, where the net force is downward. Since the elevator started at rest, the elevator must move downward, speeding up as it does so.

To find out how much it speeds up, we use the force and motion equation:

$$\Delta\vec{v} = \frac{\vec{F}_{\text{net}}}{m}\Delta t$$

We are given the values of m and Δt (which are 1000 kg and 3 seconds, respectively). And since we have the two forces (8000 N upward and 9800 N downward), we know that $\vec{F}_{\text{net}}$ is equal to 1800 N downward (the difference between the two).[xii]

When you plug these values into the force and motion equation, you get a change in velocity of 5.4 m/s downward. Since it started at rest, it must end up with a velocity of 5.4 m/s downward at the end of the three seconds.

✓ *Checkpoint 15.10: In the scenario described above, why is the net force on the elevator* *not* *8000 N?*

What if the string, rope or cable breaks?

If no mention is made of any limit to the tension, we just assume the string, rope or cable will never break. However, a problem could state that there is a maximum tension. In that case, we'd have to compare the value that is needed with the maximum tension value.

For example, suppose a particular string breaks if the tension goes above 50 N. Can this string be used to hold up a 5-kg box at rest?

The answer is yes. The gravitational force on the box has a magnitude of 49 N, as can be determined by multiplying the mass of the box (5 kg) by Earth's gravitational field strength (9.8 N/kg). Since the box remains at rest,

[xii] If you were tempted to use 8000 N as the net force, you may be focusing on the equations rather than the physics that is being represented by the equations. While 8000 N is a force, it isn't the *net* force.

the net force on it must be zero, which means the magnitude of the force due to the string must equal the magnitude of the gravitational force (49 N). The string doesn't break because 49 N is below the maximum of 50 N.

✓ *Checkpoint 15.11: A rope breaks if its tension goes above 500 N. As such, it cannot be used to hold up a 100-kg box at rest. Why would two ropes be okay, assuming they share equally in the load (i.e., each provides half the force needed to balance the gravitational force)?*

15.4 Drag and terminal speed

Way back in section 2.6, it was mentioned that fluids like air and water exert a force called **drag** on objects that move through them. That means that when a ball flies through the air there is drag on the ball due to the air, which opposes the motion of the ball. The faster an object moves through the air, the greater the drag.

When an object is released at rest, drag will be very small and so the forces are much like in part (a) of Figure 15.1, where the only force acting is the gravitational force. This is the free fall situation described in chapter 14. Due to the force imbalance being downward, a downward moving object will speed up.

As the object speeds up, the drag becomes greater and greater. At some point, we can no longer ignore it in comparison to the magnitude of the gravitational force.

As long as the drag (pushing upward) is smaller in magnitude than the gravitational force (pushing downward), as illustrated in part (b) of Figure 15.1, there is still a downward imbalance and the object continues to speed up as it falls. The only difference is that the object will not speed up as quickly as it would in the absence of drag (i.e., free fall).

At some point, though, a falling object will reach a speed where the drag becomes *equal* in magnitude (but opposite in direction) to the gravitational force, as shown in part (c) of Figure 15.1. At that point, we have balanced forces and the net force on the object is zero.

Since the object is *already* moving downward, a zero net force in this situation means the object continues to move downward but just no longer speeds up. It has reached its **terminal speed**.[xiii]

DO ALL FALLING OBJECTS EXPERIENCE THE SAME TERMINAL SPEED?

No. Light objects do not have to move as quickly in order to achieve the force balance. Consequently, light objects experience a smaller terminal speed. They also reach that speed more quickly.

So, for example, if we dropped a feather, the feather would reach its terminal speed very quickly and then float slowly down to Earth at that constant speed.

A bowling ball, on the other hand, would take a very long time to reach its terminal speed and by that time it would be traveling very fast. The longer it takes for the object to reach the terminal speed, the faster the terminal speed. A skydiver's terminal speed is around 63 m/s and it takes about 25 seconds to reach terminal speed.

The terminal speed also depends on whether the object is falling through air or water. The drag is greater when falling through a liquid like water than through air. Consequently, an object's terminal speed tends to be greater when falling through air than when falling through water.

CAN THE DRAG EVER BECOME <u>GREATER</u> THAN THE MAGNITUDE OF THE GRAVITATIONAL FORCE?

For that to happen, the object would have to be moving faster than the terminal speed.

An example of this would be a meteor that happens to be moving through empty space, where drag is minimal, and thus can be moving very, very quickly. When it reaches the atmosphere, it suddenly experiences a significant drag, where the terminal speed is much less than the meteor's speed. At that point, the meteor slows down. Such a situation is shown in part (d) of Figure 15.1.

As for ordinary objects, the only way for an object to reach a speed greater than the terminal speed is if it is thrown or launched with an initial speed greater than the terminal speed (like a bullet being shot out of a rifle). If the

[xiii] The **terminal velocity** has a magnitude equal to the terminal speed but is a vector and so it also has a direction (equal to the direction of motion).

object is simply dropped, and the fluid it is passing through doesn't change, then it will never reach a speed greater than the terminal speed because once it reaches that speed, it won't go any faster.

> If the object is thrown downward with an initial speed greater than the terminal speed then the object will slow down until it reaches the terminal speed, where the two forces once again become equal in magnitude.

Example 15.1: Suppose we dropped a light object, like a Ping-pong ball, that had the same shape and size as a heavy object, like a lead ball. Which object reaches its terminal speed quicker? Explain.

Answer 15.1: From your experience, you probably can guess that the Ping-pong ball reaches its terminal speed in a shorter amount of time. But why?

If they are the same shape and size, the drag should be the same on each. The magnitude of the gravitational force, however, is smaller on the Ping-pong ball than on the lead ball. A small drag force can more easily counter the gravitational force on the Ping-pong ball. The lead ball, on the other hand, must first have a large speed in order to experience a drag comparable to the gravitational force on it.

Thus, since the heavier ball will have to reach a faster terminal speed before the drag can balance out the gravitational force, it takes longer for the heavier ball to reach it.[xiv]

> For short fall (< 1 m; 0.5 s), drag isn't significant on a Ping-pong ball and so, for that distance, the Ping-pong ball tends to fall at the same rate as the lead ball (when both are released at rest).

What if the two objects have the same mass but are shaped differently, like a crumpled up piece of paper vs. a flat piece of paper?

Drag not only depends upon the object's speed but also on how aerodynamic the object is. For example, drag is less on a crumpled up piece of paper than a flat piece of paper.[xv] For most of our discussion, we'll assume a spherical shape unless specified otherwise.

[xiv] All objects, even massive ones, will eventually reach a terminal speed if falling with drag.

[xv] A bullet has an aerodynamic shape. Consequently, its terminal speed can be quite

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✓ *Checkpoint 15.12: Suppose we dropped two pieces of paper that are identical in every way (same mass) except that one is scrunched up into a ball. If they are released at rest, which reaches terminal speed first? Explain your reasoning in terms of the law of force and motion.*

Can we determine the displacement of a falling object if we are given its terminal speed?

Only if the object started at its terminal speed, because then the velocity won't change as it continues to fall and so we can use that value as its average velocity, which then can be used to determine the displacement.

However, let's suppose it starts at rest and eventually reaches its terminal speed. In that case, its average speed is <u>not</u> the midrange between the initial (zero) and final (terminal speed value).

Why not?

The average speed is only equal to the midrange value when the speed changes in a uniform way. The speed only changes in a uniform way (i.e., the acceleration is constant) when the net force acting on the object is constant.

During the fall, the gravitational force is constant but the drag is not – the drag increases as the object speeds up. Consequently, the *net* force is not constant as the object speeds up and the average speed will not be equal to the midrange value of the initial and final speeds.

✓ *Checkpoint 15.13: Suppose an object is dropped from rest and drag is not negligible. Consequently, after falling for quite a while, it reaches a terminal speed of 30 m/s. Is the average velocity equal to 15 m/s downward (midrange of the initial of zero and final of 30 m/s downward)? If so, why? If not, is the average closer to zero or closer to 30 m/s downward? Provide your reasoning.*

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high. In fact, it can be high enough that a falling bullet can kill people and cause significant property damage. There have been many instances of people getting killed by celebratory gunfire, where bullets are fired into the air and then fall onto people.

Summary

This chapter examined three types of contact forces: drag, surface forces, and forces due to strings, ropes and cables.

The main points of this chapter are as follows:

- It is a good idea to draw a force diagram, with labels indicating which force is which.
- As an object speeds up, the drag increases, leading to a smaller change in velocity. When the object no longer speeds up, we say it has reached its terminal speed.
- Strings, ropes and cables can only pull, not push.
- The surface force prevents objects from sinking into the surface and is always perpendicular (normal) to the surface.

By now you should be able to identify when contact forces are being applied and determine the direction of tension and surface forces.

Frequently Asked Questions

Why is the normal direction called the normal direction?

See page 240.

Is the normal direction always upwards?

No. It depends on the orientation of the surface. This is discussed in the supplemental readings (see section on inclines).

If there is an additional force pulling up on an object, does that mean the gravitational force on the object is less?

No. The gravitational force is independent of whatever else is acting on the box. We do not consider the other forces to be "part" of the gravitational force.

When an elevator moves upward, even at constant speed, there must be forces acting upward on it, right?

Yes, but for our rising elevator, there are *two* forces: the cable pulling the elevator up and gravity pulling the elevator down. According to the law of force and motion, there would be no "net force" acting upon the elevator if the elevator is moving with a constant velocity. Consequently, both forces continue to act on the elevator but they "counter-act" each other, leading to no *net* force.

If the gravitational pulls downward, why is it harder to push sideways on a heavy object?

This has to do with friction, which will be discussed in section 17.1.

Does the terminal speed depend upon the shape and mass of an object?

Yes. For a skydiver, the terminal speed depends on whether the skydiver opens up the parachute or not (the terminal speed is less with the parachute). Something like a feather would have a much lower terminal speed.

Does it take the same amount of time to reach terminal speed in all cases?

No. Objects like a feather, which have a very low terminal speed, reach that speed very quickly upon release. It depends a great deal on whether there is a lot of drag and how that drag impacts the object.

What happens to a falling object if the drag never gets big enough to exactly balance the gravitational force?

As long as there is a force imbalance in the direction of the motion, the object will continue to speed up.

Terminology introduced

Drag	Stokes' law
Force diagram	Tension
Newton's resistance law	Terminal speed
Resistance	Terminal velocity

Additional Problems

Problem 15.1: Right now you are probably at rest, sitting in a chair, dutifully reading (and enjoying!) your physics textbook. There are several forces acting on you, including the force due to the chair pushing you upwards and the gravitational force due to Earth pulling you downwards. Are the forces balanced? If so, why? If not, why not?

Problem 15.2: I am holding a rock in my hand. The rock is at rest. There are two forces acting on the rock: the contact force due to my hand (holding the rock up) and the non-contact force we call the gravitational force due to Earth (pulling the rock down). What is the net force on the rock?

Problem 15.3: A 2.0-kg box is sitting at rest on a level, frictionless floor. What is the magnitude of the surface force on the box?

Problem 15.4: A rock is on the floor of an elevator that is moving upward at a constant speed of 3 m/s. If the surface force acting on the rock is 29.4 N, what is the mass of the rock?

Problem 15.5: An elevator is supported by a cable. In each of the following situations, which is larger: the gravitational force pulling down on the elevator or the tension in the cable pulling up on the elevator?
(a) The elevator is at rest.
(b) The elevator is moving upward at a constant speed.
(c) The elevator is moving downward at a constant speed.
(d) The elevator is moving upward and slowing down.
(e) The elevator is moving downward and speeding up.

Problem 15.6: A 1000-kg elevator is moving downward at 2 m/s. The elevator is hanging from a single cable. What tension in the cable would be sufficient to stop the elevator in 4 seconds?

Problem 15.7: Consider a 1000-kg elevator that starts to move upward, reaching a speed of 2 m/s five seconds after it started (from rest). Compare the tension in the cable holding up the elevator with the magnitude of the gravitational force on the elevator (due to Earth). Are they the same? If so, why? If not, which is larger and by how much?

Problem 15.8: A person throws a rock straight up into the air. Draw a force diagram for the moment when the rock is still in the person's hand and another force diagram for when it has left the person's hand. What is different about the two force diagrams? What is the same?

Problem 15.9: A 5-kg box is sitting on a level, frictionless floor, as before, but with two additional forces on the box, due to me and a friend: a downward force of 10-N and a leftward force of 20-N force. Does the box stay at rest? Why or why not?

Problem 15.10: A 4-kg box is on a frictionless, level surface. Identify the magnitude of the surface force (due to the floor) in the following cases:
(a) Without any additional forces acting on the box.
(b) With an additional force of 30 N downward acting on the box.
(c) With an additional force of 30 N upward acting on the box.
(d) With an additional force of 50 N upward acting on the box.
(e) With an additional force of 30 N acting horizontally on the box.
In which of the cases does the box remain at rest?

Problem 15.11: Suppose we have a 10-kg box is moving with a velocity of 20 m/s leftward on a horizontal frictionless surface. If I exert a 20-N force rightward on the box for 30 seconds, what is the velocity of the box at the end of the 30 seconds?

Problem 15.12: Suppose we have a 10-kg box is moving with a velocity of 10 m/s rightward on a horizontal frictionless surface. If I exert a 40-N force rightward on the box for 5 seconds, what is the velocity of the box at the end of the 5 seconds?

Problem 15.13: Suppose we have a 10-kg box at rest on a horizontal frictionless surface. What force would give the box a velocity of 2 m/s northward if applied for 5 seconds?

Problem 15.14: Suppose we have a 5-kg box is moving with a velocity of 10 m/s rightward on a horizontal frictionless surface. A force is then exerted upon it such that 10 seconds later it is moving with a velocity of 5 m/s leftward (i.e., the opposite direction). What was the magnitude and direction of the force that was applied?

Part D

Two dimensions

16. Projectile Motion

Puzzle #16: We know that an object in free fall experiences a downward acceleration, which means an object moving upward will slow down and an object moving downward will speed up. What happens if the object is thrown *sideways*?

Introduction

The puzzle describes a type of motion called **projectile motion**. Projectile motion is what happens to an object when it is thrown (or kicked or launched) horizontally or at an angle.

Previously, we have examined objects that were thrown, but they were either thrown directly upward or directly downward (or released from rest). Our philosophy is to solve as many different problems as possible with the same set of underlying physical principles. Consequently, it is natural to ask if we can use our same set of laws and definitions for objects thrown in directions other than straight up or straight down.

The answer, of course, is that we can.

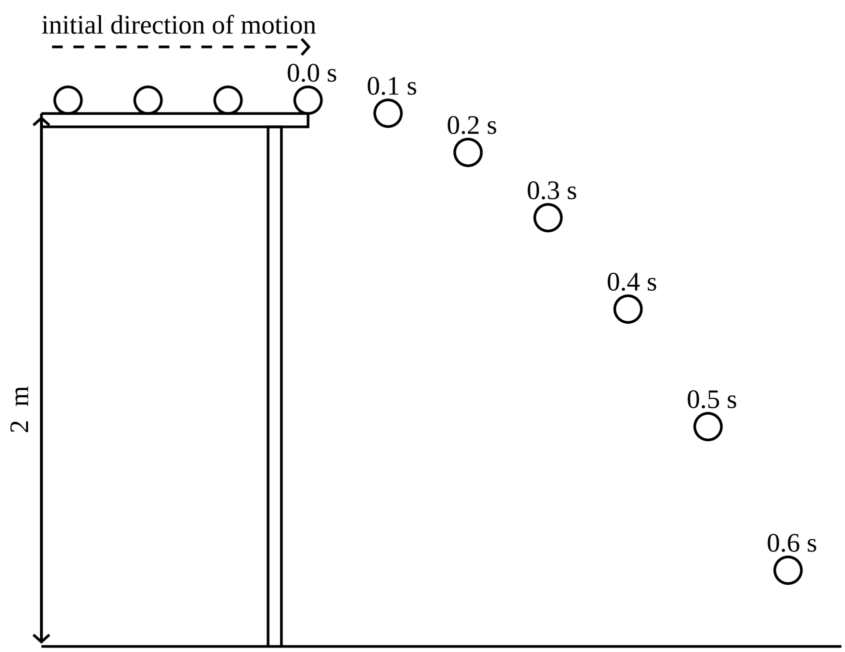
To do so, we must first recognize what is happening when a thrown object travels through the air. Once we do that, we can then see how the law of force and motion applies. Keep in mind that our task is to identify how projectile motion has similar characteristics to what we've seen before. It is the similarities that allow us to apply the same laws and definitions as before.

To simplify things, we'll restrict our analysis in this chapter to situations where the only force acting on the object is the gravitational force, which acts downward. As mentioned in chapter 14, we refer to such situations as free fall. An object need not be moving downward. Any motion is considered free fall, as long as there are no other forces acting other than gravity, which means there is no drag or air resistance.

16.1 Horizontal vs. vertical motion

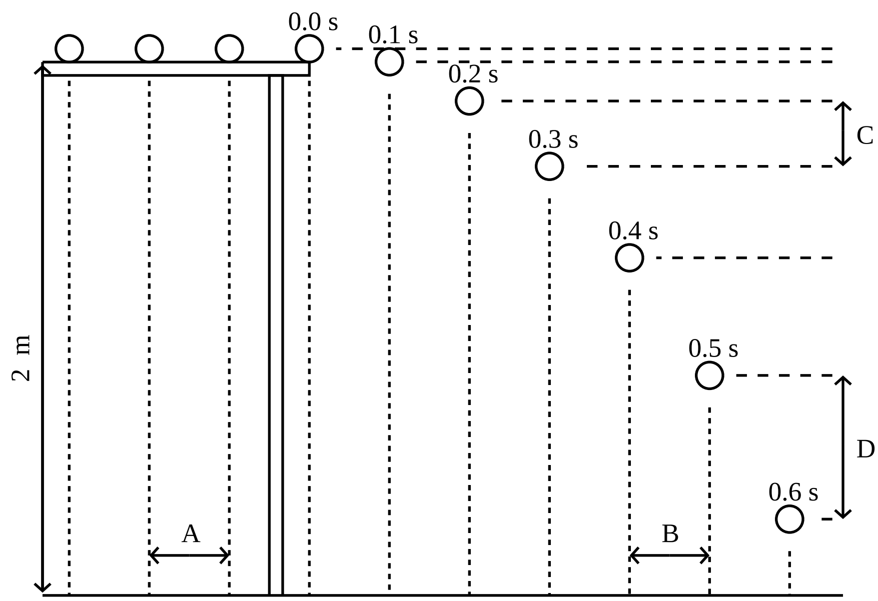
Before we can apply the law of force and motion to the situation described in the puzzle, we first have to distinguish between the horizontal motion and the vertical motion, and explain what we mean by each.

Consider a ball that rolls off a horizontal 2-m high table. The ball falls to the ground much too quickly to tell what is going on with the naked eye, but a high-speed camera can reveal what is happening, as shown in the time-lapse picture below. The timer starts at the moment the ball leaves the table, which is when the ball starts free fall.



As you can see in the time-lapse picture, the ball moves rightward while it is on the table. In comparison, while it is in the air the ball moves both rightward and downward *at the same time.*

To separate out the difference, let's repeat the time-lapse picture but add some dotted and dashed lines (see below). The dotted lines, oriented vertically, indicate how the *horizontal* position of the ball changes as it falls. The dashed lines, oriented horizontally, indicate how the *vertical* position of the ball changes as it falls.



Based on the dotted lines, one can see that the ball continues to move rightward as it falls, without any change in its *horizontal* speed. This is evidenced by the *horizontal* spacing between each image of the ball (taken every tenth of a second). Indeed, the spacing is the same in the air as when the ball is on the table (compare lengths A and B, for example).

Since the time interval is the same between each image, the same change in *horizontal* position means the *horizontal* velocity is the same throughout the fall.

❢ During free fall, an object's horizontal motion doesn't change as it falls.

On the other hand, based on the dashed lines, one can see that the *vertical* displacement during each 0.1-second *changes* as the ball falls (compare lengths C and D, for example), which means the ball's *vertical* velocity changes as it falls. This is unlike the horizontal velocity, which stays the same as the ball falls.

This is important – as the ball falls, the ball doesn't stop moving rightward. On the contrary, it continues to move rightward in the same manner it was moving rightward while on the table. I'll explain why in the next sections.

✓ *Checkpoint 16.1: Suppose the ball in Figure 16.1 is moving at a speed of 3 m/s along the table. What are the distances traveled during the time intervals indicated by (a) A and (b) B in the figure?*

16.2 Explaining the motion

In the previous section we examined the motion of a ball that has rolled off a horizontal table. While the ball is in the air (after rolling off the table and before it hits the floor), the vertical motion changes as it falls, getting faster and faster, but the horizontal motion doesn't – the ball continues moving rightward at the same rate even as it falls. Why does the vertical motion change but the horizontal motion doesn't?

As with everything else we've examined so far, the answer has to do with the net force acting on the ball. In this case, there is only one force acting on the ball after it leaves the table: the gravitational force. And that gravitational force is directed *downward*.

Notice that the net force (after the ball leaves the table) is vertical, not horizontal. Also notice that it is only the ball's *vertical* motion that changes – the ball's *horizontal* motion doesn't change. This is not a coincidence. The law of force and motion not only tells us that the motion will change, but it also tells us something about the direction of that change.

To better understand what is going on, I'll examine the horizontal motion separately from the vertical motion.

✍ Note that the two directions here – horizontal and vertical – are perpendicular to each other. Each direction is considered a **dimension**[i] , so we are basically looking at two-dimensional situations in this part of the book. The two directions need not be horizontal and vertical, but for our purposes they do need to be perpendicular to each other.

16.2.1 Horizontal motion

To explain why the ball continues to move rightward, even as it falls, we apply the law of force and motion. The law of force and motion states that the object's change in velocity will depend upon the force imbalance (and how long the force imbalance is present).

In this case, the force imbalance is directed downward. Horizontally, there is *no force imbalance*. That means that *horizontally* there is no change in velocity.

Notice what I did there. I'm applying the law of force and motion to *only* the horizontal part of the motion. The change in the *horizontal* velocity (i.e., the rightward motion) depends only upon the *horizontal* force imbalance. It doesn't matter if there is a vertical imbalance (which there is during free fall). The *horizontal* velocity will not change unless there is a *horizontal* force imbalance.

❢ An object's horizontal velocity won't change unless there is a horizontal force imbalance.

For free fall, there aren't any horizontal forces acting. The only force acting is gravity, and that acts downward, which doesn't impact the *horizontal* motion.

Since there are no *horizontal* forces acting on the ball, the ball's *horizontal* velocity doesn't change, consistent with the law of force and motion.

✍ We are assuming free fall, which means there is no drag. Drag opposes motion, so for a ball moving horizontally, drag (if present) will act to slow down the horizontal motion.

Let's now illustrate this with some numbers for the ball that has rolled off the table (see illustration on page 262):

> A ball rolls along a 2-m high horizontal table with a speed of 3 m/s. A quarter of a second after it leaves the table, how far away from the table is it (horizontally)?

To solve the problem, we need to first recognize that it is asking for a *horizontal* displacement. To answer the question, then, we need to consider the *horizontal* forces. Any vertical forces won't impact the answer.

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[i] The word "dimension" actually refers to an idea that is a bit more general than just a direction but it is sufficient to treat it as that for our purpose here.

While the ball is in the air, there are no horizontal forces acting on it (we are assuming). Consequently, the horizontal motion will be the same as it was on the table (3 m/s).

To find the horizontal displacement, we use the definition of average velocity:

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{s}}{\Delta t}$$

We are given the horizontal velocity (3 m/s; rightward as seen in the figure) and the time (0.25 s). Plugging in, we find that the horizontal displacement is 0.75 m (rightward). That means the ball is 0.75 m away from the table.

Notice that we don't need to consider how it is moving *vertically*, as the problem asked us for the *horizontal* displacement.

✓ *Checkpoint 16.2: A ball rolls off the edge of horizontal table with a velocity of 2 m/s rightward. Assume the table is high enough so that the ball doesn't hit the floor. What is the ball's horizontal displacement during the first 0.2 seconds it is in the air?*

What if the ball is thrown upward or downward as well as horizontally?

As long as the problem asks for the *horizontal* motion, the ball's motion upward or downward has no bearing on the answer (assuming no horizontal forces, as before). Remember, as long as we are only asked for something horizontal, we only consider the horizontal motion and the horizontal forces.

To illustrate, let's consider the same scenario as before except with a ball launched at an angle.

> A ball is thrown at an angle such that it is initially moving 4 m/s upward and 3 m/s rightward (i.e., it is moving both horizontally and vertically at the same time). If the ball is released two meters above the ground, how far has it moved horizontally during the first 0.25 seconds of its flight?

The only difference between this problem and what we had before is that the object is initially moving upward, as well as horizontally. However, the problem is still asking about the *horizontal* motion.

From the horizontal viewpoint, this is exactly the same as before. The ball will continue to move horizontally at 3 m/s. Consequently, during the 0.25 seconds the horizontal displacement is 0.75 m, as before.

The vertical motion has no bearing on its horizontal motion.

✍ This idea (that the vertical motion is separate from the horizontal motion) was formally developed by Isaac Newton (1643-1727), although Galileo (1564-1642) was close to documenting this relationship many years earlier.

✓ *Checkpoint 16.3: A football is initially resting on the ground. It is then kicked such that its initial velocity is 5 m/s upward and 10 m/s rightward. What is its horizontal displacement during the first 0.5 seconds of its flight?*

16.2.2 Vertical motion

To explain why the ball's vertical motion changes as it falls, we again apply the law of force and motion. In this case, the force imbalance is directed downward (vertically). That means that vertically there *is* a change in velocity.

Just as the change in the *horizontal* velocity (i.e., the rightward motion) depends only upon the *horizontal* force imbalance, the *vertical* velocity (i.e., the downward motion) depends only upon the *vertical* force imbalance. It doesn't matter what the *horizontal* force imbalance is.

This means that the vertical velocity should change in a way dictated by the vertical forces acting on the object. In this case, the only vertical force is the gravitational force. Consequently, we can attack the vertical motion just as we did in chapter 14.

In other words, the vertical velocity changes in the same way as for any object in free fall: it accelerates at 9.8 m/s^2 downward.

❢ In free fall, an object's horizontal velocity doesn't impact how the object's vertical velocity changes.

✓ *Checkpoint 16.4: Compare the lengths C and D indicated in the illustration on page 263. Should they be the same? Why or why not?*

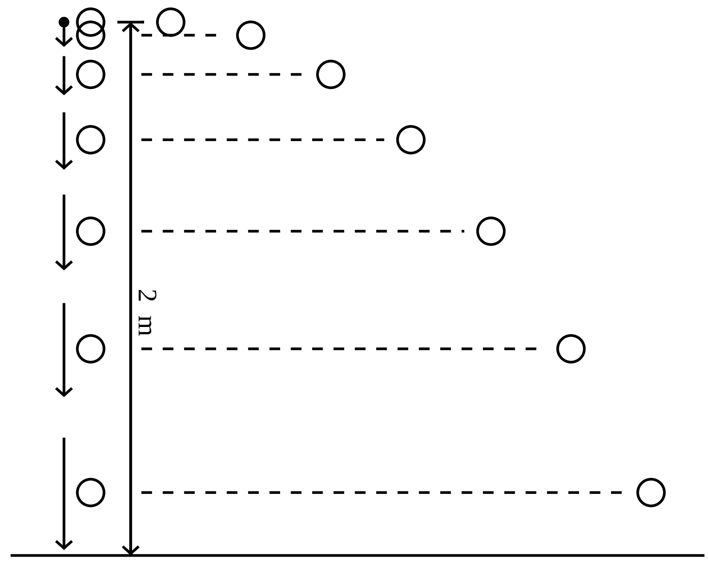


Figure 16.1: Time-lapse picture of a ball rolling off the edge of a horizontal table 2 meters high along with a ball that is dropped from rest at the exact same time the first ball leaves the table. Images are taken 0.1 seconds apart.

Since the vertical motion is independent of the horizontal motion, we can answer questions about the vertical motion without needing to know how fast the object is moving horizontally. The object will move downward at a speed that increases with time regardless.

Consider, for example, two identical balls. One ball has rolled off a horizontal table, as discussed before. Another ball is dropped from rest at the exact same moment that the first ball leaves the table. In other words, both enter free fall at the same moment and with the *same* initial vertical velocity.

Keep in mind that while on the table, the ball is moving horizontally, not vertically. That means its vertical velocity on the table is *zero*. At the moment the ball leaves the table, its velocity is the same as what it was when it was rolling on the table, which means its vertical velocity, at the start of free fall, is zero, just like the object dropped from rest.

❢ in free fall, the time to fall is independent of the horizontal motion.

If we look at both together, then, we get the time-lapse picture shown in Figure 16.1. The time-lapse picture shows that *both* balls experience the *same* changes to their *vertical* motion, since the *vertical* forces are the same on both.

Furthermore, we already know that objects in free fall experience a downward acceleration of 9.8 m/s^2 that is independent of the mass. That means the two objects don't even need to be identical. Two objects will hit the ground at the same time as long as they are released at the same time and with the same initial vertical velocity.

Why does the initial vertical velocity have to be the same?

If the initial vertical motion is different than they would not hit the ground at the same time. For example, an object thrown down will take less time to his the ground than an object dropped from rest (even though both experience the same downward acceleration of 9.8 m/s^2).

Keep in mind that we are assuming free fall. That means the objects can't be moving so fast that drag becomes significant. Indeed, if we could ignore drag (and also had a perfectly straight horizontal surface), even a bullet fired horizontally from a gun would hit the ground at the same time as bullet dropped from rest (assuming one bullet was dropped at the same time the other bullet left the gun).

✓ *Checkpoint 16.5: At the exact moment that ball A is dropped from rest, ball B is shot horizontally out of gun (i.e., no upward or downward motion initially) from the same height above the ground. Assuming no drag, which will hit the ground first? Explain.*

Because we can ignore the horizontal information when solving for vertical motion, problems asking for vertical information can be solved in a manner similar to what we did with purely vertical motion.

To illustrate, we'll again use some numbers with the scenario we've considered before (see time-lapse picture on page 262):

> A ball rolls at 3 m/s along a 2-m high horizontal table. How far below the table top is it 0.25 s after leaving the table?

Since the problem asks for "how far below the table top," we know that we are being asked for a vertical displacement. As such, we can ignore any horizontal information (like the horizontal velocity on the table top).

To find the vertical displacement, we use the definition of average velocity ($\vec{v}_{\text{avg}} = \Delta\vec{s}/\Delta t$) and solve for $\Delta\vec{s}$. However, while we are given the time

(0.25 s), we are not given the velocity. Remember, the 3 m/s provided in the problem is a *horizontal* velocity and has no bearing on the *vertical* displacement.

Instead, we need to know the average *vertical* velocity. And, as discussed in chapter 14, the vertical velocity is *changing* – there is no single value. To solve this, then, we need to use additional relationships, just like any other problem involving free fall.

The process is the same as what we used in solving the problem described on page 230, which asked for how far Rabbit fell in 12.5 seconds. Basically, we use the force and motion equation to determine the change in *vertical* velocity and then, from that, determine the average *vertical* velocity (which would be the midrange value of the initial and final *vertical* velocities). At that point we can use the definition of average velocity to get the *vertical* displacement.

Repeating the process, but with the numbers, we get the following. First, the law of force and motion (with the law of gravity) tells us that during free fall the ball's *vertical* velocity changes by 9.8 m/s downward every second (since the net force per mass is 9.8 N/kg). That means that during 0.25 s of free fall the vertical velocity changes by 2.45 m/s downward (multiply 9.8 m/s^2 by 0.25 s).

At the moment the ball leaves the table it is neither moving up nor down so its initial *vertical* velocity is zero. Since the vertical velocity changes by 2.45 m/s downward, that means the ball's final vertical velocity (i.e., at the end of the 0.25 s) is 2.45 m/s downward.

Since it started at rest and ended at 2.45 m/s downward, its average *vertical* velocity during the 0.25 seconds is 1.225 m/s downward (i.e., the midrange value of zero and 2.45 m/s downward). Using the definition of average velocity, we can multiply this average *vertical* velocity by the time to get the *vertical* displacement (0.31 m downward).

Notice that we don't need to consider how it is moving *horizontally*, as we are asked only for the *vertical* displacement.

How do we know that the initial vertical velocity is zero?

The time of interest is when the ball is in the air, which starts immediately upon leaving the table. Since the initial time is the moment the ball leaves the table, we can assume that the velocity of the ball at that moment is the same as it was when it was on the table, which was entirely horizontal, with no vertical motion.

✓ *Checkpoint 16.6: A ball rolls off the edge of horizontal table with a velocity of 2 m/s rightward. Assume the table is high enough so that the ball doesn't hit the floor. To find the ball's vertical displacement during the first 0.2 s it is in the air, what value do I use for the initial vertical velocity? Why?*

What if the ball is thrown upward or downward as well as horizontally?

The approach is exactly the same as before. As long as we are only asked for something vertical, we only consider the vertical motion and forces.

To illustrate, let's consider the same scenario as before except with a ball launched at an angle.

> A ball is thrown at an angle such that it is initially moving 4 m/s upward and 3 m/s rightward (i.e., it is moving both horizontally and vertically). If the ball is released two meters above the ground, how far has it moved vertically during the first 0.25 seconds of its flight?

The only difference between this problem and what we had before was that the object is initially moving 4 m/s upward *as well as* 3 m/s rightward.

However, the problem is still asking for the ball's vertical displacement during 0.25 seconds while it is in the air. From the vertical viewpoint, this is different from what we had before because now the initial vertical velocity is 4 m/s upward (instead of zero).

The process, though, is the same as before, since we have the same known and unknown quantities as before. In particular, we can ignore the horizontal aspects of the problem since it asks for the vertical displacement. As we know, the vertical displacement is independent of its horizontal motion.

The situation is illustrated in the time-lapse picture shown in Figure 16.2. For comparison, the time-lapse picture for an object with *no* horizontal motion is also plotted. Notice how the vertical motion is the same for both objects.

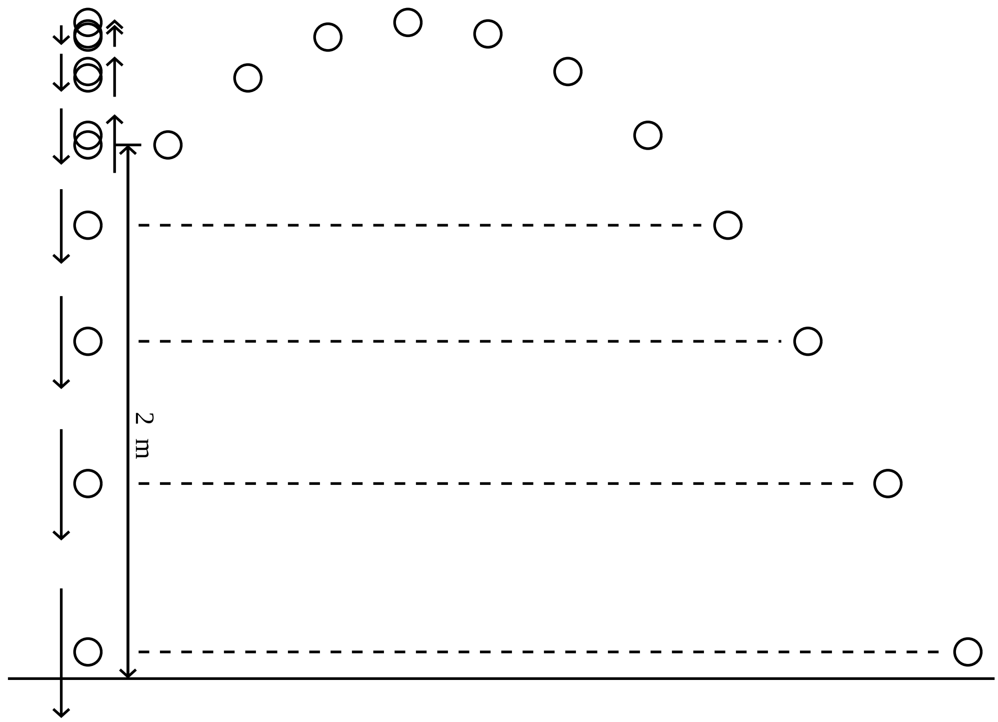


Figure 16.2: Time-lapse picture of two balls thrown up in the air. [left] Ball thrown straight upward. [right] Ball thrown horizontally as well as upward. Images are taken 0.1 seconds apart. For clarity, dashed lines indicate the heights of the balls for the last four images.

Since we can focus solely on the vertical motion, the problem is solved the same way it would be solved for an object with *no* horizontal motion.

While in the air, the ball experiences free fall, which means its *vertical* velocity changes by 9.8 m/s downward every second (since the net force per mass is 9.8 N/kg). That means that at 0.25 s the ball's *vertical* velocity has changed by 2.45 m/s downward.

Since it started with an upward velocity of 4 m/s, that means its velocity at the end of the 0.25-s time period must be 1.55 m/s (i.e., it is slowing down).[ii] The average of 4 m/s and 1.55 m/s is 2.775 m/s. Consequently, its average *vertical* velocity during the 0.25 s is 2.775 m/s upward.

[ii]The initial vertical velocity is upward while the change in vertical velocity is downward. This means the object slows down.

Using the definition of average velocity, we can multiply this average *vertical* velocity by the time to get the *vertical* displacement (0.69 m upward).

✓ *Checkpoint 16.7: In the scenario described above, a ball is thrown at an angle such that it is initially moving 4 m/s upward and 3 m/s rightward, respectively. It was found that the ball will move 0.69 m upward during the first 0.25 seconds of its flight. How would the answer change if the initial horizontal motion was 5 m/s rightward instead of 3 m/s rightward? Explain.*

16.3 Separating components

Just as a car engine can be thought of as having multiple **components**, we can also describe a projectile's motion in terms of two components: a horizontal component and a vertical component. The same can be said of forces, although for projectile motion the net force only has one component – a downward component.

When given a scenario involving a projectile, it is important to recognize that the object's motion consists of these two components, and that each component of the motion is impacted only by the corresponding component of the net force.

For example, consider the following scenario. As you read through it, think about what is being requested.

> Suppose you kick a football such that it lands 30 m away. If the entire flight takes 3 s, what was the ball's initial velocity, assuming the ball started on the ground and there was no drag during the flight?

In this case, we are asked to solve for the initial velocity. The first step is recognize that the initial velocity has two components: a vertical component (which is why it goes up in the air) and a horizontal component (which is why it lands 30 m away).

The second step is to apply the law of force and motion to each component direction (horizontal and vertical), knowing that the only force acting is the downward gravitational force. This means that the horizontal component of the football's velocity won't change while the football is in the air, while the vertical component will.

We thus have two separate problems: a horizontal problem (involving the horizontal component of the velocity and zero horizontal force) and a vertical problem (involving the vertical component of the velocity and the gravitational force).

The *horizontal* problem states that the ball moves 30 m horizontally in 3 s. That means the average horizontal velocity is 10 m/s (divide 30 m by 3 s). Since there are no forces acting horizontally, the horizontal velocity must remain the same during the entire flight. So the *initial* horizontal velocity must likewise by 10 m/s.

The *vertical* problem states that the ball moves upward and then downward in 3 s. The total vertical displacement is zero so the average vertical velocity is likewise zero. Since this is free fall, the vertical velocity must be changing at a rate of 9.8 m/s downward every second. During the 3 seconds, then, the vertical velocity has changed by 29.4 m/s downward.

The only way the average vertical velocity can be zero yet still be experiencing a change of 29.4 m/s downward is if the initial vertical velocity was 14.7 m/s upward and the the final vertical velocity was 14.7 m/s downward.

Thus, we have that the initial velocity was 10 m/s horizontally and 14.7 m/s upward.

✓ *Checkpoint 16.8: In the scenario described above, why is a displacement of zero used when figuring out the vertical component of the initial velocity?*

16.4 Galilean relativity

As noted before, one of the things we observe with projectile motion is that, regardless of the horizontal motion, the object continues to fall in the same way it would without any horizontal motion.

That this is indeed true can be seen by dropping an object while on a train, plane or car that is moving horizontally at a constant speed. As the train moves horizontally, everyone on the train is likewise moving horizontally, as is every part of the train.

So, suppose we hold a ball in our hands above a strategically placed cup on the floor of the train, directly beneath the ball. Both the ball and the cup are moving horizontally with the train. If we let go of the ball, it will continue to move horizontally as before, keeping it directly above the cup the entire time it is falling. As a result, it will fall into the cup.

It doesn't matter if you are on a plane moving at 300 mph at 30,000 ft or on a plane at rest on the airport tarmac. The horizontal motion of the plane (and object) has no impact on the vertical motion of the object.

✍ This assumes no drag. If the train is just a flat bed with no walls, there might be quite a bit of wind when the train is moving quickly. In that case, there may be a significant horizontal force acting (drag) and the ball may be blown "backwards," missing the cup.

The idea that we can't use our observations of the falling ball to determine the horizontal motion of the plane, train or car was used by Galileo Galilei way back in 1632 to argue that we likewise aren't able to use our observations of a falling object to determine if the Earth is moving or not. Not surprisingly, then, this idea (that there is no absolute motion and that we are only able to measure motion relative to other things) is called **Galilean relativity**.

Galilean relativity actually extends beyond what we've examined. Although we've only considered cases where the horizontal motion was constant, it turns out that we can't use observations (like falling objects) to determine if we are moving at a constant velocity in *any* direction.

For example, suppose we were in an elevator. Our observations of a falling ball would be the same whether the elevator was stopped at a floor or moving upward or downward at a constant speed. The laws of physics are the same whether we are stationary or moving at a constant velocity.[iii]

✓ *Checkpoint 16.9: A plane flies eastward at 50 m/s at an altitude of 500 m. At the exact moment the plane is over a pool, a piano is dropped from the plane. Does the piano fall into the pool? Does it matter if there is drag?*

[iii] This will be discussed further in section 20.5).

Summary

This chapter applied the law of force and motion to projectile motion.

The main point of this chapter is that two-dimensional motion (like projectile motion) can be solved as two separate problems – one for each component direction. This means that the change in an object's horizontal velocity depends only on the horizontal force imbalance, while the change in an object's vertical velocity depends only on the vertical force imbalance. When applied to projectile motion (free fall), this means that the horizontal velocity doesn't change at all, while the vertical velocity changes at a rate of 9.8 m/s^2, with the time to fall being independent of the horizontal motion.

Frequently asked questions

When a ball rolls off the edge of a table, wouldn't it take longer to fall than one that is dropped from rest since it has to move horizontally as well as vertically?

No. See page 268.

During free fall does the velocity transition from being horizontal to being downward?

No.

While it gains downward motion, it does not lose its horizontal motion. To stop the horizontal motion, we need a force acting horizontally. For free fall, there is no leftward or rightward force, only a downward force.

✍ As noted before, if object is moving quickly, there may be drag. In that case, the horizontal net force on the object is no longer zero. The object would no longer be in "free fall."

Wouldn't an object initially moving horizontally take longer to fall since it has to move horizontally as well as vertically?

Not if there is no drag. The object moves downward in the same way it would if dropped from rest.

Think of it this way. First, consider what happens if the ball is initially moving upward. It should then take *longer* to fall than if released from rest.

Then consider what happens if the ball is initially moving downward. It should then take *less time* to fall than if released from rest. Thus, having an initial horizontal velocity (i.e., neither upward nor downward), it should take something in between – the *same* amount of time as when released from rest.

Will the horizontal net force always be zero during free fall?

Yes, if forces like drag are negligible.

Is an object undergoing projectile motion still considered to be in free fall?

Yes, I will continue to refer to this situation as free fall, even though the object is moving horizontally as well as vertically. Free fall doesn't mean the object is only moving downward. It simply means that the gravitational force is the only force acting on the object.

Terminology introduced

Components
Dimension
Galilean relativity
Projectile motion
Reference directions

Problems

Problem 16.1: (a) Suppose I am on a stationary train. I hold a rock that is at rest relative to me. I then drop it. As it drops, the net force on it is directed downward. In what direction does the rock start to move?
(b) Suppose I am on a train that is *moving* with a constant velocity. I hold a rock that is at rest relative to me. I then drop it. As it drops, the only force on it is directed downward. In what direction does the rock start to move, relative to me?
(c) Suppose the moving train has no windows. Can I tell how fast the train is moving just by measuring how quickly the rock falls to the floor? Explain. Hint: Is the rock's acceleration any different?

Problem 16.2: You throw a ball to a friend. Answer the following questions for the time period when the ball is in the air, assuming drag is negligible. Explain your reasoning for each case.
(a) What forces, if any, are acting on the ball? Are any of the forces directed in the direction of the ball's motion (e.g., due to the ball's inertia)?
(b) In what direction is the net force acting on the ball?
(c) In what direction is the ball's velocity changing?
(d) How does the horizontal component of the ball's velocity change (i.e., does it decrease, increase or stay the same)?

Problem 16.3: Suppose I throw a rock straight up into the air with an initial speed of 4.0 m/s. If the rock was released from a height 2 meters above the ground, where is the rock 0.7 s later?

Problem 16.4: A ball rolls off a horizontal table 1 meter high with an initial horizontal speed of 2 m/s.
(a) After 0.2 seconds (i.e., 0.2 seconds after leaving the table), what is the ball's horizontal velocity and its vertical velocity?
(b) How far (vertically) has it fallen in 0.2 s?
(c) How long does it take to fall 1 meter (vertically) to the floor?
(d) How far has the ball traveled horizontally during the time in (c)?

Problem 16.5: A ball is thrown at an angle such that its initial velocity has a vertical component equal to 5 m/s upward and a horizontal component equal to 3 m/s. Assume the release point is high enough so that the ball doesn't hit the floor. After 0.3 seconds,
(a) what is the ball's horizontal velocity?
(b) what is the ball's vertical velocity?
(c) what is the magnitude of the ball's velocity?

Problem 16.6: A football, initially on the ground, is kicked such that it lands 13.8 m away. If the entire flight takes 1.4 s, what was the vertical and horizontal components of the ball's initial velocity? Assume no drag.

Problem 16.7: A football, initially on the ground, is kicked such that its initial velocity has an upward component of 5 m/s and a horizontal component of 10 m/s. Assume no drag.
(a) What is its vertical displacement during the first 0.5 seconds of its flight?
(b) How far away does the football land?

17. Motion Along Surfaces

Puzzle #17: So far, we have only dealt with frictionless surfaces. Can we predict the motion of an object when friction is present?

Introduction

So far, all of our surfaces have been frictionless. This is not because frictionless surfaces are commonplace. They aren't. Rather, the frictionless assumption was made just to simplify things. Friction complicates matters and so it helps to be able to analyze the situation in some idealized sense prior to making it more realistic.

In this chapter, we examine surfaces that have friction. The reason friction complicates things is because friction depends upon the surface repulsion force (see section 15.2). The difficulty arises because friction acts in a direction that is *parallel* to the surface whereas the surface repulsion force acts *perpendicular* to the surface.[i]

To see the issue, suppose we have a box sliding across a horizontal floor. From chapter 16, we know that the horizontal motion (along the floor) depends upon the horizontal forces. Friction is one of those forces. However, the complicating factor is that friction, in turn, depends on the surface repulsion force, which is directed perpendicular to the surface.

So, whereas in projectile motion we could predict the horizontal motion without worrying about the vertical forces, with friction we can't. To determine the horizontal motion of the box, we first have to figure out the surface repulsion force on the box and then, in turn, find the friction. Only then can we predict the horizontal motion.

[i]As discussed in section 15.2.1, the surface repulsion force is directed away from the surface, which is *perpendicular* to the surface.

17.1 Direction of the friction force

If friction is present, the surface provides both a force *perpendicular* to the surface (which I've been calling the **surface repulsion force**[ii]) and a force *parallel* to the surface (which we call the **friction force**). Even though I am treating them as two separate forces, they are both related since they both correspond to the interaction between the object and the surface. Still, these two forces have such dramatically different behavior that it makes sense to deal with them separately.

In this section, I'll examine the *direction* of the friction force. Later in the chapter I'll examine the *magnitude* of the friction force.

Whereas the surface repulsion force prevents motion into the surface, the friction force opposes sliding along the surface.

❢ Friction is a force that opposes sliding.

For example, consider a box that is so heavy that it remains stationary even when I try to push it across the floor. In this case, friction acts between the box and the floor, opposing the direction in which it would otherwise slide. So, if I was trying to push the table leftward, the surface (via friction) would be pushing it rightward (opposite the direction it would otherwise slide).

What if you didn't push at all – would there still be friction?

Friction only acts if it has to. If the box wouldn't slide anyway, friction wouldn't be needed to oppose it. Consequently, there would be no friction if you didn't push on the box.

What if you push really hard so that the table moves – would there still be friction?

Yes, friction still acts. One can tell friction is acting whenever it is harder to slide than it would otherwise be without friction. Friction not only opposes the *potential* for sliding it also opposes the *actual* sliding.

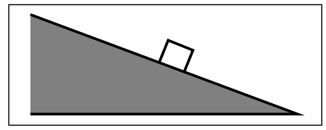
In both cases, the direction of the friction is parallel to the surface and opposite the direction of the sliding (or the direction the object would slide if there were no friction).

So, even with friction, the *first* step in a problem is still to identify all of the things that are interacting with our object. The only difference now is that

[ii] It is common to call this force the **normal force**, since it acts in the normal direction (perpendicular to the surface; see section 15.2.1).

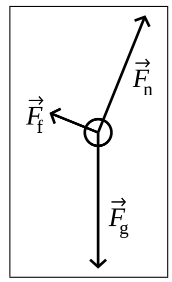
we have two forces associated with a single interaction: the surface repulsion force (directed perpendicular to the surface) and the friction force (directed parallel to the surface).

Consider, for example, a box on an inclined surface as illustrated to the right. Let's now assume there is friction, which is preventing the box from sliding down the ramp.



What forces are acting on the box?

The answer is three, as shown in the force diagram I've drawn to the right. Why these three, and why in the directions shown?



To determine the forces, we need to identify the things that are interacting with the box. The downward pointing arrow represents the gravitational force, as the box is attracted downward to Earth.

All other forces are due to contact. In this case, the box is touching the surface, so we have a surface force. Since there is friction, there are *two* forces associated with this surface: the surface repulsion force and the friction force.

The surface repulsion force is directed perpendicular to the surface and directed away from the surface (in the normal direction; see figure). The friction force is directed up the ramp because the box would otherwise slide down the ramp without friction.

✍ The friction force is indicated in the force diagram by the arrow with label $\vec{F}_f$ (where the subscript "f" indicates that it is friction). The surface repulsion force is indicated in the force diagram by the arrow with label $\vec{F}_n$ (where the subscript "n" indicates that it is directed normal, or perpendicular, to the surface; see section 15.2.1).

✓ *Checkpoint 17.1: A table sits at rest on horizontal floor. Although I push on it with a rightward force, it remains at rest because of friction between the table and the floor.*
(a) What is the direction of the friction force on the table?
(b) If I don't push on the table at all, what is the frictional force exerted on the table?

What if the box is sliding down the ramp?

If friction is present but not strong enough to keep the box at rest, the box may slide down the ramp. Or, someone may be pushing it downward, leading to it sliding down the ramp. Either way, friction on the box would be directed up the ramp, opposing the direction of the sliding.

What if the box is sliding UP the ramp?

In that case, the friction on the box would be directed *down* the ramp, since the box is sliding up the ramp.

✍ For a box to be sliding up the ramp, someone or something must be pushing it up or have started it upward at some point.

✓ *Checkpoint 17.2: Consider a box on a ramp, as shown on page 281. Would the force diagram look any different for a box sliding* down *the ramp as opposed to sliding* up *the ramp? If so, how (draw a diagram)? If not, why not?*

As a special case to illustrate how tricky it is in some cases to determine the direction of friction, consider the following scenario:

> A box is at rest on the back of a truck. When the truck starts moving, the box stays on the back of the truck because of friction between the box and the bed of the truck. In this case, what is the direction of the friction force on the box?

To answer this, first consider what would happen to the box if the bed of the truck was frictionless. Without friction, it would slide off the back of the truck. Since friction opposes this, it must act in the opposite direction, which is forward, toward the front of the truck.

How can friction be directed in the direction of the motion? I thought friction always opposed the motion.

To figure out the direction of friction, first consider which way the object would slide if there were no friction. In this case, the box would slide to the back of the truck (due to its inertia) if there were no friction. To avoid sliding, there must be friction that opposes this sliding. In this case, that means the friction must be directed forward, toward the front of the truck.

Figuring out the direction of friction can be tricky, but the key is to compare what happens with friction vs. without. For example, friction is necessary for a car to slow down and come to a stop. Without friction (like on an icy road), the car would continue to slide *forward*, even with the wheels locked. That means friction must be directed *backwards* (opposite the direction of motion) when trying to stop.

What about starting?

This is a little trickier to see, but without friction the wheels would just spin and the car wouldn't go anywhere. Think of what would happen if you placed the tire on something slippery like a banana peel. The peel would go flying backward when the tire started to spin. That means the tires pushes the banana peel backward. As stated by the law of interactions, the banana peel must be pushing the tire forward but, due to the higher mass of the tire, it has little effect on it.

Now replace the banana peel with the ground. The same is true in that the tire pushes the ground backward and the ground pushes the tire forward. However, the tire is now less massive and so the force has a bigger impact on the tire than the ground. One can also think of the friction as preventing the wheels from slipping and, to do that, there must exert a forward force on the wheels.

How about turning?

When turning, friction keeps the car on the road. Without friction, the car would slide *off* the side of the road so friction must act in the opposite direction, pushing the car *onto* the road (i.e., *from the side* of the road toward the road). We'll talk more about turning in chapter 20.

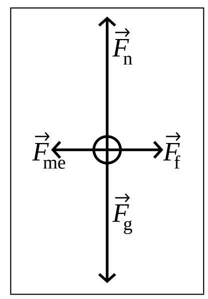
The key point is that friction always opposes the sliding, whatever direction that happens to be. Friction is not always opposite the motion.

✓ *Checkpoint 17.3: (a) On the lab floor, I can use friction to start running (because friction prevents me from slipping). What is the direction of the friction acting on me?*
(b) How about when I try to stop? Hint: Consider what would happen if you step onto a patch of ice. Your foot would go flying forward and you'd fall on your back. Remember that friction opposes the slipping (i.e., prevents your foot from flying forward).

17.2 Magnitude of the friction force

As we know from section 15.2, the surface repulsion force prevents objects from sinking into the surface and has whatever magnitude is necessary to prevent motion into the surface. Unlike the surface repulsion force, there is a maximum magnitude that the friction can have. This means that friction can keep an object at rest against an applied horizontal force but only up to a point.

To illustrate, consider our 5-kg box on a horizontal surface but suppose there is now friction present and we are told that the friction can have a maximum magnitude of 30 N. Does the box remain at rest when I exert a horizontal force of 20 N leftward on the box?



For it to remain at rest, the forces have to balance. We already know that the *vertical* forces will balance because the surface repulsion force will ensure that is so (see $\vec{F}_{\text{n}}$ and $\vec{F}_{\text{g}}$ in the diagram).

Horizontally, we have two forces. There is the 20 N leftward (due to me; indicated as $\vec{F}_{\text{me}}$) and there is the friction force (due to the floor; indicated as $\vec{F}_{\text{f}}$). The friction force is rightward because it opposes the sliding (which would otherwise be leftward due to the applied force).

To keep the box at rest, the friction force needs to have a magnitude equal to 20 N (i.e., balance the 20-N force due to me). Since the 20 N is less than the maximum possible magnitude for friction in this case, the box does indeed remain at rest.

WHAT IS THE MAGNITUDE OF THE FRICTION FORCE IN THIS CASE?

The friction has a magnitude of 20 N in this case, since that is sufficient to oppose the force I am exerting on the box.

WHAT IF YOU APPLY A FORCE WITH A MAGNITUDE OF 40 N?

According to the problem, the friction force cannot have a magnitude greater than 30 N. Consequently, the friction wouldn't be enough to balance the applied force of 40 N and the box would not remain at rest.

WHAT HAPPENS IF YOU DON'T PUSH ON IT AT ALL?

There will be no friction force at all. The friction force is present only when there is motion to oppose. If I don't push on the box, there is no friction force.

✓ *Checkpoint 17.4: A 25-kg table sits at rest on horizontal floor. Although I push on it with a force of 5 N rightward, it remains at rest because of friction between the table and the floor. If I don't push on the table at all, what is the friction force exerted on the table?*

17.2.1 Dependence on surface repulsion force

Is the maximum friction always have a magnitude of 30 N?

No. It depends on a couple of things.

❢ The maximum magnitude of the friction depends upon the magnitude of the surface repulsion force.

For example, through repeated measurements, one finds that the maximum magnitude of the friction depends on the magnitude of the surface repulsion force. This should make sense, since the surface repulsion force represents how much the two materials are pressed together.[iii] The more the two materials are pressed together, the greater magnitude of friction that is possible. You can show this yourself by pressing your hands together. The more you press your hands together, the harder it is to get one hand to slide along the other.

✍ Since the friction force (directed parallel to the surface) depends on the surface repulsion force (directed perpendicular to the surface), this means that the two directions are not completely independent. In other words, we'll need to know something about the vertical forces in order to predict the horizontal motion.

Does the maximum magnitude of friction depend on the area of contact between the object and the surface?

For the most part, no. Assuming the same surface repulsion force (i.e., same force pushing the object and surface together), a greater surface area will

[iii] To see why the surface repulsion force indicates how much the two materials are pressed together, consider a box on a floor. By the law of interactions, the surface repulsion force on the box due to the floor is equal and opposite to the surface repulsion force on the floor due to the box. The greater the surface repulsion force, the greater the box and floor are "pressed together."

mean a lower pressure, and a lower pressure cancels out the effect of the larger surface area.

✓ *Checkpoint 17.5: If you move your hand over the table such that your hand doesn't touch the table, will there still be a friction force? Why or why not?*

17.2.2 The law of friction

❢ The maximum magnitude of the friction is proportional to the magnitude of the surface repulsion force.

Not only do repeated measurements suggest that the maximum magnitude of friction depends on the magnitude of the surface repulsion force but, it turns out, the maximum magnitude of the friction is roughly *proportional* to the magnitude of the surface repulsion force.[iv] In other words, as the surface repulsion force increases, the maximum friction force increases in a proportional way, such that the ratio of the two remains the same.

✍ By being proportional, that means if the magnitude of the surface repulsion force doubles, the maximum magnitude of the friction force likewise doubles. In general, if the magnitude of the surface repulsion force increases (or decreases) by a factor n, the maximum magnitude of the friction force likewise increases (or decreases) by a factor n.

This relationship, together with the finding that the friction is independent of the surface area of the material in contact with the surface, is known as the **law of friction**.[v] Mathematically, this idea can be expressed in terms of a friction equation as follows:

$$|\vec{F}_{\text{f,max}}| = \mu|\vec{F}_{\text{n}}| \tag{17.1}$$

[iv] This relationship doesn't necessarily hold for all materials or for all surface repulsion forces. However, it should hold for most materials and allows us to explore situations that are a little more realistic than purely frictionless ones we'd otherwise be restricted to.

[v] As with the other laws, the name is based on what the law describes rather than the scientist who is credited with first identifying it. In this case, the law is sometimes known as **Amontons' law of friction**, as it was first identified in 1699 by Guillaume Amontons, a French scientist who lived from 1663 to 1705. Amontons actually wrote the law of friction in several parts. The first part stated that the friction is independent of surface area, a finding sometimes called Amontons first law of friction or da Vinci's law of friction since da Vinci had identified it 500 years before Amonton. This dependence on the surface repulsion force, sometimes called Amontons' second law of friction, was actually the third or fourth part of his law of friction.

where μ (i.e., the Greek letter "mu") is a number called the **coefficient of friction** that represents how large the friction is compared to the surface repulsion force. The greater the coefficient μ, the "stickier" the boundary between the two materials (i.e., the greater the magnitude of the friction force can be for a given magnitude of surface repulsion force). In section 17.2.3, we'll examine how the coefficient depends upon the materials that are involved.

Before continuing, I want to point out that the value of μ is typically written without any units. This is because $\vec{F}_{\text{f,max}}$ and $\vec{F}_{\text{n}}$ are both forces and, since they have the same unit (newtons), the ratio of the two (μ) can be written without a unit (i.e., the newtons cancel):[vi]

$$\mu = \frac{|\vec{F}_{\text{f,max}}|}{|\vec{F}_{\text{n}}|}$$

In this way, you can consider the coefficient of friction to essentially be a fraction that indicates how big the friction force magnitude can be compared to the surface repulsion force magnitude. A value of one means the magnitude of the friction force can be as large as the magnitude of the surface repulsion force. A value of 0.5 (i.e., 50%) means the magnitude of the friction force can be as large as half the magnitude of the surface repulsion force, but no larger.

Example 17.1: Consider a 25-kg table on a horizontal floor, where the coefficient of friction between the table and the floor is 0.20. What is the maximum value the magnitude of the friction force can have?

Answer 17.1: As expressed in the law of friction, the maximum magnitude of the friction force depends upon the magnitude of the surface repulsion force.

To find the surface repulsion force, we use the physics idea that the forces must balance in order to keep the box at rest on the floor. In this case, that means the surface repulsion force must balance the gravitational force.

[vi] Quantities like μ are called dimensionless. Dimensionless quantities can be written without units, but they could also be written with units like percent (or parts per million). In the case of μ, it is conventional and convenient to use no units because that way we don't have to worry about it in any subsequent calculation, much like we don't have to worry about multiplying by one in any calculation.

Since the object is near Earth's surface, the gravitational force has a magnitude equal to the table's mass (25 kg) times the factor of 9.8 N/kg. That gives a gravitational force of 245 N downward.

Since the surface repulsion force must balance the gravitational force, that means the surface repulsion force is also 245 N but upward.

According to the statement of the problem, the coefficient of friction is 0.20, which means the magnitude of the friction force can be as large as 20% of the magnitude of the surface repulsion force (see friction equation 17.1). Multiplying 0.20 by the magnitude of the surface repulsion force (245 N) gives a maximum friction force of magnitude 49 N.

Keep in mind that the 49-N value obtained in the example is *not* the friction force. It just happens to be the maximum possible magnitude that the friction can have (in this case). The actual value may be less than that, depending on what is needed to oppose the sliding. It just can't be more than that.

✓ *Checkpoint 17.6: In the example, the coefficient of friction between the 25-kg table and the floor is 0.20.*
(a) Suppose the table is at rest when a 40-N horizontal force is exerted on the table. What would be the friction force in that case? Would the table start to move?
(b) Suppose the table is at rest when a 60-N horizontal force is exerted on the table. What would be the friction force in that case? Would the table start to move?

Let's put all of these ideas together and show how they can be used to solve a problem. Consider, for example, the following situation, which is similar to what we've already discussed. The only difference is that it asks something that requires us to apply several different ideas in order to answer.

> A 25-kg table is on a horizontal floor, where the coefficient of friction between the table and the floor is 0.20. How hard do I have to push horizontally on the table in order to get it to start sliding?

You might already know the answer, based upon what we've done so far. However, I am going to solve it as though this is the first time we've ever seen this problem.

We start, as always, with the law of force and motion, which tells us that to get the table to start sliding, there has to be an imbalance in the forces. However, in this case, we have friction, and friction can have any value up to a some maximum value that depends on the coefficient of friction (given as 0.20 in the problem) and the surface repulsion force (not given). Once we find the maximum value, the answer to the problem is that we have to push harder than that maximum value.

The problem thus becomes one of finding the surface repulsion force.

This confuses some people, since the problem doesn't ask for the surface repulsion force – it asks for how hard we have to push on the table. However, we can't arbitrarily look at the problem and match the solution to what words are used in the problem. We need to use the physics. The physics will tell us the way.[vii]

So how do we find the surface repulsion force?

We again use the law of force and motion, which tells us that for the table to stay on the floor the vertical forces must balance. There are only two vertical forces (the gravitational force due to Earth and the surface repulsion force due to the floor), so these must be equal and opposite.

The problem thus becomes one of finding the gravitational force.[viii]

Now that we've figured out how to solve the problem, we use the numbers.

The gravitational force is 245 N (multiply the mass by 9.8 N/kg). As mentioned previously, the surface repulsion force must equal the gravitational force (245 N). Finally, knowing the surface repulsion force, we can determine the maximum magnitude of the friction by multiplying the magnitude of the surface repulsion force (245 N) by the coefficient of friction (0.20). This gives a maximum friction force of magnitude 49 N.

Since that is the maximum friction force, we have to push more than 49 N in order to have an imbalance of forces and start the table moving.

[vii] Trying to solve a physics problem without using physics is like trying to find your way without a map.

[viii] Isn't this exciting?

✍ Remember that the surface repulsion force is not always equal to the gravitational force. It just happens to equal the gravitational force in this example because there are no other forces acting perpendicular to the surface (and the object's motion is not changing).

✓ *Checkpoint 17.7: A 2-kg book is at rest on a desk. The coefficient of friction between the book and the desk is equal to 0.3. Will the book start to slide if I apply a force of 6 N parallel to the desk? Why or why not? Be sure to identify the physics you are using (as opposed to simply copying the procedure I used above).*

17.2.3 Dependence on materials

As mentioned earlier, the maximum friction depends on the materials involved. Some materials (like ice) are very slippery and have low friction while other materials (like rubber) have high friction.

❢ The maximum magnitude of the friction depends upon the materials involved.

For example, slide your hand along the table. The friction inhibits your motion along the table (i.e., it makes your hand move slower than it would otherwise go). Now rub some vegetable oil on the table and slide your hand along the table again. The friction is still there but it is significantly less.[ix]

This is all just another way of saying that the coefficient of friction, μ, depends on the materials involved. Some materials are slippery while others aren't.

✍ For a given pair of materials, the coefficient μ has the same value no matter what the value of the surface repulsion force.

❢ The coefficient of friction is not a force. It is a dimensionless coefficient representing how large the maximum magnitude of the friction is compared to the magnitude of the surface repulsion force.

Keep in mind that the coefficient μ is <u>not</u> a force and does not have units of force. The coefficient simply indicates how large the maximum friction is compared to the surface repulsion force. A large number, like 1.2, means the maximum magnitude of the friction is large (a little larger than the magnitude of the surface repulsion force). A small number, like 0.02, means the maximum magnitude of the friction is small (a small fraction of the surface repulsion force magnitude).

[ix] Remember to wash your hands and the table after you're done.

μ	**Materials**
0.9-1.0	glass and glass
0.8-1.0	skin and metals
0.62	wood and concrete
0.6	wood and brick
0.58	steel and steel
0.2-0.6	wood and metals
0.27-0.38	leather and oak
0.2-0.4	wood and stone
0.05-0.5	ice and ice
0.04-0.4	snow and waxed hickory
0.04	Teflon and Teflon

Table 17.1: Source: http://physics.info

Because the maximum friction is dependent on the materials, the coefficient has to be determined empirically by measuring the maximum friction force for the two materials that you have and then dividing by the surface repulsion force.

Do we have to do lots of experiments in order to find out what μ is?

Fortunately, other people have already done the experiments. We can just use the results of their experiments. Some of these results are listed in Table 17.1.

Notice how slippery materials like ice have low coefficient values.

Why are there two materials listed for each coefficient value?

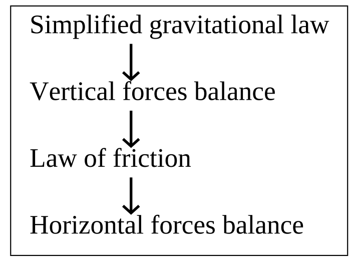
There are two materials listed because the coefficient does not just depend on the surface but also on what material is placed on the surface.

With the help of the table, we can solve problems where the coefficient is not explicitly provided. Consider the following scenario:

> A 7-kg wood box is placed on a concrete floor. I exert a force of 10 N rightward. Does the box remain at rest?

As always, we start with the physics: the box remains at rest only if the forces balance. That means the 10 N (due to me) must not be greater than the maximum friction force (due to the floor). The maximum magnitude of the friction force, in turn, is equal to the product of the coefficient of friction and the surface repulsion force magnitude (which, in turn, must balance the gravitational force since the box is not moving into the floor or rising above it).

The process is illustrated in the diagram at right. We get the gravitational force from the *simplified gravity equation* then, using the idea of *vertical force balance*, we get the surface repulsion force. Knowing the surface repulsion force, we use the *friction equation* to get the friction force and then *horizontal force balance* to determine if the box remains at rest.



The magnitude of the gravitational force on the box is 68.6 N (multiply 7 kg by 9.8 N/kg) so, in this case, the magnitude of the surface repulsion force is also 68.6 N.

Unlike in past cases, the coefficient of friction isn't provided. However, we are told that the box is made of wood and the floor is made of concrete. Since the materials are given, we know what the coefficient of friction is. From table 17.1, we find that the coefficient is 0.62.

Multiply the surface repulsion force by the coefficient (0.62) to get 42.5 N as the maximum friction force.

This means the friction force can have a magnitude equal 10 N (the magnitude of my applied force), since that is less than 42.5 N. Being 10 N each, the forces are balanced and the box remains at rest.

✓ *Checkpoint 17.8: Suppose the 5-kg wood box is placed on a different floor of unknown material. I find the box remains stationary when I exert a force of 20 N but it moves when I exert a force of 25 N. What could the coefficient of friction be for these materials?*

17.3 Sliding objects

If the object is sliding and there is friction, then we'll assume the magnitude of the friction force is given by the *maximum* value[x], as determined by 17.1:

$$|\vec{F}_{\text{f,max}}| = \mu|\vec{F}_{\text{n}}|$$

We will also assume that the magnitude of the friction force is the same, regardless of how fast the object slides along the surface.[xi]

❢ If the object is sliding and there is friction, we'll assume the magnitude of the friction force is given by the maximum value and is independent of the sliding velocity.

To illustrate, consider the following scenario.

> Suppose we have a 10-kg box at rest on a horizontal surface where μ is equal to 0.3. What force would we have to apply to the box in order to give it a velocity of 2 m/s rightward if applied for 5 seconds?

As always, start with the physics. The law of force and motion tells us that there must be a rightward force imbalance if the object experiences a change in velocity rightward. The amount of the force imbalance can be determined from the force and motion equation.

In this case, we know $\Delta\vec{v}$, Δt and m as 2 m/s rightward (since it started at rest), 5 seconds and 10 kilograms, respectively. Plugging into the force and motion equation, I get 4 N rightward (multiply the change in motion by the mass and divide by the time).

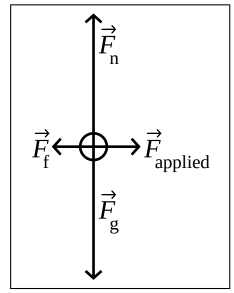
However, this is the *net* force (i.e., the leftover force), <u>not</u> our *applied* force.

---

[x] In practice, the magnitude of the friction force when sliding is a little less than the maximum value given in the equation. This is because more bonds form between surfaces when there is no sliding and so more bonds need to be broken. For this reason, most physicists use a different value of μ when the object is sliding.

[xi] This finding is also part of Amontons' original law of friction. It is sometimes called Coulomb's law of friction, named after Charles-Augustin de Coulomb, who verified Amontons' law of friction in 1781.

To avoid this oversight, it is a good idea to draw a force diagram (see right), identifying all of the forces acting on the object, *before* using equation 3.1.

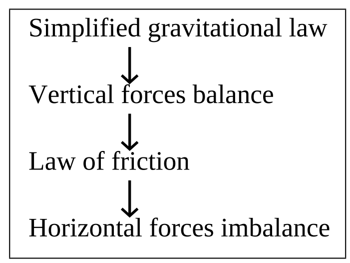


From the force diagram, we can see that not only are we applying a force on the box but so is the floor (via friction). If the net force is 4 N rightward, that only means that our applied force has magnitude 4 N *greater* than the magnitude of friction.

Knowing the horizontal force imbalance, we can find the applied force. However, that requires us to first know the friction force.

The friction force can be obtained from the friction equation ($|\vec{F}_{\rm f,max}| = \mu|\vec{F}_{\rm n}|$). However, that requires us to first know the surface repulsion force.

The surface repulsion force can be obtained from knowing that the *vertical* forces must balance (since the box is not moving up or down). However, that requires us to first know the gravitational force, which can be obtained from the simplified gravity equation (i.e., multiply the mass by the gravitational field strength of 9.8 N/kg to get the magnitude of the gravitational force).



This four-step process (working backwards) is illustrated to the right. From the simplified gravity equation, we find that the gravitational force (due to Earth) is 98 N downward. Since the vertical forces balance, we know the surface repulsion force must be 98 N upward. From the friction equation, the maximum friction force must have a magnitude equal to 30% of 98 N (i.e., 29.4 N). The friction force must oppose our force, but have a magnitude that is 4 N less (see above). Consequently, our force must have a magnitude that is 4 N more, or 33.4 N (rightward).

✍ Not surprisingly, such a problem would be very difficult without using the physics to help guide us as to which relationships to use and the order of those relationships.

✓ *Checkpoint 17.9: (a) In the scenario described above, why did we have to determine the surface repulsion force? (b) What would the box do if our applied force was less than 29.4 N rather than greater?*

Multiple steps are needed in the previous example because the friction force (parallel to the surface) depends on the surface repulsion force (perpendicular to the surface). That means that we needed to apply the force and motion equation *twice*: once to the direction perpendicular to the surface and again to the direction parallel to the surface. We applied it in the perpendicular direction in order to get the surface repulsion force. Then, after using that to figure out the friction, we applied the force and motion equation to the parallel direction.

Note that this multiple step process is only needed when we are given the coefficient of friction rather than the friction force itself. After all, we dealt with friction way back in chapter 2. The difference here is that we are provided with the coefficient of friction instead of the friction force. That means we need to first get the surface repulsion force.

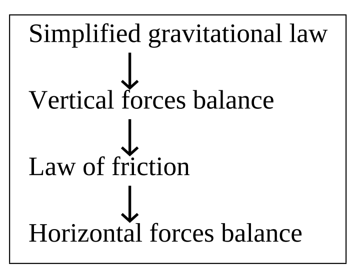
To illustrate, consider the following scenario, which is similar to one we examined on page 30:

> A 25-kg table is on a horizontal floor, where the coefficient of friction between the table and the floor is 0.20. I then push it to the right such that it slides across the floor. Once I get it moving at 1 m/s, how hard do I have to push in order to *keep* it sliding at 1 m/s, without slowing down, speeding up or changing directions?

As before, we know that to *start* the table moving, there has to be a force imbalance (i.e., I have to push harder than friction) but to *keep* it moving, once it is already moving, the forces have to balance (i.e., I have to push with the same magnitude as friction).

The difference in this case is that we aren't provided with the friction force. Instead, we have to figure it out.

The process, then, is the same as with the other problems examined in this chapter, and indicated in the diagram.



From the simplified gravity equation, we find that the gravitational force has a magnitude of 245 N (multiply the mass by 9.8 N/kg).

From the balance of the vertical forces, then, the surface repulsion force must have the same magnitude as the gravitational force (245 N). Knowing the surface repulsion force, then, we can get the maximum magnitude of the friction force to be 49 N (multiply the magnitude of the surface repulsion force by the coefficient of friction).

Finally, from the balance of the horizontal forces, my applied force must be 49 N also, equal in magnitude to the friction force but opposite in direction.

Keep in mind that we didn't solve for the force needed to *start* the object moving at 1 m/s. Instead, we solved for the force needed to *keep* the object moving at 1 m/s, assuming it had already reached that speed.

Also notice that we didn't use the information about the table's speed (1 m/s, as specified in the problem) because none of the forces depended on the speed. The maximum friction force only depends on the surface repulsion force and the gravitational force only depends upon the mass.

✓ *Checkpoint 17.10: To solve the problem examined above, did we need to use the speed of 1 m/s? If so, how? If not, why not?*

Summary

This chapter examined how to apply the laws of physics to situations involving friction, where the friction force depends on the surface repulsion force. Only horizontal surfaces were examined.

The main points of this chapter are as follows:

- The friction force opposes motion parallel to the interface.
- The maximum magnitude of the friction depends upon the materials involved and the magnitude of the surface repulsion force.
- The maximum magnitude of the friction is proportional to the magnitude of the surface repulsion force.
- The coefficient of friction is not a force. It is a dimensionless coefficient representing how large the maximum magnitude of the friction is compared to the magnitude of the surface repulsion force.

- If the object is sliding and there is friction, we'll assume the magnitude of the friction force is equal to its maximum value and independent of the sliding velocity.

Frequently asked questions

If the floor isn't frictionless and an object is sitting at rest on the floor, what is the friction force if you don't push on the object?

See page 284.

Is the coefficient of friction the same as the friction force?

No. It is a dimensionless coefficient that represents how large the friction is compared to the surface repulsion force.

What is the value of μ?

The coefficient of friction, μ, depends on the surfaces. Use table 17.1 (on page 291) to find the value if it isn't already provided in the problem.

Does the magnitude of the friction depend on how much of the object is actually in contact with the surface?

Perhaps, but we will assume it does not.

Where does the friction equation (17.1) come from?

The expression, written as $\mu = |\vec{F}_{\mathrm{f,max}}|/|\vec{F}_{\mathrm{n}}|$, is empirical, meaning that it is based on measurements. We can treat it as representing a law that is always true, even though we haven't demonstrated it as such.

Terminology introduced

Amontons' law of friction	Law of friction
Coefficient of friction	Normal force
Friction force	Surface repulsion force

Additional problems

Problem 17.1: A student is running to class when she suddenly remembers that she forgot her homework. She quickly turns around and runs back to her room. Friction with the ground acts opposite her original motion to slow her down. As the student is momentarily at rest and reversing directions (from going toward class to going back to her room), what is the direction of the friction force on her?

Problem 17.2: A person pushes horizontally on a large box that sits on a horizontal floor. Suppose there is friction between the box and the floor. In order to get the box to move, does the magnitude of the force due to the person have to be greater than, equal to, or less than, the magnitude of the force exerted by friction?

Problem 17.3: A 5.00-kg box is at rest on a level surface and the coefficient of friction between the box and surface is 0.8. What is the greatest horizontal force I can apply on the box such that the box remains at rest?

Problem 17.4: Draw a force diagram showing the forces on a box that is being pulled to the right along a horizontal surface (with friction) by a rope that is attached to the box. Be sure to label each of the arrows in your force diagram, so it is clear which arrow corresponds to which force.

Problem 17.5: Suppose we have a 10-kg box at rest on a horizontal surface where the coefficient of friction is 0.2. For each of the following forces, predict the velocity of the box if the force is applied for 30 seconds:
(a) I exert a 20-N force rightward on the box for 30 seconds.
(b) I exert a 10-N force rightward on the box for 30 seconds.

Problem 17.6: Suppose we had a 2-kg box on a horizontal surface where the coefficient of friction is 0.4 between the box and the floor. An applied force leftward is exerted, speeding up the box. The force is removed at the moment the box is moving 4 m/s leftward. Once the applied force is removed, a timer is started. How far has the box slid from the time the time was started to the moment the timer reads 0.5 seconds?

Problem 17.7: At a particular instant, a 3-kg box sliding along a horizontal surface ($\mu = 0.4$) with a speed of 5 m/s.
(a) What is its speed one second later?
(b) Suppose at a certain instant the speed is 10 m/s. What is its speed one second later?

Problem 17.8: Suppose we have a 10-kg box moving at 15 m/s rightward on a horizontal surface. What force is needed to bring the box to rest in 20 seconds if:
(a) The surface is frictionless?
(b) The surface has friction with a coefficient of friction equal to 0.2?

Problem 17.9: Suppose we have a 10-kg box on a horizontal surface with a coefficient of friction equal to 0.2. A horizontal force of 30 N rightward is applied for 5 s. How fast is it going at the end of the 5 s and how long does it take, after the 5 s, to come to a stop (due to friction)?

Problem 17.10: A 2-kg book is at rest on a desk. The coefficient of friction between the book and the desk is equal to 0.3.
(a) What is the value of the surface repulsion force acting on the book (due to the desk)? Note that the book remains at rest so the net force on the book must be zero.
(b) What is the maximum value that the friction force could have?
(c) What value of force parallel to the desk is necessary to start the book sliding across the desk?
(d) Suppose the book is sliding at 0.5 m/s. What value of force parallel to the desk is necessary to keep the book sliding at 0.5 m/s?

Problem 17.11: A 25-kg table sits at rest on a horizontal floor. When I push on it with a horizontal force of 50 N, it speeds up horizontally at a rate of 0.5 m/s every second.
(a) What is the coefficient of friction between the table and the floor?
(b) If the coefficient of friction between the table and the floor is 0.3, how hard do I have to push horizontally on the table in order to start it sliding?
(c) How hard do I have to push horizontally on the table to keep it sliding at a constant speed once it is sliding?
(d) How hard do I have to push horizontally on a 20-kg table in order to make it speed up at a rate of 0.5 m/s every second?

Problem 17.12: A 10-kg box is at rest on the back of a truck. The coefficient of friction between the box and the truck is 0.2. If the truck speeds up at a rate of 1 m/s^2, will the box slide? Why or why not?

18. Obtaining Component Values

Puzzle #18: We know that changes to the vertical motion are related to the net vertical force, and changes to the horizontal motion are related to the net horizontal force. What do we do if the velocity and/or forces are at an angle?

Introduction

The main point of chapters 16 and 17 was that we treat two-dimensional problems (like projectile motion), which involve both vertical and horizontal motion and/or forces, as two one-dimensional problems: one using only the horizontal components and another using only the vertical components.

For example, in one of the scenarios in chapter 16, an object was thrown with an initial velocity that was 4 m/s upward and, at the same time, 3 m/s rightward. The vertical component, 4 m/s upward, would be used for any questions involving the vertical motion, like how long it takes for the object to fall to the ground. The horizontal component, 3 m/s rightward, would be used for any question involving horizontal motion, like how far the object moves horizontally in a certain amount of time.

However, what do we do when the initial velocity is instead given as a speed and a direction that is oriented "at an angle", which puts it "between" the two perpendicular directions?

The answer is that we must "replace" the misaligned velocity with two perpendicular velocities that, taken together, are mathematically equivalent to the original velocity (e.g., one vertical and one horizontal).

In this chapter, we'll explore what this replacement means and how we go about determining the replacement. Our approach will be the same for all quantities that include a direction (e.g., eastward) as well as a magnitude (e.g., 10 m/s). Such quantities are called vectors (see page 54 and the vectors section of the supplemental readings). Examples of vectors include displacement and force as well as velocity.

✍ Keep in mind that this is purely a mathematical exercise, as the physics is unchanged from what we know before. In other words, once we make the replacement we solve the problem as before (e.g., with the horizontal components done separately from the vertical components).

18.1 The language of giving directions

In chapters 16 and 17, every vector was either directed horizontally, directed vertically or was a combination of two perpendicular vectors, one directed horizontally and one vertically. For example, a ball could be thrown with an initial velocity that is 4 m/s upward and 3 m/s rightward.

This was important because the horizontal forces, if present, would influence the horizontal velocity, and the vertical forces, if present, would influence the vertical velocity.

But what if we are given an initial velocity that is directed at an angle? Or if we are given a force at an angle?

In such a case, we need to find a pair of two perpendicular values that we can use instead of the vector at an angle. That way, we can solve the problem as described in chapters 16 and 17.

Before doing the replacement, though, it is important to know the language used for indicating the *direction* of a vector at an angle. You may already be familiar with this, but I'll review it in this section just to make sure.

Direction is given in terms of an angle. There are 90 degrees between two perpendicular directions, like upward and rightward.[i]

❢ Angles are given relative to some reference direction.

The angle of a particular direction represents how far off the direction is from some reference direction. For example, upward is 90 degrees away from rightward. Downward is also 90 degrees away from rightward but in the other direction.

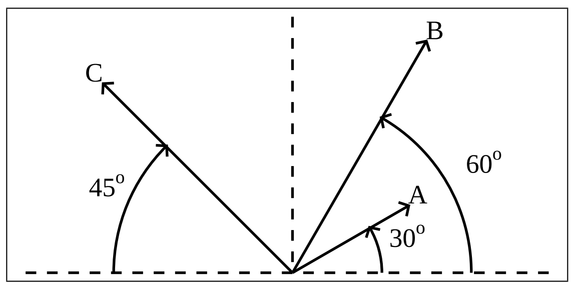
[i] As will be discussed in chapter 21, there are 360 degrees in a full circle. Thus, there are 90 degrees in one quarter of a circle (i.e., a right angle). That means that eastward and northward are 90 degrees away from each other.

Since upward and downward are both 90 degrees away from rightward, it isn't sufficient to just indicate the angle and the reference direction. We must also indicate which way we are measuring the angle. For example, we can say something like "90 degrees to the upward side of rightward" or "90 degrees to the downward side of rightward."

Of course, we wouldn't go to so much trouble to say "90 degrees to the upward side of rightward" when we can just say "upward." However, such specificity is crucial when we have directions that are, say, between upward and rightward. For example, midway between upward and rightward would be "45 degrees to the upward side of rightward."

Even then, it is easier to say "midway between upward and rightward" than "45 degrees to the upward side of rightward."

To see the real benefit of this language, consider the figure, where vector A is 30° to the upward side of rightward, vector B is 60° to the upward side of rightward, and vector C is 45° to the upward side of leftward.



For each vector direction, we indicate not only the angle but also which way the angle is being measured and also the **reference direction** from which the angle is being measured.

For example, the direction for vector A is given in terms of an angle (30°), a reference direction (rightward) and a measuring method (to the upward side). The angle (30°) doesn't mean anything unless the other two pieces of information are also given. After all, 30° to the downward side of leftward is different than 30° to the upward side of rightward.

✍ The language is similar to how you might describe where you are. For example, "ten feet to the left of the tree" is more specific than "ten feet from the tree" or just "ten feet".

Alternatively, the three vectors could represent horizontal directions on a map, with northward and eastward instead of upward and rightward. In that case, the direction of vector A would be described as being 30° to the northward side of eastward. In that case, the angle is 30° and the reference direction is eastward. And, since 30° could be to the northward or southward side of eastward, we also have to specify that it is to the northward side.

Could we also describe vector A as being "60° to the eastward side of northward"?

Yes. There are 90 degrees between eastward and northward. Consequently, 60° to the northward side of eastward is the same as 30° to the eastward side of northward. It doesn't matter as long as it is clear what the direction is.

✍ Some people prefer[ii] having eastward as their reference direction for everything (so that zero degrees indicates the eastward direction) but it is not wrong to use northward as the reference direction.

Could we just say "30° northeast"?

No, because it wouldn't be clear whether this was closer to north or closer to east.

However, since "30° to the northward side of eastward" is a mouthful, it is common to just shorten this to "30° north of eastward" (like "10 feet left of the tree"). In fact, that is the convention I will follow as well from now on.

Example 18.1: Suppose an object is moving in the direction 30° up from rightward. Is the direction closer to straight upward or is it closer to straight horizontal (rightward)?

Answer 18.1: This direction is closer to horizontal (rightward) than upward since Since "30° up from rightward" means the direction is 30° to the upward side of rightward, and 90° from rightward would be straight upward.

✓ *Checkpoint 18.1: (a) Suppose the position is in a direction that is 40° north of west. What is the angle (in degrees) between that direction and northward? (b) Suppose the direction is 20° south of west. What is the angle (in degrees) between that direction and northward?*

[ii] I believe the preference for eastward as the reference direction is due to the convention in mathematics of always using zero degrees as pointing toward the right, with the angle increasing counter-clockwise from that direction. This is by no means the only way to indicate direction, however. Meteorologists, for example, tend to set zero degrees pointing northward, with the degrees increasing *clockwise* from that direction.

18.2 The meaning of equivalency

We now know how to indicate the direction of a vector that is not aligned with one of our component directions. We still need figure out how to replace a non-aligned vector with a pair of perpendicular vectors that together are equivalent to the non-aligned vector. Those two perpendicular vectors are called **components** of the original vector.

Before we can do the replacement, we first have to recognize what it means for those two components, together, to be "equivalent" to the non-aligned vector.

To do this, consider an object that is moving at a constant velocity equal to 2 m/s northward and 3 m/s eastward. That object wouldn't move 2 m northward and *then* 3 m eastward, taking one second to move the entire 5 meters. Instead, the object would be moving *diagonally*, such that after one second it is at a position that is 2 m northward and 3 m eastward of where it started.

If we used a string to measure the straight-line distance from where the object started to where it ended up one second later, we'd find the distance is 3.6 meters and it would be in a direction that is 33.7 degrees north of east.

In other words, the end result of moving 2 m/s northward and 3 m/s eastward is the same as moving 3.6 m/s in a direction 33.7 degrees north of eastward.

How did you get 3.6 m and 33.7 degrees north of eastward?

How I obtained the angle is unimportant but you can see the footnote if you are interested.[iii]

The distance was determined via the **Pythagorean theorem**, which we can write mathematically as follows:

$$A^2 = A_x^2 + A_y^2 \tag{18.1}$$

If you plug in 3 and 2 for A_x and A_y, respectively, you get 3.6 for A.[iv]

[iii] The angle was determined using the inverse tangent function. For information on how the inverse tangent was used, see the supplemental readings.

[iv] It is common to use x and y to indicate two perpendicular directions, usually rightward and upward. I don't know why we use x and y but it is less confusing than r (rightward) and u (upward) or using v (vertical) and h (horizontal).

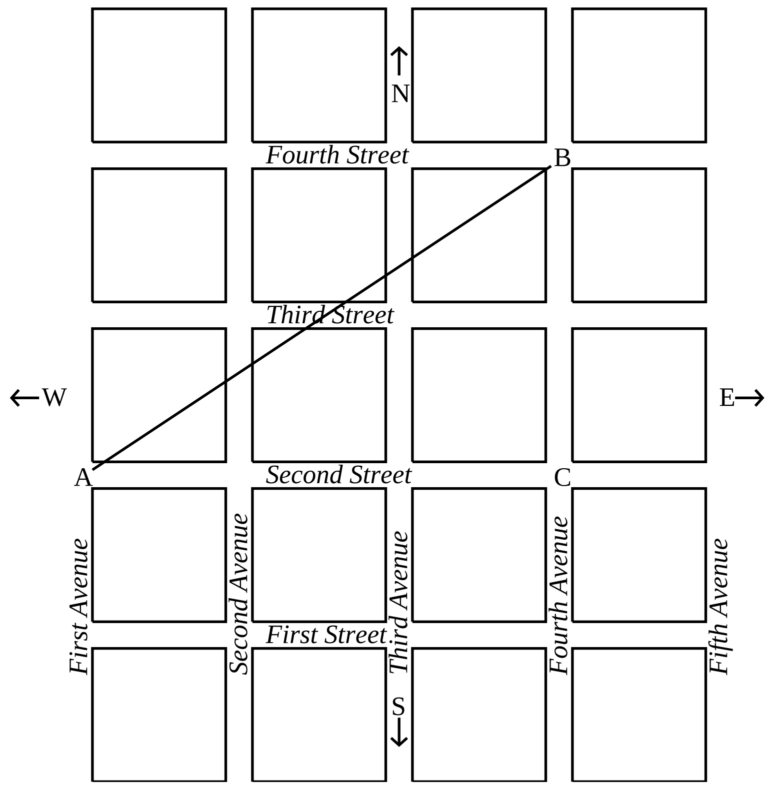


Figure 18.1: A map of streets (going east-west) and avenues (going north-south). Point A is at the intersection of second street and first avenue. Point B is at the intersection of fourth street and fourth avenue. A straight line connects the two points. East is toward the right.

Isn't the Pythagorean theorem used with triangles?

Yes. Basically, we treat the two components as though they are two legs of a right triangle, with the magnitude of the vector equal to the hypotenuse of that triangle.

This might be easier to see with the map shown in Figure 18.1.

If you've ever lived in a city like Manhattan, you're likely already familiar with streets that are laid out in a grid, as illustrated in the figure. In the case of the map in the figure, each street is a block apart, with the east-west streets called "First Street," "Second Street," "Third Street" and so on as you move north and the north-south streets called "First Avenue," "Second Avenue," "Third Avenue" and so on as you move east.

Suppose we want to travel from the corner of Second Street and First Avenue (see point A on the map) to the corner of Fourth Street and Fourth Avenue (see point B on the map). Most people would say to go three blocks eastward and two blocks northward. Or, visa-versa, they might say to go two blocks northward and then three blocks eastward.

In other words, when describing the displacement they would provide two displacements — one in each perpendicular direction. In this case, one displacement is eastward and the other is northward.

However, the *straight-line* distance from point A to point B is *less than* 5 blocks, since that distance is less than the 5 blocks one would travel from point A to point C (3 blocks) and *then* to point B (2 more blocks).

For our purposes, there are two important things you need to recognize. First, both ways are equivalent in terms of the end result. Moving 3 blocks eastward and 2 blocks northward is equivalent to moving 3.6 blocks in a direction 33.7 degrees north of east.

Consequently, if we are ever provided with the latter (3.6 blocks in a direction 33.7 degrees north of east), we can replace that with the former (3 blocks eastward and 2 blocks northward). By doing the replacement, we can then solve the problem as we would in chapters 16 and 17.

❢ Vectors at an angle can be written as the sum of two perpendicular vectors, one in each component direction.

❢ The two perpendicular replacement vectors will necessarily have magnitudes that sum together to give a value larger than the magnitude of the original vector.

Second, the magnitude of the combination is less than the sum of the component values (unless there is only one component to start with). This means that the replacement process won't be as simple as figuring out which two numbers add up to another number.

For example, for the map in Figure 18.1, the lengths of the two perpendicular displacements (i.e., 3 blocks eastward and then 2 blocks northward) together are longer than the diagonal displacement length (3.6 blocks) because the two perpendicular displacements follow a longer path. The "end result" is the same but the eastward displacement moves "to the right" of the direct line indicated by the diagonal line and then the northward displacement has to "bring it back to the left."[v]

If the vector is a force instead of a displacement, it is like the two perpendicular forces are countering each other slightly, one pulling one way and the

[v] If you are interested in the distance traveled then the two components are *not* equivalent to the original vector. However, for our purposes, we are only interested in *where you end up* and, in that sense, they *are* equivalent.

other pulling the other way. As a result, the "amount" of force associated with the two perpendicular forces may be larger but the "net effect" of the two together is the same as the single force vector at an angle.[vi]

✓ *Checkpoint 18.2: When a ball is thrown with an initial velocity of 4 m/s upward and 3 m/s rightward, we find that 0.25 seconds later its velocity is 1.55 m/s upward and 3 m/s rightward. At that moment, how fast is it going?*

18.3 Determining components

In the previous section, the Pythagorean Theorem was introduced as a way of finding the magnitude of a vector from the two component values. While useful to know, it doesn't allow us to solve problems when faced with vectors at an angle. To solve such problems, we need to do the *reverse*. We need to know how to take a vector that is non-aligned with our component directions and identify the two perpendicular vector components that are mathematically equivalent to it.

I'll show you three different ways to do this. The first two ways are qualitative, which means that they give you an estimate of what the two perpendicular vector components are, but won't give you a specific value for them. Only the third way will give you a specific value. However, you need at least one of the first two ways in order to use the third way.

The first way uses the language of section 18.1 to identify which component is larger. The second way uses drawings to do the same thing. Once you've identified which component is larger and which is smaller, you can use the third method, which uses trigonometric functions, to figure out the values.

[vi]It may take more effort to exert two separate perpendicular forces than the single force. So, if you are interested in the effort exerted then the two perpendicular forces are *not* equivalent to the original force. However, for our purposes, we are only interested in *net* force and, in that sense, they *are* equivalent.

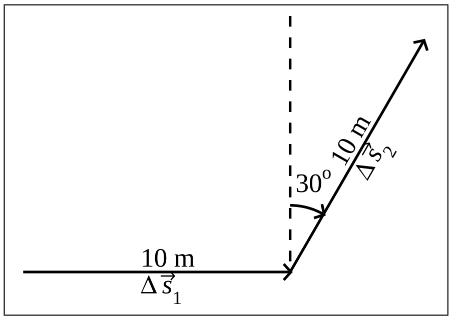


Figure 18.2: Two displacements indicated by arrows. The first displacement (identified with the label "$\Delta\vec{s}_1$") is 10 m eastward. The second displacement (identified with the label "$\Delta\vec{s}_2$") is 10 m in a direction 30 degrees east of north. East is toward the right.

18.3.1 Qualitative analysis

Before doing any math, you need to first recognize that it isn't always necessary to replace a vector with an equivalent set of two perpendicular vectors. If the vector is already aligned with one of the two perpendicular directions then you don't need to do anything – we already know how to deal with vectors that are aligned. We only need to make the replacement when a vector is "between" the two perpendicular directions.

For example, in Figure 18.2, the first displacement is already aligned with the eastward direction. We don't have to replace that vector. It is fine as is.

The second displacement, on the other hand, is between northward and eastward and so we'd need to rewrite it as the sum of two perpendicular vectors: one northward and another eastward. The two perpendicular vectors are the components of the displacement.

The second thing to do is to recognize that, unless the vector direction is exactly "midway" between the two component directions, one component will be larger than the other. Whichever reference direction the vector direction is closer to, the component for that direction will be larger.

For example, the direction of the second displacement in Figure 18.2 is closer to being northward than eastward. As such, we expect the northward component of that displacement to be larger than the eastward component.

❢ The "larger" component will correspond to the direction that is closer to the direction of the original vector.

✍ The second displacement has a direction that is closer to northward than eastward. That means the *angle* between it and northward is smaller than the angle between it and eastward. It also means the northward *component* is larger than the eastward component. Notice how "component" does not mean angle.

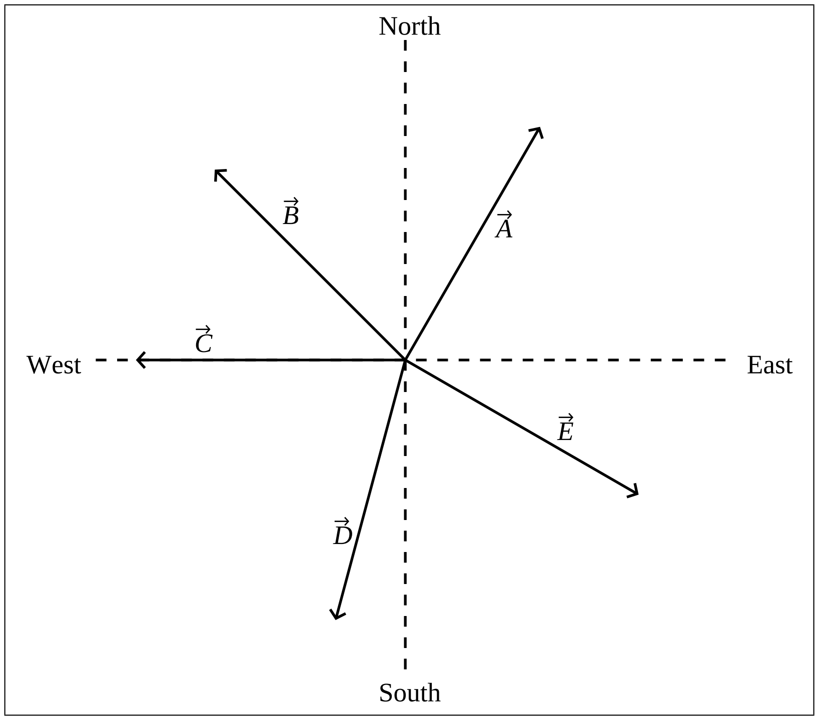


Figure 18.3

Now let's do a couple for practice. I have drawn five vectors in Figure 18.3. The one indicated as $\vec{A}$ is similar to the second displacement in Figure 18.2. As such, it can be written as the sum of a northward vector and an eastward vector, with the northward component being larger (since the direction of $\vec{A}$ is closer to being northward than eastward).

How about vector $\vec{B}$?

That vector has a direction that is between northward and westward. As such, it can be written as the sum of a northward vector and a westward vector.

Which is larger?

Since the direction of $\vec{B}$ is midway between northward and westward, it turns out that the two vectors will have the same magnitude (i.e., the northward component will equal the westward component).

How about vector $\vec{C}$?

That vector is directed due west. As such, it doesn't need to be replaced with two perpendicular vectors. Essentially, there is a westward component but no northward or southward component.

HOW ABOUT VECTOR $\vec{D}$?

That vector has a direction that is between westward and southward. As such, it can be written as the sum of a westward vector and a southward vector. The southward component is larger, since the direction of $\vec{D}$ is closer to being southward than westward.

✍ There are lots of perpendicular vectors that, added together, equal a single vector. We are assuming that a particular set of component directions have already been chosen and so we are restricted to selecting a pair of vectors that align with that set of component directions. The set of component directions we choose depends on the problem (see the inclines section of the supplemental readings for more on this).

✓ *Checkpoint 18.3: Examine vector $\vec{E}$ in Figure 18.3.*
(a) In which two directions (eastward, northward, westward and/or southward) do the perpendicular components of $\vec{E}$ point?
(b) Which component is larger?

18.3.2 Graphical analysis

Another method for estimating which component is larger and which is smaller is to make a drawing of the original vector and the two perpendicular replacement vectors.

For example, in Figure 18.4 I have drawn the vector $\vec{A}$ three times (this is the same vector $\vec{A}$ that was drawn in Figure 18.3). In part (a), I have drawn this vector alone.

In (b), I have added a rectangle (dotted lines) with two sides aligned with the axes and with the vector $\vec{A}$ being equivalent to diagonal of the rectangle. It turns out that when you drawn the rectangle in this way, the vector components, drawn in gray in part (c), are equivalent to the two sides of the rectangle that are aligned with the axes.

The reason this works is because the relationship between the sides and diagonal of a rectangle is mathematically equivalent to the relationship between the components of a vector and the vector itself.

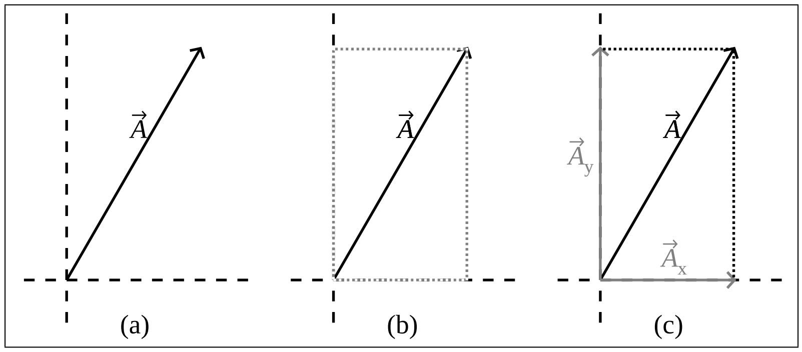


Figure 18.4: In (a) an arrow is drawn representing the vector $\vec{A}$ in Figure 18.3. In (b), a rectangle is drawn (dotted lines), aligned with the axes, with the vector $\vec{A}$ along the diagonal of the rectangle. In (c), the vector components, in gray, represent the two sides of the rectangle that are aligned with the axes.

Notice how two adjacent sides of a rectangle are perpendicular to one another. The vector components are likewise perpendicular to one another.

Notice also how the height of the rectangle in Figure 18.4 is larger than the width of the rectangle. In the same way, the vertical component of the vector, indicated as $\vec{A}_y$ in the figure, has a greater magnitude than the horizontal component of the vector, indicated as $\vec{A}_x$ in the figure.

✍ One can't draw a rectangle with the vector as the diagonal in this way if the vector lies along one of the component directions. However, if the vector already lies along one of the component directions, you don't need to replace it with two perpendicular vectors so you don't need use the graphical method (or any other method). That is why we are only considering vectors that don't already lie along one of the component directions.

✓ *Checkpoint 18.4: (a) For each of the vectors drawn in Figure 18.3, draw the set of perpendicular vectors that align with the component directions indicated (dashed lines), as illustrated in Figure 18.4d. Make sure your pair is aligned with the those component directions <u>and</u> are perpendicular to one another, like a rectangle.*
(b) Is it possible for one of the two perpendicular vectors to have a magnitude larger *than the magnitude of the vector it is meant to replace? Explain.*

18.3.3 Using sine and cosine

Now that we know what it means to have two perpendicular vectors that are mathematically equivalent to a given vector, and can figure out which component is larger and which is smaller, let's look at how we can use **trigonometric functions** to determine the value of each component.

To use those functions properly, you first have to recognize that the value of each component is going to be some fraction of the original vector magnitude. By this I mean that each of the perpendicular vectors will have a magnitude smaller than the original vector magnitude.

So, for example, if the original vector is 10 m/s in a direction 60° north of east, the eastward component will have a magnitude less than 10 m/s. Let's say the eastward component is 5 m/s. That would mean the eastward component is half the magnitude of the original vector (i.e., the fraction is 50% or 0.5).

The northward component will also be some fraction of the 10 m/s magnitude (in the example). It would not also be 50% (0.5), though, since the components will not add up to the original magnitude (see section 18.2).

✍ As seen in section 18.3.2, the perpendicular vectors along with the original vector can be drawn as a triangle. With a triangle, the length of the two sides of a triangle won't add up to the length of the hypotenuse. For example, consider a 3-4-5 triangle (a triangle where one side is length 3, one side is length 4 and the hypotenuse is length 5). The two sides (3+4) do not add up to the hypotenuse (5). The actual relationship is a bit more complicated (as discussed on page 305).

Second, you need to recognize that the fractional value depends upon how far (or close) the vector direction is to each component direction. Whichever component direction the vector direction is closer to, the greater that component. This means that the fractional value depends upon the angle between the vector direction and the component directions.

To find the fractional values, you use the **sine** and **cosine** functions.

✍ These functions will be abbreviated as sin and cos in mathematical equations. They are usually abbreviated the same way on a calculator.

❢ A two-dimensional vector can be "decomposed" into two perpendicular vectors by using the cosine and sine functions.

The sine and cosine functions are found on most calculators. To find the fractional values, you need to know the angle between the vector direction and the component directions.

Get a calculator now and try it out. Type in an angle at random. Then press either the "sin" key or the "cos" key. You'll find that the calculator provides you with a number between −1 and +1.

WHAT DOES IT MEAN?

Let's look at our example of a position that is 10 m (from our origin) in a direction 30 degrees east of north. We'll use the angle of 30 degrees. Using your calculator, you can find that the cosine of 30 degrees is 0.866 (86.6%) and the sine of 30 degrees is 0.5 (50%).[vii]

These two fractions, 0.866 and 0.5, give an indication of the relative size of the two components, in terms of a fraction of the original magnitude.

I GET 0.154 WITH THE COSINE AND −0.988 WITH THE SINE. WHY?

If you don't get 0.866 and 0.5, chances are your calculator is interpreting your "30" as being in a unit other than degrees. As will be discussed in chapter 21, there are several units one can use to indicate an angle. Degrees is just one way. If you are using the angle in degrees, you need to make sure the calculator is interpreting it that way. How you do this varies from calculator to calculator.

ARE THESE TWO NUMBERS THE VALUES OF THE COMPONENTS?

No. They represent the *fraction* of the vector magnitude for each component. Each component can't have an absolute value that is larger than the magnitude of the vector. This is why the sine and cosine will always give a value between −1 and +1 (or equal to −1 or +1).

In order to get the value of the components, we need to *multiply* the fractions by the original magnitude. In this case, the magnitude is 10 m and so I get 8.66 m and 5 m for the two components.

[vii] The 0.866 has been rounded to the thousandth place. This is just to make it convenient for me to write. I'm not following any particular method, like the method of significant digits (see the supplemental readings). In general, one should keep as many digits as possible during intermediate calculations and then round for the final answer, which why I will tend to keep a couple more digits than that suggested by the method of significant digits.

Which component is 8.66 m and which component is 5 m?

In this case, the vector is closer to the northward direction than the eastward direction, so the larger component in this case must be the northward component.

Will the cosine value always correspond to the larger component?

No.

The cosine happens to give the fraction corresponding to the direction *from which the angle is measured*. In this case, the angle (30 degrees) is measured from *north*. That is why the cosine value corresponds to the *northward* component.

Consequently, the fraction given by the cosine will be larger only if the angle is less than 45 degrees.

How do I remember which is the sine and which is the cosine?

There is no need to remember which is which. You just need to figure out which component is bigger (by either considering the direction or making a drawing). Then, calculate both components using the sine and the cosine and compare the values with what you expect to determine which component is which.[viii]

By the way, as a check of your results, you can use the Pythagorean theorem (see equation 18.1 on page 305). If your component values are correct, the Pythagorean theorem should give you a value that matches that of the original vector magnitude.

✓ *Checkpoint 18.5: Pick a number at random and then use your calculator to determine the cosine and sine of that number.*
(a) Do you get numbers that are between $+1$ *and* -1*? Should you?*
(b) When you add up the two values, do you get a number that is equal to one? Should you?

[viii]Some people just use cosine all the time by always using the angle from the component direction you are looking for (e.g., using the angle between *eastward* and the vector direction in order to get the *eastward* component). However, you don't need to do that if you understand what the components represent.

Could we have instead used the angle from east instead of from north?

Yes.

The direction is 60 degrees from east, though. When you take the cosine and sine of 60 degrees, you find the values are reversed. In other words, the cosine of 60 degrees is 0.5 and the sine of 60 degrees is 0.866, the opposite of what they were when 30 degrees was used.

The reason for the reversal is that the sine now corresponds to the northward component, not the cosine.

Remember that you can always obtain the direction and the *relative* sizes of the two component values simply by knowing the direction of the vector. We only need the sine and cosine values to get *numerical* values for the components.

Example 18.2: A force of magnitude 4.0 N is directed 50° north of west. Determine the component values along west and north.

Answer 18.2: Without doing any calculations, we know that the two perpendicular vectors are directed westward and northward. Furthermore, since the force is directed closer to northward than westward (remember that 45° would be half-way between the two component directions), we expect that the northward component will be larger than the westward component.

The cosine and sine of 50° are 0.643 and 0.766, respectively. Multiplying by the magnitude of the force vector (4.0 N), we get 2.57 N and 3.06 N, respectively.

Since we know that northward component must be larger, the northward component must be 3.06 N, with the westward component being 2.57 N.

✓ *Checkpoint 18.6: An object is moving with a velocity of 5.0 m/s in a direction 20° north of east. Determine the eastward and northward components.*

Summary

This chapter examined how we use trigonometry to replace a vectors at an angle to the component directions with a pair of perpendicular vectors that are aligned with the component directions. This is done via the use of the sine and cosine functions. The chapter also looked at how we incorporate that trigonometry within the solution of a physics problem.

The main points of this chapter are as follows:

- Angles are given relative to some reference direction.
- Vectors at an angle can be written as the sum of two perpendicular vectors, one in each component direction.
- The "larger" component will correspond to the direction that is closer to the direction of the original vector.
- The two perpendicular replacement vectors will necessarily have magnitudes that sum together to give a value larger than the magnitude of the original vector.
- There are an infinite number of vector pairs mathematically equivalent to a vector but only one that is aligned with the component directions.
- A two-dimensional vector can be "decomposed" into two perpendicular vectors by using the cosine and sine functions.

Terminology introduced

Cosine
Pythagorean theorem
Reference direction
Sine
Trigonometric functions

Frequently asked questions

HOW DO WE DETERMINE THE MAGNITUDE FROM THE COMPONENTS?

Use the Pythagorean theorem. See page 305.

How do we know which component is based on the sine and which is cosine?

My recommendation is to first figure out which component is bigger (based on the direction or from a drawing), without doing any math. Then, calculate both components using the sine and the cosine and compare the values with what you expect. You can then look at the drawing and see which component is which.

You'll find that the correspondence depends on how the angle is measured.[ix] So, rather than memorize which is which, it is easier to just use your understanding of what the components represent.

Is "30 degrees east of north" the same as "60 degrees north of east"?

Yes. See page 304.

How do we identify the components of a vector graphically?

See page 311.

Shouldn't the magnitudes of the two perpendicular vectors add up to the magnitude of the original vector?

No. In general, the magnitudes of the two perpendicular vectors will add up to be *more* than the magnitude of the original vector. In a similar way, the sine and cosine functions (which are each between $+1$ and -1) do not add up to one, and the two sides of a right triangle do not add up to the hypotenuse of the triangle!

If a force vector has a magnitude of 10 N and one component is 5 N, why isn't the other component 5 N?

See section 18.3.2.

Additional problems

Problem 18.1: A force is directed 25 degrees west of south. Which component is larger: the westward component or the southward component?

[ix] When the angle is measured from horizontal the cosine gives the horizontal component. When the angle is measured from vertical the cosine gives the vertical component.

Problem 18.2: A ball is thrown at an angle such that its initial velocity has a vertical component equal to 4 m/s upward and a horizontal component equal to 2 m/s rightward. What is the magnitude of the ball's velocity?

Problem 18.3: An object undergoes a displacement of 100 m in a direction of 18 degrees west of north (i.e., 108 degrees counter-clockwise from east as seen from someone looking down).
(a) Draw a picture of this displacement, indicating the northward and westward directions.
(b) Determine the values of the two perpendicular components: one northward and one westward.

19. Applications in Two Dimensions

Puzzle #19: Assuming no drag, at what initial angle should a ball be thrown in order to travel the farthest horizontally (for the same initial speed)?

Introduction

Chapter 18 showed how we can replace a vector by a set of two perpendicular vectors that, taken together, are equivalent to the original vector.

The reason for doing this replacement is because of how we solve two-dimensional problems, like those that involve both vertical and horizontal motion and/or forces. Such a problem would be split into two: one using only the horizontal components and another using only the vertical components. Any vector not aligned with the horizontal or vertical directions (i.e., at an angle) would have to be *replaced* by a set of two perpendicular components, one vertical and one horizontal, before we could solve the problem.

This process is illustrated in this chapter by examining situations we've already examined but, this time, we'll consider examples that involve vectors at an angle.

19.1 Projectile motion

Consider the following scenario:

> A ball is thrown with an initial speed of 10 m/s at an angle of 37 degrees above the right. If the ball is released 1 meter above the ground, how far above the ground is it 1 second later?

We've already discussed projectile motion in chapter 16 and the scenario provides a projectile motion situation similar to those provided before. The only difference is that the initial velocity is not aligned with the horizontal and vertical directions.

So what is different about how we solve it?

❢ Before doing any calculations, first do the necessary trigonometry.

The difference is that, before applying the physics (like the law of force and motion) we must first do some trigonometry since our initial velocity is at an angle. Below I highlight the "trigonometry part" of the problem by indenting the relevant paragraphs, with a line along the left side of the text. I'm doing that to emphasize the part of the solution that is math, not physics.

The initial velocity is directed between upward and rightward. So, we need to replace that with two perpendicular velocities: one directed upward and one directed rightward. Furthermore, since the initial velocity is more rightward than upward, we expect the rightward component to be larger than the upward component.

> Taking the sine and cosine of 37°, I get 0.602 and 0.799. The larger one (the cosine in this case) must correspond to the rightward component (in this case) since the rightward component is larger than the upward component (in this case).
>
> Multiplying by the magnitude of the initial velocity, I get 6.0 m/s (upward) and 8.0 m/s (rightward).

From this point on, the process is the same as what was described in chapter 16. I'll repeat that process here, just to refresh your memory.

In this case, the scenario asks for the *vertical* displacement, which means I carry out the physics using only the *vertical* components (like the vertical component of the initial velocity and the vertical component of the force per mass).

Since the object is in free fall, the *vertical* acceleration is 9.8 m/s^2 downward. That means the ball's *vertical* velocity has changed from 6.0 m/s upward (its initial vertical velocity) to 3.8 m/s downward one second later (a difference of 9.8 m/s downward).

The average of 6.0 m/s upward and 3.8 m/s downward is 1.1 m/s upward.[i]

[i] You can get the average by adding +6 and −3.8 and then dividing by two, or you can take half of the difference (4.9) and subtract that from 6.

Multiply the average *vertical* velocity (1.1 m/s upward) by the time (1 s) to get the *vertical* displacement (1.1 m upward).

Since the ball started 1 m above the ground, this means that 1 second later the ball is 2.1 m above the ground (add the *vertical* displacement to the initial *vertical* position).

DID WE NEED TO USE THE RIGHTWARD COMPONENT OF THE INITIAL VELOCITY?

Not to determine the *vertical* displacement. If, instead, we were asked for the *horizontal* displacement, we would use the *horizontal* component of the initial velocity (8.0 m/s in this case). During 1 second of flight, the object would move 8 m horizontally.

It is interesting to note that the higher the angle of the initial velocity, the greater the initial upward velocity and the smaller the initial rightward velocity. The larger initial upward velocity means more time in the air. However, the smaller rightward velocity (which doesn't change during the flight) means the ball covers less ground.

IS THERE A PARTICULAR ANGLE THE BALL CAN BE THROWN SUCH THAT THE BALL TRAVELS THE FARTHEST WITHOUT HITTING THE GROUND?

To cover the greatest distance, you need a large horizontal velocity *and* a large time in the air. The balance between these two occurs when the initial velocity is directed at an angle of 45 degrees above the horizontal, half way between vertical and horizontal.

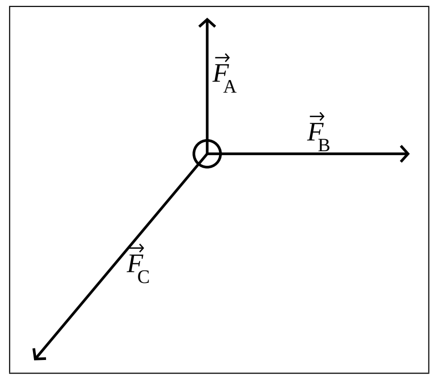
✍ This assumes no drag. You might want to think about how the "best" angle is different when drag is present.

✓ *Checkpoint 19.1: In the scenario described above, we needed to find how far above the ground the ball was one second after being released. Why did I use the vertical component of the initial velocity (6.0 m/s) and not the horizontal component of the initial velocity (8.0 m/s)?*

19.2 Net force

Projectile motion is not the only example of a situation where we had to consider the direction of vectors. Consider, for example, the process of identifying the net force on an object. Forces in opposite directions add differently than forces in the same direction.

With forces at an angle, we must do some trigonometry prior to determining the net force. For example, suppose we have an object at rest on a horizontal surface with the following three forces are acting on it (see force diagram; east is to the right):
(A) 2.0 N northward.
(B) 3.0 N eastward.
(C) 4.0 N 50° south of west.



❢ Think about the physics and do the trigonometry prior to carrying out calculations.

Does the object remain at rest?

The physics of the problem is contained in the law of force and motion, which states that the object remains at rest if the forces are balanced, which means we have to add up the forces. However, in order to order to add up the forces, we must first do some trigonometry on the force that is at an angle.

Force C is directed between south and west. So, we will replace that one with a pair of perpendicular forces: one directed southward and one directed westward. Furthermore, since the force is more south than west (45 degrees would be midway), we expect the southward component to be larger than the westward component.

> Taking the sine and cosine of 50°, I get 0.766 and 0.643. The larger one (the sine in this case) must correspond to the southward component (in this case) since the southward component is larger than the westward component (in this case).
>
> Multiplying by the magnitude of force C (4.0 N), I get that the original force C is equivalent to 3.1 N southward and 2.6 N westward.

We can now add up the forces. Along the east-west direction, the net force is 0.4 N eastward (given 3.0 N eastward from force B and 2.6 N westward from force C). Along the north-south direction, the net force is 1.1 N southward (given 2.0 N northward from force A and 3.1 N southward from force C).

Since the net force is non-zero, that means the forces are not balanced and the object does not stay at rest.

✓ *Checkpoint 19.2: Suppose force C had a magnitude of 5.0 N instead of 4.0 N. Would the forces then be balanced, since 2.0 N (force A) and 3.0 N (force B) add up to 5.0 N (force C)? If not, why not?*

19.3 Net displacement

The process of adding two-dimensional forces is the same process used to add any two-dimensional vector. Namely, we must use trigonometry whenever one (or more) of the vectors is not aligned with the two perpendicular directions.

For example, suppose we have two displacements:
(1) 5.0 m directed 20° south of west
(2) 2.0 m directed northward
What is the total displacement?

We already know that we need to add the north-south displacements separately from the east-west displacements. Since the first displacement is at an angle, that means we must first replace the first displacement with a pair of two perpendicular displacements.

The first displacement is directed between westward and southward. So, we expect two components: westward and southward. Furthermore, since the displacement is closer to westward than southward, we expect the westward component to be larger than the southward component.

> Taking the sine and cosine of 20°, I get 0.342 and 0.940. The larger one (the cosine in this case) must correspond to the westward component (in this case) since the westward component is larger than the southward component (in this case).
>
> Multiplying by the length of that displacement, I get 1.7 m southward and 4.7 m westward as the two perpendicular displacements that are equivalent to the original displacement.

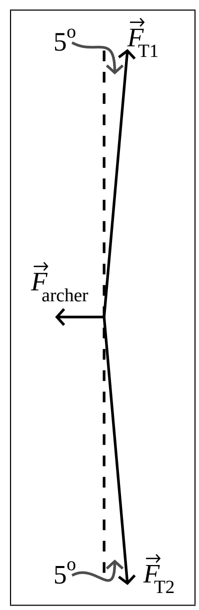
I can now carry out the requested task, which is to add the displacements. The physics tells us that we should do each component direction separately.

For the east-west direction, I have a total of 4.7 m westward (only the first displacement, as the second displacement is northward). For the north-south direction, I have a total of 0.3 m northward (the first displacement provides 1.7 m southward and the second displacement provides 2.0 m northward).

✓ *Checkpoint 19.3: What is it about the scenario above that tells us we must first do some trigonometry prior to adding the displacements?*

19.4 Law of force and motion

The force and motion equation involves vectors and, as such, must be applied twice when dealing with two component directions. For each component direction, it is crucial that you make sure you are taking into account *all* of the forces that are acting in that direction and not ignoring forces that are at an angle. It is for this reason that I recommend that you do the physics and trigonometry *before* actually solving the problem.



Consider, for example, an archer using a bow to shoot an arrow. Let's suppose the archer pulls the arrow against the bow string such that the string makes an angle of 5 degrees with respect to the vertical. The forces on the arrow are illustrated in the figure. The archer, not shown, is pulling the arrow leftward while the string pushes the arrow rightward, toward the target (not shown).

What is the tension in the string if the archer must exert a force of 100 N to keep the arrow stationary? Assume the mass of the arrow is negligible.

As always, start with the physics. The physics says that for the arrow to remain stationary, the net force exerted upon it must be zero. That means a portion of the force due to the bow string (indicated as $\vec{F}_{\text{T}}$) needs to pull rightward (pushing the arrow forward) in order to balance the force due to the archer pulling leftward (indicated as $\vec{F}_{\text{archer}}$).

Next, do the trigonometry on the forces that are at an angle (the force due to the string). Doing the trigonometry focuses our attention on that portion of the force due to the string that balances $\vec{F}_{\text{archer}}$.

There are two forces acting due to the string. Both have the same magnitude (same string) but one is directed upward and rightward, while the other is directed downward and rightward. In both cases, the force is more vertical than horizontal, so we expect the vertical component to be larger than the horizontal component.

> Taking the sine and cosine of 5°, I get 0.087 and 0.996. The larger one (the cosine in this case) must correspond to the vertical component (in this case) since the vertical component is larger than the horizontal component (in this case).

At this point, we don't know the magnitude of the forces due to the string (i.e., the tension). After all, that is what we are trying to solve for. It will be sufficient for the time being just to recognize that the horizontal part is 0.087 of whatever the magnitude is.

We can now carry out the physics.

Horizontally, the forces have to balance in this case. What horizontal forces do we have?

There is the applied force due to the archer, which is 100 N leftward. This has to be balanced by the force due to the string, so the force due to the string is 100 N rightward. There are two forces due to the string, so each one must be 50 N rightward.

However, only a portion of that force is horizontal. In fact, only 0.087 of the magnitude is horizontal, so the magnitude itself must be 575 N (divide 50 N by 0.087).

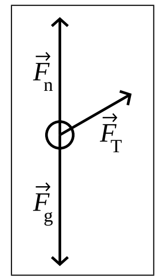
✓ *Checkpoint 19.4: Why is the horizontal portion of the force due to the string only 50 N if the applied force (due to the archer) is 100 N?*

19.5 Frictionless surfaces

When dealing with surfaces, there is a surface repulsion force that pushes the object in a direction that is away from and perpendicular to the surface.

The surface repulsion force acts to balance out all of the other forces acting perpendicular to the surface, keeping the object on the surface (rather than sinking into the surface).

To determine the surface repulsion force, then, we restrict our focus to only the vertical forces, as illustrated by the following scenario: A 2-kg box is sitting on a level, frictionless floor. I then pull on the box (via a string attached to one corner) with a force equal to 10 N in a direction 30° upward from the rightward direction (see force diagram to the right). What is the surface repulsion force on the box?



The physics of the problem is contained in the law of force and motion, which states that since the box does not move into the floor or rise above it, the vertical forces must balance.

However, in order to check if the forces are balanced, we must first do some trigonometry on the force that is at an angle.

✍ Be careful. It is tempting to ignore that force at an angle. However, it exerts a vertical force also. As such, we cannot ignore it when applying the second law in the vertical direction.

My applied force is directed between upward and rightward. So, we can replace that with a pair of perpendicular forces: one upward and one rightward. Furthermore, since the applied force is more rightward than upward, we expect the rightward component to be larger than the upward component.

Taking the sine and cosine of 30°, I get 0.5 and 0.866. The larger one (the cosine in this case) must correspond to the rightward component (in this case) since the rightward component is larger than the upward component (in this case).

Multiplying by the magnitude of my applied force (10 N), I get that my applied force is equivalent to 5 N upward and 8.66 N rightward.

Now we can carry out the physics.

Vertically, the forces have to balance. We have my applied force, which provides an upward force of 5 N, and the gravitational force (due to Earth), which is 19.6 N downward (multiply the mass of the box by 9.8 N/kg downward). Both of those together are 14.6 N downward. For the forces to balance, the surface repulsion force needs to be 14.6 N upward.

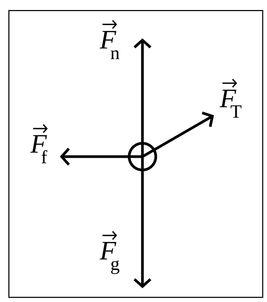
✍ If we were instead asked for the *net* force, we could have ignored all of the vertical forces (including the upward component of the force due to me) since we know the net force has to be horizontal. In this case, it would simply be 8.66 N rightward, since that is the only horizontal force acting on the box in this case. That means the box will start to move rightward.

✓ *Checkpoint 19.5: In the scenario described above, why isn't the surface repulsion force equal in magnitude to the gravitational force?*

19.6 Friction

As mentioned in chapter 17, the friction force depends on the surface repulsion force. For a horizontal surface, that means the horizontal information is not completely independent from the vertical information. Regardless, any forces at an angle must first be replaced with two perpendicular components prior to solving the problem.

For example, suppose I pull on a box (via a string) in order to make it slide along a level floor at a constant velocity of 1 m/s rightward. The string is attached to one corner and I pull on it with a force of 10 N in a direction 30° upward from the rightward direction (see force diagram to the right).



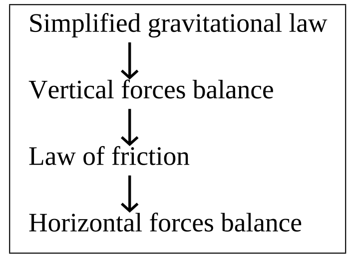
If the coefficient of friction is 0.2, what is the mass of the box?

What does the mass have to do with anything?

This is actually a good question to ask since you must first think of the physics before doing anything else.

The mass is important because the more massive the box, the harder it will be for the box to slide. More specifically, the more massive the box, the greater the gravitational force (from the law of gravity). And, the greater the gravitational force, the greater the surface repulsion force has to be (from the law of force and motion). The greater the surface repulsion force, the greater the friction (from the law of friction). Finally, the greater the friction, the harder it will be for the box to slide (from the law of force and motion).

This tells us that the process is just the opposite of what we did in chapter 17. Indeed, it is illustrated by the diagram to the right, which is the same diagram I used in chapter 17 except that that arrows are reversed. The only other difference is that we first need to use some trigonometry on the force applied at an angle.



The force due to the string is directed between upward and rightward. So, we'll need to replace that force with a pair of perpendicular forces: one upward and one rightward. Furthermore, since that force is more rightward than upward, we expect the rightward component to be larger than the upward component.

> Taking the sine and cosine of 30°, I get 0.5 and 0.866. The larger one (the cosine in this case) must correspond to the rightward component (in this case) since the rightward component is larger than the upward component (in this case).
>
> Multiplying by the magnitude of my applied force (10 N), I get 5 N upward and 8.66 N rightward.

From this point on, the process is the same as what was described in chapter 17.

Since the box moves at a constant velocity, the horizontal forces must balance. That means the *horizontal* component of my applied force (8.66 N rightward) must be balanced by friction (as there are no other horizontal forces acting). That means that the friction force must be 8.66 N leftward.

Since the magnitude of the friction force is 20% of the magnitude of the surface repulsion force (as indicated by a coefficient of friction of 0.2), that means the surface repulsion force must have a magnitude of 43.3 N (divide 8.66 N by 0.2).

Since the vertical forces also balance (i.e., the box stays on the horizontal surface). This means that the gravitational force must be 48.3 N downward (i.e., it must balance the surface repulsion force, 43.3 N upward, and the vertical component of the force due to the string, 5 N upward).

> ✍ Do *not* forget that the force due to the string also has a vertical component!

Finally, knowing the gravitational force, we can solve for the mass (since g is 9.8 N/kg). That gives a mass of 4.93 kg (divide 48.3 N by 9.8 N/kg).

Did we need to know the velocity of the box?

No. All we needed to know was that the velocity wasn't changing. From that we know that the forces balance.

✓ *Checkpoint 19.6: In order to get the value of gravitational force acting on the box, we needed the vertical components of the other forces acting on the object. For each, indicate its value and describe how that value was obtained.*
(a) $\vec{F}_{\mathrm{n,y}}$
(b) $\vec{F}_{\mathrm{f,y}}$
(c) $\vec{F}_{\mathrm{T,y}}$

It is interesting how small changes to a problem can make the difference between making something easy to solve vs. hard to solve.

For example, just switching the unknown to the magnitude of the applied force can make the problem relatively hard to solve. Consider the following:

> A 10-kg box is on a level floor. A string is attached to one corner and I pull on it in a direction 30° upward from the rightward direction. If the coefficient of friction is 0.2, how hard do I have to pull in order to keep the box sliding toward the right at a constant speed of 1 m/s?

Let's think about the physics to see why this is more difficult.

I have to apply a force in order to counter the friction that is acting to slow down the object. So, the magnitude of the applied force depends upon the friction.

The friction, in turn, depends on the surface repulsion force.

The surface repulsion force, in turn, depends on both the gravitational force <u>and</u> the applied force (the more force applied, the less the surface repulsion force has to do).

In other words, my applied force depends on the friction force, which depends on the surface repulsion force, which depends on my applied force. It is circular. How can I determine my applied force if, eventually, it depends on itself?

In cases like this, you cannot solve the problem sequentially. You must identify all of the relationships and combine them.

This doesn't make it *harder* to solve, I suppose, but it requires more algebra to do so, as you must assign quantity abbreviations to quantities you don't yet have values for.

Since this ends up being more a matter of mathematics than physics, I'll pass on solving it here. I only present it to illustrate how the physics can tell us about the solution.

✓ *Checkpoint 19.7: Both of the scenarios described above (i.e., solving for the mass given the applied force vs. solving for the applied force given the mass) require the use of the same relationships, so why does one require more algebra to solve than the other?*

Summary

This chapter looked at how we incorporate trigonometry within the solution of a physics problem.

The main points of this chapter are as follows:

- Before doing any calculations, first do the necessary trigonometry.
- Think about the physics and do the trigonometry prior to carrying out calculations.

By now you should be able to identify the physics involved in two-dimensional problems and use that physics to solve the problems, using trigonometry to deal with vectors that are not aligned with the component directions.

Additional problems

Problem 19.1: Suppose $\vec{v}_1 = 20$ m/s northward and $\vec{v}_2 = 20$ m/s in a direction that is 30° east of north (i.e., 60° north of east).

(a) What is the magnitude of $\vec{v}_1 + \vec{v}_2$?
(b) What is the magnitude of $\vec{v}_1 - \vec{v}_2$?

Problem 19.2: Suppose an object undergoes two displacements: A displacement of 20 m in a direction 45 degrees north of east and a displacement of 30 m in a direction due east. What is the magnitude of the object's total displacement?

Problem 19.3: Suppose the following two forces are acting on an object: A force of magnitude 20 N acting in a direction 45 degrees north of east and a force of magnitude 30 N acting in a direction due east. What is the magnitude of the net force?

Problem 19.4: Suppose the following three forces are exerted on a 3-kg object:
$\vec{F}_1 = 120$ N at 60°
$\vec{F}_2 = 200$ N at 186.83°
$\vec{F}_3 = 160$ N at 330°
What are the components along 0 and 90 degrees for each of these force vectors?

Problem 19.5: Suppose we have two forces: (1) 4.0 N directed 50° north of west and (2) 5.0 N directed eastward. What is the net force?

Problem 19.6: Suppose we have the following three forces:
Force 1 is 3 N northward.
Force 2 is 4 N eastward.
Force 3 is 5 N 20° south of west.
Determine the net force.

Problem 19.7: Determine the net force acting on an 8-kg box that is being pushed on by three people. One person holds it from below and exerts an upward force on the box equal to 100 N, another person exerts a force of 40 N to the right, and a third person exerts a force of 40 N at an angle of 30 degrees downward from the leftward direction.

Problem 19.8: A 10-kg box is sitting on a level, frictionless floor. A string is attached to one corner and I pull on it with a force of 10 N in a direction 60° above the horizontal. What is the net force acting on the box?

Problem 19.9: A 25-kg table sits at rest on the floor. When I push on it with an applied force of 30 N in a direction 60° below the horizontal, it accelerates horizontally with a magnitude of 0.1 m/s^2.
(a) What is the magnitude of the net force exerted on the table?

(b) What is the magnitude of the surface repulsion force exerted on the table due to the floor?
(c) What is the magnitude of the frictional force exerted on the table?

Problem 19.10: A bird is accelerating northward at a constant rate of 2 m/s^2. If its initial velocity is 10 m/s in the northeast direction (i.e., 45° north of east), what is the northward and eastward components of the bird's velocity 4 s later?

Problem 19.11: A ball is thrown with a speed of 25 m/s at an angle of 20 degrees above the horizontal. The ball is released 4 meters above the ground.
(a) Is the ball still above the ground 2 seconds after being released? If so, how far above the ground is it?
(b) How far did the ball move in the horizontal direction during the 2 seconds?

Problem 19.12: I throw a rock up into the air with an initial velocity of 5 m/s upward in a direction 30 degrees above the horizontal. What is the ball's speed 1.5 seconds later?

Problem 19.13: A 2-kg object has the following three forces exerted on it:
$F_1 = 120$ N eastward
$F_2 = 200$ N at 36.87 degrees west of north
$F_3 = 160$ N at southward
What is the object's change in velocity during a 3-second period?

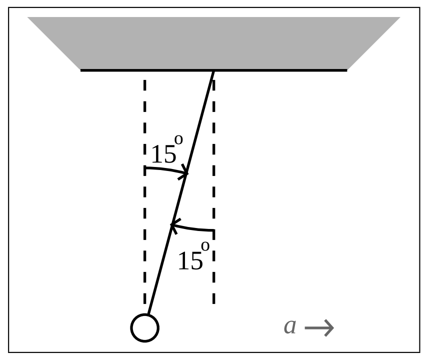
Problem 19.14: A 10-kg box is sitting on a level floor with a coefficient of friction equal to 0.25. A string is attached to one corner and I pull on it in a direction 30° above the horizontal. The string will break if the tension goes above 30 N. Can I get the box to slide without breaking the string? First draw a free body diagram. Hint: first assume the tension is 30 N and figure out what the surface repulsion force is. From that, you can calculate the maximum friction force (from $\mu = 0.25$). Is the maximum friction force enough to keep it from sliding even when the tension is 30 N?

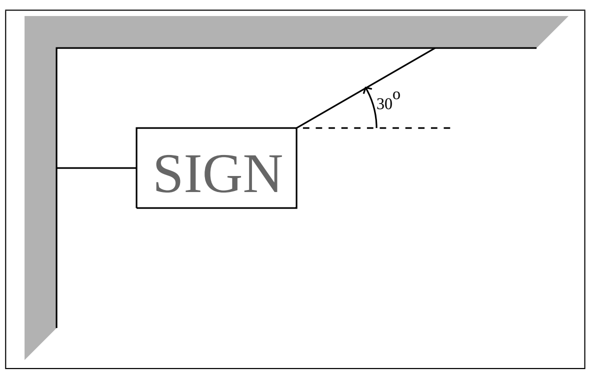
Problem 19.15: A 2-kg box is on a horizontal, frictionless surface with an initial horizontal velocity of 2 m/s leftward. A constant force of 12 N rightward is applied to the box for 0.1 seconds.
(a) What is the displacement of the box during the 0.1 seconds?
(b) Suppose the mass of the box was 1-kg instead of 2 kg. What would be the displacement of the box during the 0.1 seconds?

(c) Suppose the force on the 2-kg box was 20 N instead of 12 N (still rightward). What would be the displacement of the box during the 0.1 seconds?

Problem 19.16: A 5-kg box is on a level surface and the coefficient of friction between the box and surface is 0.5. Suppose an applied force is exerted on the box in a direction 60°below the horizontal. What is the greatest magnitude of the applied force such that the box remains at rest?

Problem 19.17: A 0.1-kg ball is hung from a string from inside a car, as illustrated in the diagram at right. When the car accelerates forward (which is rightward in the diagram), the ball swings toward the back of the car (leftward in the diagram). Suppose the ball is held as shown, where the string makes an angle of 15 degrees with respect to the vertical. With what horizontal acceleration will the ball remain at this angle when released?





Problem 19.18: A 10-kg sign is held stationary when it is hanging from two strings, at the angles indicated in figure above. The tension in the string pulling at an angle of 30 degrees above the horizontal is 196 N. What is the tension in the other string? Keep in mind that the sign is stationary and that there is a gravitational force pulling down on the string.

Problem 19.19: A small block is placed on a frictionless, horizontal surface inside a moving vehicle. When the vehicle accelerates forward, the block slides to the back of the vehicle. It is found that the block does not slide if the surface is inclined at an angle of 15°. What is the vehicle's acceleration?

Part E

Circular Motion and Rotation

20. Circular Motion

Puzzle #20: Why is it difficult to drive around a curve when it is icy?

Introduction

In this part of the text, we examine objects that are spinning or moving in a circle. We'll start with objects moving in a circle (this chapter).

Why study circular motion?

Circular motion turns out to be quite common. In fact, right now, as you read this book, you are moving in a circle. The Earth, after all, is spinning on its axis. Consequently, an object at rest with respect to Earth's surface (like you) is not really at rest. It is revolving around Earth's axis along with Earth's surface. If we could trace your path as the Earth turns, we would find it traces out a big circle. It wasn't your imagination - you really are going in circles!

Even in cases where the object doesn't travel in an exact circle, the motion might be close enough to a circle that making predictions about circular motion can be extended to these other motions. For example, the moon also travels in a circle (roughly) as it orbits Earth. Likewise, Earth travels in a circle (roughly) as it orbits the sun.

We'll start our investigation by examining ways of describing circular motion. We'll then apply the law of force and motion in order to predict what is necessary to keep an object moving in a circle.

20.1 Notation

If an object is moving in a circle that means the object remains a certain fixed distance from the center of the circle. That distance is called the **radius** of the circle.[i]

Since the object follows the circular path, we can describe its motion in terms of its distance around the circle. A distance equal to the **circumference** of the circle means that the object has moved completely around the circle and back to its original position.

✍ This is different than what we considered for straight-line motion. For straight-line motion, an object that returns to its starting position undergoes a displacement of zero since any positive movement must be balanced by an equal amount of negative movement (see, for example, the discussion on page 143). For circular motion, on the other hand, one completion around the circle corresponds to a non-zero displacement equal to the circumference of the circle, since the movement was in one direction the entire time.

Basically, we simply indicate how far "around" the circle the object moves, either **clockwise** or **counter-clockwise**. Clockwise refers to the direction that the hour and minute hands spin on a clock. Counter-clockwise is the opposite direction.

✍ As originally mentioned in section 10.3, we can use positive and negative to indicate opposite directions. Consequently, we can use positive and negative to indicate the direction around the circle. For example, positive velocities can correspond to counter-clockwise movement around the circle, with negative velocities meaning clockwise motion.[ii]

The length of a circle's circumference depends on its diameter. The larger the diameter, the larger the circumference.

It turns out that the circumference is equal to a little more than three times the **diameter**. The actual proportion is not a integer or simple fraction so it is indicated as the letter π, where π stands for the number of diameters that make up the circumference.

✍ The actual value of π can be rounded to 3.14159269. It has no units, since it is just a number.

[i] A line representing the radius of a circle connects the center of the circle with a point on the circle itself, much like the spokes of a wheel. The Latin word for *spoke* (of a wheel) is radius.

[ii] One can use the reverse convention as long as it is clear which one you are using.

The circle's radius is half its diameter. Consequently, the circumference is equal to 2π times the radius.

❢ The circumference of a circle is equal to 2π times the radius.

In general, I'll use r to indicate distances. In this case, r indicates the distance from the center of the circle to where the object is, which happens to be the radius of the circle (in this case). Consequently, the circumference of the circle is equal to $2\pi r$, and the object will travel that distance when it completes one revolution around the circle.

✍ Later on, we may need to know the radius of the object. To distinguish that value from the distance r described above, I'll an upper-case R for the radius of the object. Distinguishing between the two will become important later on (see page 391).

To indicate how far the object has traveled around the circle, I'll use Δs_{circ}.

Why not use $\Delta\vec{s}$?

$\vec{s}$ is a vector, which means that $\Delta\vec{s}$ is technically zero when an object moves completely around the circle and ends up back where it started. For circular motion, we don't want that. Instead, we want the distance to continue to increase as the object moves around the circle.

Notice also that I've removed the arrow on top of the s. We can still use positive and negative values to indicate the direction around the circle (e.g., positive for the counter-clockwise direction and negative for the clockwise direction), as we've done before, but I removed the arrow on top of the s because s_{circ} is not a vector in the same sense that $\vec{s}$ is.

✍ As long as it is clear that the motion is circular, you don't really need to include the "circ" subscript. I'm doing so just to make it clear to you when we are discussing circular motion.

Since circular motion is a type of periodic motion, like the oscillations studied in chapter 11, we can use terminology similar to what we used for oscillations. One complete **revolution** around the circle is similar to one complete **cycle**, since the object returns to its initial position when it completes one revolution around the circle. Consequently, we can say that the **period** of the motion is the time it takes for the object to complete one full revolution around the circle.

Example 20.1: Suppose the person in Figure 20.1 is 1.5 m from the center

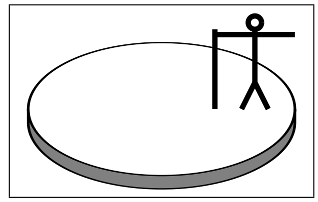


Figure 20.1: A diagram of a large rotating platform. A person stands near the edge. When the platform spins, the person holds onto the post (attached to the platform) to avoid falling off.

of the platform and it takes 8 seconds to make one complete revolution. Assuming the rate at which the platform spins is constant (i.e., it doesn't go faster or slower), how far does the person move in 3 seconds?

Answer 20.1: In this case, the period is 8 seconds. In 3 seconds, it goes 3/8 of the way around. In this case, the circumference is 9.425 m (use $2\pi r$, where r is 1.5 m). Three-eighths of that is 3.5 m.

✓ *Checkpoint 20.1: Suppose a person moves in a circle that has a radius of 1.3 m and it takes 5 seconds to make one complete revolution around the circle.*
(a) What is the circumference of the circle the person travels around?
(b) What is the period of the person's motion around the circle?
(c) How far (Δs_{circ}) does the person move in 2 seconds?

20.2 Velocity

I'll make a similar notation change for the velocity as I did for displacement. In particular, instead of $\vec{v}$, I'll use v_{circ}, where v_{circ} indicates how fast the object moves around the circle.

The definition of average velocity can then be written as follows:

$$v_{\text{circ,avg}} = \frac{\Delta s_{\text{circ}}}{\Delta t}$$

Other than the slight change in notation, the meaning is the same. For example, let's suppose we have a person located 2 meters from the center on a rotating platform that rotates once every six seconds (see Figure 20.1). From this information, how fast is the person moving (i.e., what is the person's velocity around the circle)?

To find the speed, we need to know how far the person traveled in a given amount of time. In this case, we know that the person traveled a distance equal to the circumference in six seconds.

Since the circumference is $2\pi r$, that means the person moves a distance equal to $2\pi(2\text{ m})$ in 6 seconds. From the definition of average velocity, we have:

$$v_{\text{circ,avg}} = \frac{\Delta s_{\text{circ}}}{\Delta t}$$

and plugging in, we get an average speed of 2.1 m/s.

Next, suppose we want to know how *far* the person has moved in two seconds. Since the average speed remains at 2.1 m/s the entire time, we can again use the definition of velocity with $v_{\text{circ,avg}}$ and Δt equal to 2.1 m/s and 2 s, respectively, then solve for the distance around the circle Δs_{circ}. I get 4.2 m.

A distance of 4.2 m is one-third of the circumference (since two seconds is one-third of the period of six seconds). So, if all we really wanted to know was the distance, we wouldn't need to find the speed first (see Example 20.1, for an example of that).

✓ *Checkpoint 20.2: Suppose the person in Figure 20.1 is 1.3 m from the center of the platform and it takes 5 seconds to make one complete revolution. Assume the rate at which the platform spins is constant (i.e., it doesn't go faster or slower). How fast is the person moving around the circle?*

20.3 Net force

It can be tricky to identify the net force needed to make an object move in a circle. To simplify things, we'll only consider objects that are moving in a circle at a constant speed, a situation called **uniform circular motion**.

To identify the force that is needed to move in uniform circular motion, we can use the law of force and motion. The law of force and motion states that an object's velocity changes only when the net force on an object is not zero.

❢ The law of force and motion applies to circular motion, as with all other types of motion.

Since an object in uniform circular motion is not speeding up or slowing down, you might think the net force on it must be zero. However, an object moving in a circle is constantly changing *direction* and, by definition, an object's velocity includes both the speed *and* the direction of motion. That means the velocity is changing and, based on the law of force and motion, the net force can't be zero. If it were, the object would move in a *straight line* at a constant speed.

❢ For *uniform* circular motion, the object's speed is constant.

Since circular motion is not straight-line motion, we know that the net force can't be zero. That means that a force is needed to make a car round a curve. That force is provided by friction with the ground. If you remove the friction (because of ice, for example), the car can't go around the curve and will slide off the roadway (as it goes straight instead).

But in what direction is the net force? After all, if you push on an object from behind, it speeds up, and if you push it backwards, it slows down. Given that, how do you get an object to move in a circle?

To *get* it started moving, you have to push it forward. However, once in motion, you just need to make it change directions and to do that you need to push it in a direction *perpendicular* to the motion. For an object moving in a circle, never getting closer to the center or farther away from the center, that *perpendicular* direction is *inward* – toward the *center* of the circle.

✍ As noted, you'll need to apply a force in the direction of motion to *start* the object moving in a circle. However, since uniform circular motion is the case where the speed is constant, we are <u>not</u> looking at when the object is *starting* or *stopping*.

❢ The net force must be directed toward the center of the circle in order for an object to undergo uniform circular motion.

To illustrate what it means to *keep* an object moving in a circle (once already in motion) by constantly pushing the object toward the center of the circle, consider the situation in Figure 20.2.

In the figure, an object moves counter-clockwise around a circle, from a to b to c and so on. At every location, the net force must be inward, as indicated by the arrows. If there was no net force (i.e., the forces on the object are balanced), the object would not be able to turn and would instead go in a straight line (see, for example, the dashed arrow in Figure 20.2).

For example, suppose a ball was attached to a string you were holding and you were using the string to spin the ball around your head such that, from a viewpoint above your head, the ball appears to go in a counter-clockwise circle like that shown in Figure 20.2.

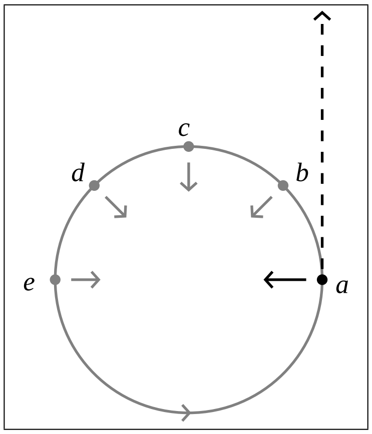


Figure 20.2: A diagram showing the path taken by an object moving counter-clockwise around a circular path at a constant speed. The arrows pointing to the center represent the direction of the force exerted on the object as it moves around the circle. For example, when the object is at point a, the force on the object is leftward. When the object is at point c, the force on the object is downward. If no force was acting on the object at point a, the object would have moved in a straight line (dashed path), not a circular one.

Once the ball is moving in the circular path, you still have to pull on the string because otherwise the ball would go off in a straight line. And if you let go of the string, that is exactly what would happen: at the moment you let go, the ball no longer curves.

Example 20.2: Suppose a ball was attached to a string you were holding and you were using the string to spin the ball around your head such that, from a viewpoint above your head, the ball appears to go in a counter-clockwise circle like that shown in Figure 20.2. Where would the ball go if you let go of the string at point a?

Answer 20.2: Without the string around to provide a force toward the center of the circle, the ball would travel in a straight line. At point a, the ball is moving up the page. Consequently, without any net force, the ball will continue to move up the page (see dashed line in the figure).

✓ *Checkpoint 20.3: Suppose you were spinning a ball around your head such that, from a viewpoint above your head, the ball appears to go in a counter-clockwise circle like that shown in figure 20.2. If you let go of the string at point c, in what direction would the ball then travel from that point onward? Ignore the vertical motion of the ball (which doesn't show up from the viewpoint above your head). Explain your reasoning.*

20.4 Acceleration

According to the force and motion equation, the direction of the net force on an object must also be the direction of the object's acceleration. Thus, since the net force is inward for an object in uniform circular motion, the acceleration must likewise be inward for an object in uniform circular motion.

Let's investigate what this means.

❢ For objects undergoing uniform circular motion, the object's speed is constant but the object is still accelerating because its velocity is changing directions.

❢ An object undergoing circular motion has an acceleration that is inward.

When an object is in uniform circular motion, it is not speeding up or slowing down, so you might think the acceleration is zero. However, an object moving in a circle is constantly changing *direction*. Since, by definition, an object's velocity includes both the speed *and* the direction of motion, that means the object's velocity is *changing*. Since acceleration is defined as the rate at which the velocity is changing, that means the acceleration is not zero.

In other words, in physics the word "acceleration" does not just mean how quickly an object is "speeding up", it means how quickly the object is undergoing *any* change in velocity, whether it be speeding up, slowing down *or* changing directions. For an object moving in a circle at a constant speed, it is not speeding up or slowing down. It is still accelerating, though, toward the center of the circle, as is the direction of the net force exerted on that object, consistent with the force and motion equation.

For example, using the ideas in chapter 12, one can show that the moon's acceleration (due to the gravitational force attracting it to Earth) should be 2.7×10^{-3} m/s^2. The moon's acceleration, in this case, does not mean it is speeding up or slowing down. Rather, since the acceleration is directed inward, toward Earth, it corresponds to the rate at which the moon changes its direction as it moves in a circular path around Earth.[iii]

Example 20.3: As Earth orbits the sun, in which direction is the gravitational force on Earth (due to the sun)? What about its acceleration?

Answer 20.3: Both the gravitational force on Earth (due to the sun) and its acceleration are directed toward the sun (at the center of Earth's orbit).

[iii] At the same time, one can show that Earth's acceleration (due to the gravitational force attracting it to the moon) should be 3.3×10^{-5} m/s^2. This serves to make the Earth also move in a circle, albeit much smaller than the moon's, resulting in a slight wobble for Earth. This is discussed further in section 22.6.

What if the object is speeding up or slowing down while moving in a circle?

If the object is speeding up while moving in a circle then we say that the object is not only accelerating inward (as it turns) but also accelerating in the direction of motion (as it speeds up). Similarly, if the object is slowing down while moving in a circle then we say that the object is not only accelerating inward (as it turns) but also accelerating in a direction opposite the motion (as it slows down).

Notation-wise, if we only want to refer to the acceleration in the direction of motion, we can use a_{circ}. That notation is consistent with the notation we used before for the speed of the object around the circle, v_{circ}, and allows us to still use the definition of acceleration:

$$a_{\text{circ,avg}} = \frac{\Delta v_{\text{circ}}}{\Delta t}$$

Just keep in mind that this is *not* the actual acceleration of the object (which must also have an inward part if the object is turning).

✓ *Checkpoint 20.4: Suppose you are running around a circular track at a constant speed. Using the notation we are using, would $a_{\text{circ,avg}}$ be zero? What about $\vec{a}$?*

20.5 Reference frames

As discussed already, there must be a force imbalance acting on an object in order for it to go in a circle, and that force must be inward (like an arrow pointing to the center of the circle). Otherwise, the object would go in a straight line.

While this appears pretty straightforward, it can be a bit tricky to analyze if the *observer* is also moving in a circle.

For example, suppose you are seated in a moving car and you close your eyes so you can't see outside. Even though you can't use visual cues to determine which way the car is going, you can still tell when the car turns because you'll feel as though you are being pushed to one side of the car.

But to which side are you pushed when the car turns?

It turns out you are pushed to the side that is *opposite* the direction the car is turning. For example, if the car turns toward the right, you'll feel as though you are being pushed to the left side of the car.

The reason for this how to do with the way you are experiencing the situation.

We know from before that there must be an inward force on the car to keep it on a circular path. For a car turning to the right, that means the ground must exert a rightward force on the car.

However, the ground doesn't act on *you* – just the car. Consequently, *you* continue to go straight. As the car turns to the right, and you continue straight. In other words, the car moves to *your* right – which means you move to the *car's* left.

Consequently, your interpretation of the situation (i.e., seemingly being pushed to the left) is really just a result of the car turning to the right (due to the force on it) and you going straight (since there is no force on you).

Basically, if you treat the car as moving straight when it really is accelerating rightward, then you'll see yourself as accelerating leftward when you are really just going straight. And, by seemingly accelerating leftward, it may appear as though there is a force pushing you leftward, when really there is a force on the car pushing it rightward.

In general, such "mysterious" forces may appear whenever we interpret a situation from the point of view that is, itself, accelerating (like the car in the example above). For this reason, it is important that the motion be interpreted from the point of view that is not itself accelerating.

We call the "point of view" one's **reference frame**. So, stated another way, it is important that the motion be interpreted from a non-accelerating reference frame.

In the example with the car, it is natural to use the car as our reference frame (rather than the ground) but by doing so we will appear to be accelerating (even though there is no force on us) when it is actually our reference frame (based on the car) that is accelerating.

If we are to use the law of force and motion appropriately, we need to make sure we are interpreting the situation from a reference frame that is not, itself, accelerating. We call a non-accelerating reference frame an **inertial reference frame**.

To illustrate what I mean, consider a ball that is stationary in a wagon. If the ball is placed at the front of the wagon and someone suddenly pulls the wagon forward, the ball will move to the back of the wagon.

Many observers, even ones that are stationary, will inadvertently use a reference frame that accelerates with the wagon. If so, the ball will appear to accelerate backward, even though there is no force pushing it backward. That runs counter to the law of force and motion.

❢ The law of force and motion should be applied only if an object's motion is measured relative to a reference frame that is, itself, not accelerating.

Only if the observer uses an inertial reference frame, one that is stationary with the ground, will the ball also be interpreted to remain relatively stationary as the wagon moves underneath it, consistent with the fact that there are no horizontal forces acting on the ball.

Is it wrong to use an accelerating reference frame to interpret the motion?

No, although you will need to include additional "fictitious" forces if you want to make the law of force and motion work.

For example, suppose someone is standing near the edge of a large spinning platform (see, for example, Figure 20.1 on page 342). That person is moving in a circle and so there must be an inward force acting on the person. Typically, the person is holding onto the platform and so the inward force is due to the platform.

However, to people on the platform itself, who tend to interpret the motion from a reference frame that is spinning with the platform, it will appear as though there is a force pushing the person *away* from the center (which they must balance by holding onto the platform).

✍ The Earth is spinning and, like the spinning platform, that can impact the way we apply the law of force and motion. For most situations, we can ignore Earth's rotation. However, on a larger scale, that rotation has an effect on how we view things, like the rotation of hurricanes (to be discussed on page 478) and the orbit of the moon (to be discussed in section 22.6).

✓ *Checkpoint 20.5: I walk in a circle on a level, horizontal floor. There is* *friction between me and the floor that allows me to walk in the circle. What is the direction of the friction force on me? Toward the center (making me turn that way), away from the center (making me turn the other way), in my direction of motion (making me go faster) or opposite my motion (making go slower)?*

20.6 Terminology

At this point, I want to make a comment about some terminology. As mentioned above, in order for an object to move in a circle, the net force needs to be inward (directed to the center of the circle).

Inward isn't a word that most people use to describe the direction. I'm only using it to be consistent with the way we refer to directions in general, like upward and downward. Most people use the word **centripetal** (literally, "center seeking") instead of inward. For example, we can observe that the moon orbits Earth in a large circle, consistent with Earth exerting a gravitational force on the moon that is directed toward Earth, situated at the center of that circular orbit. For the moon, the gravitational pull toward Earth is in the centripetal direction.

In terms of this language, we can say that for every case of circular motion, the direction of the net force is centripetal. In some cases, there is a single force that is contributing to the net force. In other cases, there are lots of forces that contribute to the net force.

It is important to realize, however, that there is *not* an additional force, called the centripetal force, that acts in addition to the other forces that acting. A **centripetal force** is simply a force that happens to be directed inward. There may be several forces that are directed, totally or partially, in the centripetal direction.

A relatd term is **centrifugal**, which is *opposite* to centripetal. Whereas centripetal means "towards the center" (inward), centrifugal means "away from the center" (literally, "center fleeing"). For the situation described in section 20.5, the person on the platform appears to experience a centrifugal force, pushing the person *away* from the center of the spinning platform.

✓ *Checkpoint 20.6: Right now you are moving in a circle as Earth rotates around its axis. Which of the following is the centripetal direction? Explain the rationale behind your choice.*
(a) Upward
(b) Downward
(c) Toward the North
(d) Toward the South
(e) Toward Earth's axis (this would be both downward and toward the north for someone situated in the northern hemisphere)

20.7 Local vs. global gravity

In section 13.1, it was mentioned that the gravitational field strength is 9.8 N/kg due to Earth for an object on or near Earth's surface. If you look up the value of g for a location like Pennsylvania[iv], you'll find that the actual value of g is about 9.802 N/kg. The difference is due to rounding, since 9.802 rounds to 9.8.

However, if you instead try to calculate g by plugging the mass and radius of Earth into the gravitational field strength expression (equation 13.3) for M and r, respectively, you'll get a gravitational field strength of 9.8205 N/kg, not 9.802 N/kg.

You might think the difference is just due to rounding, but it isn't, as 9.8205 N/kg does not round to 9.802 N/kg. Instead, the reason for the difference has to do with the fact that the Earth is spinning and the fact that the universal law of gravity equation in chapter 12 represents something slightly different than the simplified gravity equation of mg.

To distinguish the two, we could call g the **local** gravitational field strength, whereas the value given by equation 13.3 (GM/r^2) is the **global** gravitational field strength.

What does the spinning Earth have to do with it?

[iv] See, for example, https://www.sensorsone.com/local-gravity-calculator/.

Something that is stationary with respect to Earth's surface is actually not stationary with respect to the stars. Rather, the object is moving in a circle around Earth's rotation axis. As we've learned in this chapter, objects in circular motion must experience an inward net force, and in section 20.5 it was mentioned how there appears to be a force pushing outwards when we observe something from a rotating reference frame.

The same is true here – relative to our reference frame (which spins with Earth), there appears to be a force pushing things away from Earth's axis. This extra force appears to act in addition to the gravitational force that pulls objects in toward the center of Earth. The two counter-act slightly, leading to a combined force that has a magnitude slightly less than what one gets purely from the universal law of gravity equation alone.[v]

> ✍ The two directions, away from Earth's axis and toward Earth's center, are not completely opposite except on Earth's equator, leading to a lower value of g there (i.e., 9.78 N/kg vs. 9.80 N/kg in Pennsylvania).[vi]

Although the difference between mg and Gm_1m_2/r^2 is usually small, it is practical, when there is a difference, to use mg, particularly if you are an architect or an engineer.[vii]

To illustrate with an extreme case, consider an astronaut on board the orbiting International Space Station. As mentioned in chapter 12, the gravitational force on the astronaut, in orbit about Earth, is pretty close to what it would be if they were on Earth because they are only about 200 miles away, which is still small compared to Earth's radius (about 4000 mi).

However, relative to their frame of reference (where they complete a circle every 90 minutes instead of every 24 hours for someone on Earth), the *local* gravitational field strength is zero. In other words, from their point of view (a non-inertial frame of reference), they experience no local gravitational field.

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[v] The difference between the local and global gravitational fields can be determined using ideas introduced in chapter 22.

[vi] It is also lower on the equator because Earth's surface there is farther from Earth's center, due to Earth not being a perfect sphere.

[vii] It is also consistent with the equivalence principle between gravitation and acceleration. The equivalence can be illustrated as follows. Suppose you are in a small box at rest on Earth's surface. If you have no access to the outside, the equivalence means that your experience inside the box would be the same as if you were in space accelerating in the "upward" direction at a value equal to g.

✓ *Checkpoint 20.7: Based on the discussion in this section, why should g be greater at the north pole than at the equator? Assume Earth is a perfect sphere of radius 6.37×10^6 m.*

Summary

This chapter examined how to describe circular motion. Similar expressions and terminology can be used for both circular motion and oscillations. This chapter also examined the force needed to keep an object moving in a circle.

The main points of this chapter are as follows:

- The circumference of a circle is equal to 2π times the radius.
- For *uniform* circular motion, the object's speed is constant.
- The law of force and motion applies to circular motion, as with all other types of motion.
- The net force must be directed inward (centripetally) in order for an object to undergo uniform circular motion.
- An object undergoing circular motion has an acceleration that is directed inward (centripetally).
- For objects undergoing uniform circular motion, the object's speed is constant but the object is still accelerating because its velocity is changing directions.
- The law of force and motion should be applied only if an object's motion is measured relative to a reference frame that is, itself, not accelerating.

By now you should be able to use the definitions of velocity and acceleration, applied to circular motion, to relate various aspects of the circular motion.

Frequently Asked Questions

How far is one full circumference?

The circumference is equal to 2π times the radius (or π times the diameter).

If an object completes one full circle, is the displacement zero?

Since the object returns to its initial position, the *vector* displacement is zero. However, the displacement *around the circle* is not zero, unless the object reversed direction and came back to where it was originally.

Keep in mind that velocity is a vector whose direction changes as the object moves around the circle. However, we can consider the motion *around the circle* to be constant in the sense that it continues clockwise or counter-clockwise the entire time.

What does it mean to say that the acceleration around the circle is zero for uniform circular motion but the vector acceleration is not?

See section 20.4.

Why worry about whether we are measuring the object's motion relative to a reference frame that is, itself, undergoing a change in motion?

The value of the law of force and motion is that it applies in general. We don't want to use different laws for different situations. As long as we measure an object's motion relative to a reference frame that is not, itself, accelerating then we can use the law of force and motion.

When you spin a ball around on a string, is there a force pushing the ball away from the center?

Not if you interpret the motion in an inertial reference frame. There may seem to be such a force, though, if you use a non-inertial reference frame. See section 20.5.

When an object moves in uniform circular motion, will there always be just a single force pushing it toward the center?

No. See, for example. section 22.6 on page 379.

Why would an object in circular motion be accelerating if it isn't speeding up?

If the velocity *around the circle* (indicated as v_{circ}) remains constant then the acceleration *around the circle* (indicated as a_{circ}) is zero. However, according to the definition of acceleration, an object is accelerating whenever the speed or direction changes. In this case, the direction is changing so there is a turning acceleration, even if the speeding up acceleration is zero.

Terminology introduced

Centrifugal	Counter-clockwise	Local
Centripetal	Cycle	Radius
Centripetal force	Diameter	Reference frame
Circumference	Global	Revolution
Clockwise	Inertial reference frame	Uniform circular motion

Additional problems

Problem 20.1: A rotating platform (see, for example, Figure 20.1 on page 342) is initially at rest and then accelerates such that a person, located 2 m from the center, moves one-quarter of the way around in 5.0 seconds.
(a) How far did the person move (Δs_{circ}) around the circle?
(b) What was the person's average speed (v_{circ}) around the circle?

Problem 20.2: Suppose we have an object undergoing counter-clockwise (looking down) uniform circular motion and, at a given time, the velocity directed toward the right. For that moment,
(a) Draw a picture of the object's path and indicate its position.
(b) Draw an arrow representing the direction of the object's velocity.
(c) Draw an dashed arrow indicating the direction of the object's acceleration.

Problem 20.3: Assuming the moon orbits Earth in a perfect circle at a constant speed, in what direction is (a) the moon's acceleration and (b) the moon's velocity?

Problem 20.4: Earth travels in a nearly uniform circular path as it orbits the sun, completing one circle every year. Since Earth's path is not straight, there must be a force acting on Earth pushing it toward the sun. What force is doing this?

21. Rotational Motion

Puzzle #21: How do we apply the law of force and motion to spinning objects?

Introduction

Like everything else, spinning objects obey the law of force and motion. For example, you have to do something to get it spinning faster or slower. And, with no friction and no outside forces, a spinning object would continue to spin forever.

However, while it is straightforward to apply the law of force and motion in this matter, we cannot use the current formulation of the force and motion equation to make specific predictions about the motion because the force and motion equation is written in terms of objects moving with a particular velocity, not things that are spinning.

To come up with a force and motion equation for rotation, we need to first come up with a way to describe rotation, and then apply that to the force and motion equation. Fortunately, to describe rotation we can use language similar to what we used for circular motion (from chapter 20). For example, the period of circular motion is the time it takes for an object to complete the circular path. Similarly, the period of rotation is the time it takes for an object to spin once around its axis.

The big difference is that we describe rotation in terms of an angle, not a distance. In this chapter, I'll go over how we describe the rotation of objects in terms of angles. In chapter 22, I'll relate the rotational description with the circular description of chapter 20, which will allow us (in chapter 23) to apply the law of force and motion to rotation.

21.1 Rotation (angular displacement)

As mentioned in the introduction, we measure how far an object rotates in terms of angle. Two common angular units are revolutions and degrees. One revolution corresponds to one complete turn of the object, and there are 360 degrees in one revolution.

✍ The unit abbreviation for "revolution" is "rev". The unit abbreviation for "degree" is °.

✓ *Checkpoint 21.1: A object rotates by one-quarter of a revolution. How many degrees is that?*

You may already be familiar from mathematics classes that it is common to use the lower-case Greek letter theta (θ) for angles. It turns out that the scientific community tends to use lower-case Greek letters for quantities that have to do with rotation, so using θ for angles is just being consistent with that convention.

And, just as we use $\Delta\vec{s}$ for displacement (change in position), we'll use $\Delta\theta$ to represent the object's **angular displacement** (i.e., how far it has rotated).

✓ *Checkpoint 21.2: What does $\Delta\theta$ represent?*

21.2 Rotation rate (angular velocity)

We describe how fast something rotates (i.e., its rotation rate) in much the same way we describe how fast anything moves. In that sense, the rotation rate is just an *angular* velocity. Indeed, from now on, I will refer to the rotation rate as the **angular velocity**.

In keeping with convention of using lower-case Greek letters for angular quantities, I will use the lower-case Greek letter omega (ω) to represent the angular velocity in equations.

WHY OMEGA?

There is no exact Greek equivalent to the Roman letter v. I'm guessing that ω (omega) is the closest letter to it.

How is the angular velocity related to the angular displacement?

The relationship between angular velocity and angular displacement is the same as the relationship between their translational equivalents, "regular" velocity and "regular" displacement. In particular, the angular velocity is defined as the rate at which the angle changes (compare to the $\Delta\vec{v} = \Delta\vec{s}/\Delta t$ relationship):

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$$

❢ The relationships between angular quantities is the same as the relationship between their translational equivalents.

✓ *Checkpoint 21.3: For constant net force, the velocity changes in a uniform way and the average velocity can be written as the average of the initial and final velocities as follows:*

$$\vec{v}_{\text{avg}} = \frac{\vec{v}_{\text{i}} + \vec{v}_{\text{f}}}{2}.$$

Suppose you have an object that is spinning faster and faster, with an angular velocity that increases in a uniform way. Rewrite the expression above in a form using angular quantities such that it describes how the average angular velocity depends on the initial and final angular velocities.

What units do we use for angular velocity?

Whereas velocity has units of a *length* per time, angular velocity has units of an *angle* per time. For this chapter, any angular unit can be used for the angle, just as any time unit can be used for the time.

For angular velocity that means you could use units like degrees per minute (deg/min) or revolutions per day (rev/day). You just need to make sure that every angular quantity used in a particular relationship is expressed in the same angular unit. Otherwise you will need to convert.

For example, when using the definition of average angular velocity you can have all of the angles in units of degrees and time in units of seconds if you'd like (i.e., θ in units of degrees and ω in units of degrees/second).

✍ Other than the fact that angular quantities are measured in units of angle, not length, the process is the same as before. Whatever unit you choose to use, just make sure your units work out with the other quantities in the expression.

To illustrate, consider the following scenario.

> Suppose you had a bicycle initially at rest with wheels of radius 30 cm. The bicycle starts moving such that 5 s later the wheels are making one revolution every 2 s. Assuming the bike speeds up at a steady rate, how far has each wheel rotated in the 2 s?

Before getting the answer, let's first recognize that the initial angular velocity is zero and the final angular velocity is 0.5 rev/s (i.e., 1 revolution every 2 seconds).

Since the wheel speeds up at a steady rate, the average angular velocity will have the midrange value between the initial and final values (i.e., $\omega_{\text{avg}} = 0.25$ rev/s). We can then use the definition of average angular velocity to get the angular displacement:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$$

Plugging in the value of the average angular velocity (0.25 rev/s) and the elapsed time (5 s), we find that the angular displacement is 1.25 rev.

Does it matter what shape the object has?

The relationships between the various angular quantities does not depend upon the shape of the object (i.e., the radius of the wheel was not used).[i]

✍ Remember that quantities like ω and θ have directions.[ii] Make sure you keep track of the direction. You can use clockwise and counter-clockwise or you can use positive and negative.

✓ *Checkpoint 21.4: A hoop of 2.5 m radius is rotating such that it makes one complete revolution every 2 s.*

[i] As we will learn in chapter 23, though, how quickly an object spins up or down *does* depend on the mass and shape of the object, as well as the net torque exerted on it.

[ii] Technically, the abbreviations in the equations should use arrows, like $\vec{\omega}$, to remind of this but they are left off so as to not overly complicate the equations.

(a) What is the hoop's angular velocity?
(b) Suppose the hoop starts spinning faster and faster such that 3 s later, it is making one complete revolution every 1.25 s. What is the hoop's angular velocity at the end of the 3 s?
(c) What was the angular displacement of the hoop during the 3 s? Assume it speeds up at a steady rate.

21.3 Angular acceleration

Just as we have an angular equivalent of displacement and velocity, we also have an angular equivalent of acceleration.

The **angular acceleration** describes how quickly an object spins up or down. Angular acceleration is represented in equations by the lower-case Greek letter alpha (α), which is the first letter of the Greek alphabet, just like the Roman letter a is the first letter of the Roman alphabet.

❢ We use lower-case Greek letters as abbreviations for angular quantities.

The angular acceleration is defined as the rate at which the angular velocity changes:

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$$

The angular acceleration is measured in terms of an angle per time squared, like degrees per hour squared (deg/h^2) or revolutions per second squared (rev/s^2).

To illustrate how we use the angular acceleration in the same way as the regular acceleration, consider the following scenario:

> A rotating platform (see, for example, Figure 20.1 on page 342) is rotating at a rate of 2 rev/min clockwise. The platform then starts slowing down at a uniform rate of 1 rev/min^2 counter-clockwise for four minutes. How far did the platform rotate during the four minutes?

Because the angular acceleration is opposite its initial angular velocity, that means the disk is slowing down. At a rate of 1 rev/min^2, it takes 2 minutes for the disk to stop (since it started at 2 rev/min clockwise). The disk then spins the opposite way and in two more seconds, it is spinning at 2 rev/min counter-clockwise.[iii]

The average angular velocity therefore zero and so the total angular displacement is zero also.[iv] In other words, the platform returns to its initial position.

✓ *Checkpoint 21.5: A disk is spinning at a rate of 2 rev/s clockwise when it experiences an acceleration of 1 rev/s^2 counter-clockwise for three seconds. How fast is it spinning at the end of the 3 seconds and what was its total angular displacement during the three seconds?*

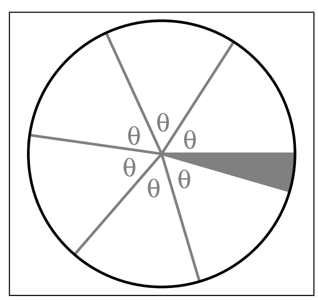
21.4 Radians

Although any angular unit can be used for indicating how far an object rotates, we'll find out in chapters 22 and 23 that the SI unit of angle is the radian. That means in equations with lots of different types of units, using all SI units, including the radian, will ensure that the units match.

So, while any angular unit can be used for this chapter, we'll be restricted to using radians in chapters 22 and 23. Given that, we might as well learn about radians now if you aren't already familiar with them.

✍| The unit abbreviation for "radian" is "rad".

There are a little more than 6 radians in a full circle. This is illustrated in the figure to the right. Think of the circle as being a pizza with seven slices. Six of the slices are of equal size, with one smaller slice (shaded). The tip of each slice makes an angle. All of the unshaded slices have the same angle (indicated by the lower-case Greek letter θ in the figure). That angle is equal to one **radian**.



[iii]Mathematically, we can treat one direction as positive and one direction as negative, particularly if the problem just states that there is motion in "one direction" and then "the other".

[iv]If the average angular velocity was not zero, we'd multiply the average velocity by the time to get the total angular displacement.

✍ As you can see, a radian is about 57.3 degrees or a little less than one-sixth of a revolution.

What is so special about the radian?

❢ A radian is the angle at which the arc length is equal to the radius.

It turns out that each pizza slice in the figure has a special property – the length of its crust is equal to the radius of the pizza. And when that happens, the angle created is equal to one radian.

The angle is proportional to the arc length – the bigger the arc length, the bigger the angle. So, like its arc length, the angle of the shaded slice is smaller than the angle of the other slices and thus represents an angle less than one radian. Since the shaded slice has an arc length (crust) that is about one-quarter the length of the radius, that slice represents an angle that is about a quarter of a radian.

It doesn't matter how big the pizza is. If you cut it into slices such that the slices have this property (length of crust equal to the radius of pizza), you will always get six slices, plus a little extra.

For example, suppose we had a pizza[v] with a radius of 18 cm (about 7 inches). A radian would be represented by a slice whose crust has a length equal to 18 cm (i.e., the radius). And you can get six of those slices out of the piece (plus a little more).

Indeed, the angle, in radians, can be defined as being the ratio of the arc length divided by the radius. For example, for our 18-cm radius pizza, an angle of 2 radians is represented by a slice whose crust (arc length) is 36 cm, twice as big as the radius.

✓ *Checkpoint 21.6: Draw a picture of a pizza slice such that the tip of the pizza slice represents an angle of 1.5 radians. Explain your choice.*

How many radians are in one full rotation?

As mentioned before, regardless of the size of the pizza, you will always get six slices (each with an angle of 1 radian) plus a little more (with an angle of about one-quarter of a radian). That means there are about six and a quarter radians in one full rotation.

❢ There are 2π radians in one full rotation (360 degrees).

[v] A 14-inch pie has a diameter of 14 inches or a radius of 7 inches.

The actual value is 2π radians in one full rotation. The lower-case Greek letter pi (π) is being used to represent a number that has a value of about 3.14159265358979. I write "about" because the actual number is irrational and goes on forever (in base 10). That is why people typically write it as π.

You may recall from geometry that one circumference equals a distance of 2π radii (i.e., $C = 2\pi r$).[vi] Using the definition of radians, we can show that the angle associated with the complete circle is the circumference ($2\pi r$) divided by the radius r, which gives an angle of 2π radians.

How do I convert between radians and degrees or revolutions?

Converting is relatively easy. There are 2π radians in one revolution. So, to convert from revolutions to radians, simply replace every instance of "rev" with "2π rad".

To convert from degrees to radians is a little trickier but still straightforward. There are 2π radians in 360°. Divide both by 360 to get that there are $2\pi/360$ radians in each degree. That means you can replace every degree by $2\pi/360$ radians.

Example 21.1: Suppose a hoop is rotated one-quarter of a revolution. What is the rotation in radians?

Answer 21.1: One revolution is 2π radians. Consequently, one quarter of a revolution is one quarter of 2π radians. The answer is $\pi/2$ radians or 1.57 rad.

✓ *Checkpoint 21.7: How many radians are there in one-half of a revolution?*

Summary

This chapter examined how angular quantities are related. Angular quantities include the angular displacement (how far the object has rotated), the angular velocity (rotation rate) and angular acceleration.

[vi] I am using r instead of R because, in keeping with the convention mentioned on page 341, it refers to the radius of the *circle*, not necessarily the radius of the *object*.

The main points of this chapter are as follows:

- We use lower-case Greek letters as abbreviations for angular quantities.
- The relationships between angular quantities is the same as the relationship between their translational equivalents.
- A radian is the angle at which the arc length is equal to the radius.
- There are 2π radians in one full rotation (360 degrees).

By now you should be able to predict various aspects of rotational motion, given other aspects of the motion.

Frequently Asked Questions

Is it important to use the "angular" descriptor when referring to the angular quantities like the angular displacement and angular velocity?

Yes. Otherwise, one might think we are referring to the linear quantities, with units like meters, miles or feet. Although the approach and mathematics are the same whether we are dealing with linear quantities or angular quantities, the units will be different.

Why do we use Greek letters for angular quantities?

That is just the convention.

What is a radian?

A radian is about 57.3 degrees or a little less than one-sixth of a revolution.

Does 1 radian equal π?

No.

The π represents a number (about 3.1415927). It is a number that relates the number of diameters present in a circumference of a circle.

Because of this, you'll frequently see the letter π when indicating a revolution, or fractions of a revolution, in terms of radians. For example, one full revolution is 2π radians. However, that doesn't mean that *every* angle in radians includes a π.

Do we have to use radians when dealing with angular quantities?

No. In chapters 22 and 23, however, we'll be examining relationships that include both angular and length quantities. In those cases we'll need to use radians. Basically, as long as the relationship does not involve a length, you can use any angular unit you want (as long as they are all the same).

Terminology introduced

Angular acceleration
Angular displacement
Angular velocity
Radian

Additional problems

Problem 21.1: The radius of Earth is 6,370 km. The Earth rotates once per day. What is the angular velocity of the Earth, in rev/s and rad/s?

Problem 21.2: An airliner arrives at the terminal, and the pilot shuts off the engines. The initial angular velocity of the fan blades is 300 rev/s, and it takes 2.0 minutes for them to come to rest. Assuming the angular acceleration of the blades is constant as they slow down, what is the angular displacement of the blades during the 2.0 minutes?

22. Circular-Angular Relationships

Puzzle #22: How do we know the mass of Earth?

Introduction

Have you ever wondered how we know Earth's mass? We don't weigh it in the usual sense. Instead, we apply the force and motion equation. In particular, we relate the force acting on the moon (which depends on Earth's mass) with the moon's acceleration (as it orbits Earth). To do this, we need a way of determining the acceleration of an object moving in a circle, and we'll do that in this chapter.

22.1 The value of radians

In chapter 21 we examined rotation. To describe rotation, we needed to use angular units, such as degrees, revolutions and radians. Outside of math and science, degrees and revolutions are used almost exclusively, so you might wonder why we use radians so much in math and science.

The reason, as mentioned in section 21.4, is that the radian is the SI unit. Consequently, as long as we use it with other SI units, we are ensured that all the units will work out. However, the radian is even more useful than that. It turns out that using the radian will ensure the units work out even with non-SI units since the radian relates arc lengths to radii.

To understand what this means, it helps to reflect back on the degree and examine the value of that unit. The advantage of the degree is that it goes evenly into one revolution. In particular, there are exactly 360 degrees in one revolution. Thus, if you want to relate an angle with a revolution or a fraction of a revolution, degrees would be your go-to unit.

However, the degree doesn't go evenly into the arc length. For example, suppose we had a pizza of radius 18 cm and we cut a slice such that the slice had a length of crust equal to 18 cm (equal to the radius). How many degrees would represent the angle at the tip of the pizza slice (the part that was at the center of the original pizza)?

It turns out this is not easy to do. You'd have to first figure out how many of these pizza slices would fit into one full pizza, and then divide the total number of degrees in a full circle (360) by the number of these slices that make up a full pizza. When you do this, you get an angle of about 57.29578 degrees. It is actually equal to 180 divided by π, where π is a number equal to about 3.1415927.

On the other hand, figuring out that angle in radians is very easy. It is one radian. Basically, the angle in radian is equal to the arc length divided by the radius. That makes it very easy to represent the angle in radians if you know the arc length and the radius.

Of course, while the radian is easy to determine when you know the arc length, you get more complicated values when you are given the fraction of the full circle. Whereas the angle in degrees involves the value of π when the corresponding arc length is known, the angle in radians involves the value of π when we are instead given a particular number of revolutions or fraction of a revolution. For example, the angle for one-quarter of a circle is 1.570796 radians, equal to the value of π divided by two.

Since the radian is defined as the arc length divided by the radius, we can write the relationship between the angle, arc length and radius as follows, if the angle is in radians:[i]

$$\Delta s_{\text{circ}} = r\Delta\theta \qquad \text{[angle in radians]}$$

where s_{circ} is the arc length (i.e., the length of the pizza crust) and r is the radius. Notice that the addition of the qualifier that the angle must be in radians. This equation won't work if the angle is not in radians.

Another advantage of the radian is that the radian unit "disappears" when

[i]In this expression, we could consider r to be a conversion factor with units of m/rad, but since it happens to have a value equal to the radius, we might as well treat it as such and abbreviate it as r.

multiplied or divided by another quantity.[ii] For example, suppose we had a pizza of radius 18 cm and wanted to know the crust length of a pizza slice whose tip had an angle of two radians. To figure it out, we'd multiply the 18 cm by two radians to get 36 cm of crust.

Notice that the radians unit doesn't appear in the answer. Given that when we multiply two quantities, we are not only multiplying the numerical values (18 and 2 in this case) but also the units (cm and rad in this case), you might wonder why the answer doesn't have units of cm·rad.

The reason the radian doesn't appear as a unit in the answer is because a radian is essentially equivalent to a meter (of arc length) per meter (of radius). Consequently, the meters cancel out. In comparison, if the angle was expressed in units of degrees or revolutions, we would need to do some kind of conversion when used in a product or quotient. With radians, we don't have to do any conversion.

✓ *Checkpoint 22.1: Suppose a platform (like the one in Figure 20.1 on page 342) rotates 0.5 rev. A person is located 2.5 m from the center. If we multiply the rotation angle (0.5 rev) and the distance (2.5 m) do we get that the person has moved a distance of 1.25 m? Why or why not?*

22.2 Relating circular and angular quantities

For every object that is rotating, each part of the object is going in a circle. In addition, the circular motion of those parts is directly related to the rotational motion of the object as a whole.

In the same way, you can use angular quantities for circular motion. Consider, for example, a car that is moving around a circular track. Now picture yourself standing in the center of the circle, watching the car. As you watch the car, you have to rotate. The faster the car goes, the faster you have to rotate.

[ii] Angle is an example of a **dimensionless** quantity. All dimensionless quantities have the same property in that if the proper unit is chosen the unit will "disappear" when being multiplied or divided by another quantity. In this way, it is much like μ, the coefficient of friction, that was discussed in chapter 17.

So, an alternate way of describing an object in circular motion is to describe how someone in the center of the circle would *rotate* in order to follow the object. By relating the two we could, for example, figure out how fast a particular point on the object is moving given the rotation rate, or visa-versa.

From the previous section, we already have a way of relating the circular displacement with the angular displacement:

$$\Delta s_{\text{circ}} = r\Delta\theta \qquad \text{[angle in radians]}$$

❢ To obtain the measurement around the circle, multiply the angular measurement in radians by the radius (of the path, not the object).

We can use this relationship, along with the definitions of angular velocity and angular acceleration to get the following relationships:[iii]

$$\left.\begin{array}{rcl} s_{\text{circ}} &=& r\theta \\ v_{\text{circ}} &=& r\omega \\ a_{\text{circ}} &=& r\alpha \end{array}\right\} \text{angle in radians} \qquad (22.1)$$

As mentioned before, the angular unit needs to be in radians in order to use these rotational-circular expressions. Remember that a radian is essentially a meter per meter. Thus, as written above, the units work out only if the angular quantity is in radians. For this same reason, we'll need to use radians in chapter 23 also.

✍ As mentioned on page 341, I'm using r to indicate a distance and R to indicate the radius of an object. In this case, I am using r because it represents the distance from the object moving in a circular path to the center of that circular path. It is not the radius of that object. This distinction will become more crucial in chapter 23 (see page 391).

To make it clear that I am using the right units, from now on I will include both the numerical values and the units when using an equation. In prior chapters, I used just one or the other and you can continue to do so if you want. However, I'll generally be including both from now on.

To illustrate this, and show how to properly use radians, consider the following scenario.

[iii] To get the relationship for v_{circ}, apply the definition of velocity (i.e., $\Delta s_{\text{circ}}/\Delta t$ and replace s_{circ} with $r\theta$. Since r doesn't change, $\Delta(r\theta)$ just becomes $r\Delta\theta$. Dividing by Δt gives $r\omega$. The same approach can be used to obtain the relationship between a_{circ} and α.

> A disk is rotating such that it makes four complete revolutions every 2 seconds. If the disk has a radius of 2 m, what is the speed of a point on the disk that is 1.5 m from the center?

The problem gives the displacement around the circle ($\Delta\theta = 4$ rev) and the elapsed time ($\Delta t = 2$ s) and asks for the velocity around the circle (v_{circ}).

To get that velocity, we can multiply the angular velocity by the radius, as shown on page 370. First, though, we must do two things: (1) we need to convert the angle into radians and (2) we need to determine the angular velocity.

We can convert the angle into radians by recognizing that there are 2π radians in a revolution:

$$\begin{aligned} \Delta\theta &= 4 \text{ rev} \\ &= 4\ (2\pi\text{rad}) \\ &= 25.13 \text{ rad.} \end{aligned}$$

To get the angular velocity, use the definition of average velocity (written in angular notation):

$$\begin{aligned} \omega &= \Delta\theta/\Delta t \\ &= (25.133 \text{ rad})/(2 \text{ s}) \\ &= 12.57 \text{ rad/s.} \end{aligned}$$

Keep in mind that the definition can be used with any angular unit but we are using radians specifically so that we can use the v equation on page 370:

$$\begin{aligned} v_{\text{circ}} &= r\omega \\ &= (1.5 \text{ m})(12.57 \text{ rad/s}) \\ &= (1.5)(12.57) \text{ m rad/s} \end{aligned}$$

which gives a velocity around the circle equal to 18.8 m/s.

WHY DID YOU USE 1.5 m AS r INSTEAD OF 2 m?

Remember that r is the radius of the *motion*. In this case, that is the distance from (a) the point indicated to (b) the **rotation axis**, the imaginary line around which the object spins (which we are assuming goes through the center of the object, like with Earth's axis or the axle of a wheel). In this case, the radius of the object is 2 m but that isn't what we want.

How did you convert from units of "m rad/s" to "m/s" in the last step?

Although the unit of radians is abbreviated as "rad" it is really a ratio of distance to distance (i.e., Δs_{circ} to r). Consequently, we can replace "rad" with "m/m", which gives the units I showed.

✍ As mentioned above, an equation like $\omega = \Delta\theta/\Delta t$ is a definition and thus holds true regardless of what unit we use. In comparison, an equation like $v_{\text{circ}} = r\omega$ is more like a conversion and depends on what units we use (i.e., radians in this case).

Example 22.1: A rotating platform, as in Figure 20.1 on page 342, is rotating at a rate of 1 rad/s. The platform then starts spinning faster and faster such that 3 s later, it is rotating at 2 rad/s. Consider a point on the platform that is 2 m from the center. Assuming the platform spins up in a uniform way, how far does that point move during the 3 s?

Answer 22.1: Since the platform speeds up in a uniform way, we know that the average rotation rate is 1.5 rad/s (halfway between 1 rad/s and 2 rad/s). Knowing the average rotation rate, we can use the definition of average velocity to obtain an angular displacement equal to 4.5 rad (multiply the average rotation rate by the time). From that, we can find the distance around the circle by multiplying the angular displacement by the distance to the center (see first expression in equation 22.1 on page 370) to get 9 m.

✍ The angular acceleration in this case is 0.33 rad/s^2, since the rotation rate increased by 1 rad/s during three seconds.

✓ *Checkpoint 22.2: A rotating platform, as in Figure 20.1, is rotating at a rate of 2 rad/s clockwise. The platform then starts experiencing a constant angular acceleration such that it slows down to a stop and starts spins in the other direction. After three seconds, it is rotating at 4 rad/s counter-clockwise. How far does a point on the platform, 2 m from the center, move during the three seconds and in what direction?*

22.3 Acceleration of an object in circular motion

In chapter 20, it was mentioned that in order for an object to move in a circle, the net force on it needs to be inward (to the center of the circle). At the time, our treatment of the situation was purely qualitative. However, using what we know about angles, we can now predict exactly how much force is needed.

To do this, we'll work backward from what we know, which is that for uniform circular motion the object's speed remains constant but the object's direction continually changes, such that after a length of time called the period the object is back where it started.

From this information, we can determine the object's acceleration. By definition, the acceleration is the rate at which the velocity is changing:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Remember that the acceleration corresponds to *any* change in the velocity, which includes the direction as well as the speed. For an object in uniform circular motion the magnitude of the velocity is not changing (i.e., constant speed), but the direction *is* changing.

In particular, the object turns completely around in a time equal to the period. A complete revolution is 2π radians. Using the letter T to represent the period[iv], that means the angle changes at a rate of $2\pi/T$.

It is not surprising, then, that the magnitude of the acceleration (inward) is equal to the product of $2\pi/T$ and v_{circ} (i.e., how fast it is going around the circle), which is $(2\pi v_{\text{circ}})/T$.

The expression may appear strange but it actually has the same form as another relationship we have already discussed but did not formally express in an equation. Recall that an object in uniform circular motion will cover a distance equal to the circumference $(2\pi r)$ in a time equal to the period (T). Consequently, its velocity around the circle is just the ratio of the two, or $(2\pi r)/T$.

❢ For objects undergoing uniform circular motion, the object's speed is constant but the object is still accelerating because its velocity is changing directions.

[iv] We are using a capital T in order to distinguish this from lower-case t, which we have used for time.

The similarity is more obvious when the two expressions are written together:

$$\left.\begin{array}{lcl} v_{\text{circ}} & = & (2\pi r)/T \\ a_{\text{cent}} & = & (2\pi v_{\text{circ}})/T \end{array}\right\} \text{ for uniform circular motion only} \qquad (22.2)$$

In this way, for an object in uniform circular motion the expression for its acceleration[v] (toward the center of the circle) has the same form as the expression for its speed (around the circle).

❢ An object moving in uniform circular motion will accelerate toward the center of the circle with a magnitude equal to $2\pi v_{\text{circ}}/T$.

What does the "cent" subscript mean?

The subscript "cent" on a indicates that the object's acceleration is directed inward, toward the center of the circle.[vi] After all, it is the acceleration associated with the turning of the object, not the acceleration associated with it speeding up or slowing down (which it isn't since its speed is constant).

To illustrate, suppose it takes me 180 seconds to make it all the way around a circular track when I run at a constant speed of 2.5 m/s. What is my acceleration?

At first glance, you might be tempted to say that my s acceleration is zero because I am traveling at a constant speed. However, I'm turning, and we know that is an acceleration also. To find out the value of the acceleration, we use equation 22.2, which is that $a_{\text{cent}} = (2\pi v_{\text{circ}})/T = 2\pi(2.5 \text{ m/s})/(180 \text{ s}) = 0.087 \text{ m/s}^2$. The acceleration is not that much because I'm not turning very quickly – 180 seconds is a long time to go all the way.

✍ Notice that we used radians to *obtain* equation 22.2 but we didn't need radians to *use* equation 22.2!

The process is straightforward when we are given the speed and the period. It is a little trickier if we are given the radius of the circle rather than the period. This is often the case when describing things going around a curve – only part of a circle – rather than an entire circle. For example, consider the scenario illustrated in Figure 22.1, where a car goes around a banked curve. Suppose the car has a mass of 1000-kg and the circular path has a radius of 200 m. If the car is traveling at a constant speed of 30 m/s, what is its acceleration?

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[v] Keep in mind that this is the acceleration due to the change in direction. The speed remains unchanged, so the acceleration *around the circle* is zero.

[vi] Many people refer to a_{cent} as the **centripetal acceleration**. Keep in mind that the word "centripetal" simply means "inward." So, saying an object undergoes a centripetal acceleration is just a fancy way of saying that the object has an inward acceleration.

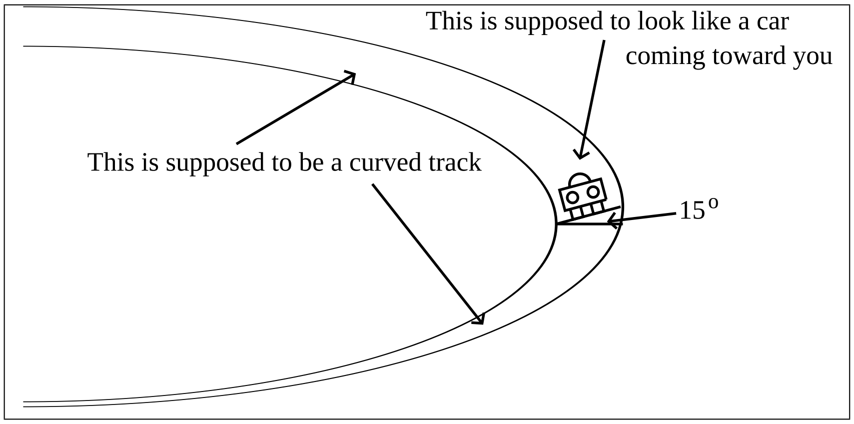


Figure 22.1: A diagram showing a car going around a curve banked at 15°.

In this case, we have the speed but we don't have the period. To use equation 22.2, we need the period.

Fortunately, we know that with circular motion the speed is equal to the circumference divided by the period, $v_{\text{circ}} = (2\pi r)/T$, and we can use that to get what the period would be if the car would go all the way around. When I do that (plugging in 30 m/s for v_{circ} and 200 m for r), I get a period T equal to 41.89 s. Now that I have the period, I can use equation 22.2, $a_{\text{cent}} = (2\pi v_{\text{circ}})/T$, to get the acceleration. Plugging in 30 m/s for v_{circ} and 41.89 s for T, I get an acceleration of 4.5 m/s^2 (inward). The car has a larger acceleration than I do (when running around a track) because the car is turning more quickly.

✓ *Checkpoint 22.3: A 30-kg dog runs around a circular track of radius 100 m at a constant speed of 5 m/s. What is the magnitude of the dog's acceleration?*

22.4 Obtaining forces from motion

If we are told the motion of an object, we can find the acceleration as described above and then use the force and motion equation to figure out the net force that must be acting. Since we must first find the acceleration, we'll use the version of the force and motion equation that equates the acceleration with the net force per mass ($a = F_{\text{net}}/m$).

For example, consider the scenario discussed before where a car goes around a banked curve of radius 200 m at 30 m/s (see Figure 22.1). Suppose the car has a mass of 1000-kg. What is the net force acting on the car?

The physics tells us that there must be a force imbalance because otherwise the car would not go in a circle. The net force in this case must be directed toward the center of the circle.

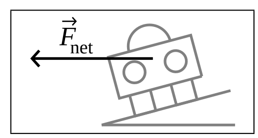
From before, we know that the acceleration is 4.5 m/s^2 inward. We can then use the force and motion equation to get the net force. Taking the acceleration and multiplying by the mass, I get 4500 N, inward, which corresponds to the force exerted on the car by the road.

✓ *Checkpoint 22.4: A 30-kg dog runs around a circular track of radius 100 m at a constant speed of 5 m/s. What is the net force acting on the dog?*

Note that the process described so far gives us the magnitude of the net force. We already know that the direction is toward the center of the circle.

However, sometimes it can be a little tricky to identify which direction is really toward the center of the circle.

The illustration at right shows the direction of the net force in the case described above. The net force arrow is directed leftward because that is the direction the car is turning toward.



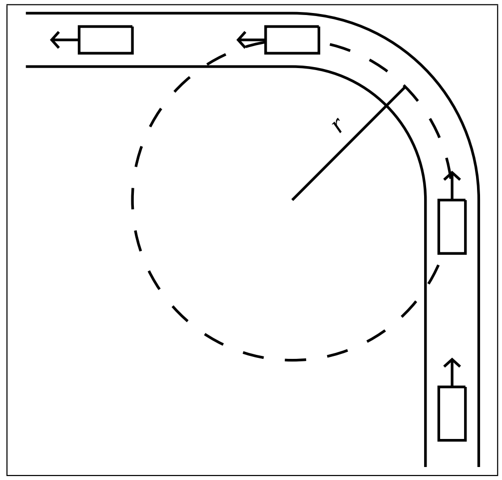
✓ *Checkpoint 22.5: Why is the inward direction not exactly parallel to the surface of the road in the case described above?*

22.5 Obtaining motion from forces

Now that we've seen how we can the net force that must act on an object to make it move in a circle, let's now examine the reverse process, where we are given the net force and from that determine information about the motion.

In general, the process will be the reverse of what we did before. Rather than starting with the motion to find the acceleration then use acceleration to find the net force, we'll do the steps in reverse order.

To illustrate the process with numbers, consider the scenario illustrated to the right, where a car goes around a curve (drawn as though we are looking down on the situation). Assuming the road is level (horizontal) and the friction on the car is 12,000 N, what is the smallest radius for the curve (r in the figure) such that a 1000-kg car can safely follow the curve at a speed of 30 m/s?



Before using any numbers, let's consider the physics.

The only reason the car can follow the curve is because friction acts with a direction that is inward (to center of circle, indicated by the dashed circle in the figure). The larger friction is, the tighter the curve can be.

First we use the force and motion equation ($a = F_{\text{net}}/m$) to get the acceleration. Plugging in 12,000 N for the net force and 1000 kg for the mass, I get 12 m/s^2 for the acceleration. We can then use that value in the a expression in equation 22.2 ($a_{\text{cent}} = 2\pi v/T$) to get the period (how long it would take for the car to go all the way around the circle). Plugging in 12 m/s^2 for the acceleration and 30 m/s for the speed, I get 15.7 s for the period.

How does this help us – we want to know the radius of the circular path?

To get the radius of the circular path, we need to use the v expression in equation 22.2 ($v_{\text{circ}} = 2\pi r/T$). Plugging in 30 m/s for the speed and 15.7 s for the period, I get 75 m for the radius.

✓ *Checkpoint 22.6: Consider the situation described above, where a car is going around a curve at 30 m/s. Suppose the friction is only 10,000 N instead of 12,000 N. Would that mean a smaller radius is possible or would a larger radius be needed? Explain.*

22.6 Orbits

Orbits, like the orbit of Earth around the sun, are examples of circular motion and so we know that there must a force acting that is directed toward the center of the orbit. For orbits, that force is the **gravitational force** (on the moon due to Earth).[vii]

❢ For an object in orbit, the gravitational force (pulling the object toward the center of the orbit) is what keeps the object in orbit.

Even though the moon and Earth are far away from each other, their masses are large. That leads to a large force, which is responsible for keeping the moon in orbit around Earth.

Since we can calculate the gravitational force, we can use that along with what was discussed in the previous section to find out things about the orbit, like the radius (which is how far the moon is from Earth). For example, from Earth, we can measure T, the time it takes for the moon to complete one orbit. This happens to be 27.32 days.[viii] According to NASA, the moon is 3.844×10^8 m from Earth. Let's use what we know to find Earth's mass.

First, we can use the radius of the orbit (3.844×10^8 m) and the period (27.32 days) to find the moon's speed ($v_{\text{circ}} = 2\pi r/T$). This gives a value of 1023.22 m/s. Remember to convert the period to seconds before plugging in.[ix] Otherwise, the units won't work out.

Then, we can use the moon's speed (1023.22 m/s) and the period (27.32 days) to find the moon's acceleration ($a_{\text{cent}} = 2\pi v/T$). This gives a value of 2.72367×10^{-3} m/s^2.

From the force and motion equation, we know that the net force on the moon must be the moon's mass times its acceleration (since $a = F_{\text{net}}/m$). We don't know the moon's mass but that is okay because we don't really need it. After all, the force is due to the the moon's gravitational attraction to Earth, must like why we fall to Earth when we jump up. And, just how

[vii] The moon also interacts with the sun. We are ignoring the sun because the gravitational force due to the sun is responsible for the moon orbiting, with Earth, around the sun, not the moon's orbit around Earth. This is why our analysis is limited to the moon's interaction with Earth only.

[viii] The time from one full moon to the next, which is called a lunar month, is about 29.53 days. This is longer than the orbital period because the Earth has continued to move around the Sun during that time and so the moon has to complete more than one orbit around Earth in order to be seen as a full moon again.

[ix] Multiply by 24 hour/day and then multiply by 3600 seconds/hour.

we accelerate toward Earth at 9.8 m/s^2 because Earth's gravitational field strength here is 9.8 N/kg, the moon's acceleration of 2.72367×10^{-3} m/s^2 toward Earth means that Earth's gravitational field strength at the moon must be 2.72367×10^{-3} N/kg.

In chapter 13, we learned that Earth's gravitational field strength depends on its mass and how far we are from Earth's center (equation 13.3):

$$g = G\frac{M}{r^2}$$

Since we know Earth's gravitational field strength at the moon ($g = 2.72367 \times 10^{-3}$ N/kg) and we know how far the moon is from Earth ($r = 3.844 \times 10^8$ m) and we know the value of G (always 6.67430×10^{-11} N $\cdot$ m^2/kg^2), we can plug these values in and solve for Earth's mass M. This gives a value of 6×10^{24} kg, very close to NASA's reported value.[x]

FOR OBJECTS IN ORBIT, WILL THE NET FORCE ALWAYS COME FROM THE GRAVITATIONAL FORCE DUE TO EARTH?

No. Each case is different. It depends on what object is at the center of the orbit.

✓ *Checkpoint 22.7: Earth travels in a nearly uniform circular path as it orbits the sun with a period of about 365.2425 days. The sun is pretty close to the center of the circle and the distance from the sun's center to Earth's center is* 1.496×10^{11} *m. Use this information to find the mass of the sun. Compare your answer to the value given in the supplemental readings.*

Summary

This chapter examined how angular and circular quantities are related.

The main points of this chapter are as follows:

[x] It is not *exactly* the same as NASA's reported value because we assumed the the Earth-Moon distance was equal to the radius of the moon's orbit. Earth is not exactly at the center of the moon's orbit, which means that the radius of moon's orbit is not exactly equal to the Earth-Moon distance. The radius of moon's orbit is actually slightly less.

- To obtain the measurement around the circle, multiply the angular measurement in radians by the radius (of the path, not the object).
- If the relationship does not involve a length, you can use any unit for the angle, as long as the same unit is used for each angular quantity.
- For objects undergoing uniform circular motion, the object's speed is constant but the object is still accelerating because its velocity is changing directions.
- An object moving in uniform circular motion has a inward acceleration with a magnitude equal to $2\pi v_{\text{circ}}/T$.
- For an object in orbit, the gravitational force (pulling the object toward the center of the orbit) is what keeps the object in orbit.

Terminology introduced

Centripetal acceleration
Dimensionless
Orbits
Rotation axis

Frequently Asked Questions

How can the gravitational force due to Earth keep the moon in orbit? Isn't the moon too far away for Earth to exert any gravitational force on the moon?

Even though the moon is pretty far away, the gravitational force on it (due to Earth) is still significant because both masses are very large. See page 378.

Additional problems

Problem 22.1: The radius of Earth is 6,370 km and rotates once per day around its axis. How fast does an object on Earth's equator move around the circle?

Problem 22.2: Suppose we have an object undergoing uniform circular motion of radius 2 m and period of 3 seconds. What is the magnitude of the object's acceleration (toward the center of the circle)?

Problem 22.3: A hoop is rotating such that it makes one complete revolution every 2 seconds. If the hoop has a radius of 1.5 m, what is the speed of a point on the hoop?

Problem 22.4: The second hand on a clock is 15 cm long and makes one full revolution every minute.
(a) Does each part of the clock's second hand rotate around the center of the clock face with the same rotation rate of one full revolution every minute? If not, why not?
(b) Does each part of the clock's second hand follow the same circular path (i.e., with the same radius)? If so, what is the second hand's speed along that path? If not, why not?

Problem 22.5: A rotating platform (see, for example, Figure 20.1 on page 342) is rotating at a constant speed. If a person, located 2 m from the center, moves one-quarter of the way around in 5.0 seconds, what is the person's speed?

Problem 22.6: Suppose a platform started at rest and then experiences an angular acceleration equal to 1 rad/s^2. Suppose we track a point on the platform that is 2 m from the center. How far (around the circle) would the point be when it reaches a speed of 4 m/s?

Problem 22.7: A rotating platform (see, for example, Figure 20.1 on page 342) is rotating such that it makes one complete revolution every 2 seconds. It then starts spinning faster and faster such that 3 s later, it is making one complete revolution every 1.25 seconds. A person is located 2.5 m from the center. Use the results from checkpoint 21.4 to answer the following questions. Remember to first convert the angular values into radians.
(a) What is the average acceleration of the person around the circle during the 3 seconds?
(b) How far did the person move around the circle during the 3 s?

Problem 22.8: Suppose you want an expression for the acceleration of an object in circular motion ($a = 2\pi v/T$) but you want it in terms of angular quantities instead of circular quantities. Show that it can be written as $a = \omega^2 r$.

Problem 22.9: An old record player plays 78's (i.e., records that are to be played at 78 RPM, where RPM means revolutions per minute). A penny (about 3 g) is placed near the rim of the record as it is playing (5 inches from the center of the record). At 78 RPM, this means that the penny has a speed of 103.7 cm/s.
(a) The penny's acceleration must be inward. What is the magnitude of this acceleration? Make sure your units work out!
(b) What is the magnitude of the net force on the penny, directed to the center of the record?

Problem 22.10: Assume the moon goes around Earth in uniform circular motion[xi] with a radius equal to 3.844×10^8 m and a period of 27.32 days (mean orbital period[xii]; see the supplemental readings). What is the moon's acceleration toward Earth?

Problem 22.11: A geostationary satellite is one that orbits Earth exactly once each day. When located above the equator, such a satellite will stay over one spot as it orbits, thus allowing a satellite dish to remained focused on one spot, regardless of the time of day. How far from the center of Earth must such a satellite be located? Hint: you'll need the mass of Earth (look it up) but you don't need the mass of the satellite.

Problem 22.12: (a) Assuming you are at rest right now with respect to the floor, what is the magnitude and direction of your acceleration, keeping in mind that you are actually moving in a circle as Earth turns on its axis.
(b) Given the acceleration in part (a), what is the magnitude and direction of the net force acting on you?
(c) Use $\vec{F}_{\rm g} = m\vec{g}$ to calculate the magnitude and direction of the gravitational force on you.
(d) Explain why the magnitude of the net force on you is so different from the magnitude of the gravitational force on you.

Problem 22.13: A 1000-kg car attempts to go around a banked curve that has a radius of 200 m (see Figure 22.1). Assuming the road is *frictionless*

[xi]For our purposes, we can assume that the moon goes in uniform circular motion even though it is not exactly true. The moon's distance from Earth varies slightly during its orbit. It speeds up as it moves closer, and it slows down as it moves away. However, the changes are not significant for our purposes.

[xii]The time from one full moon to the next full moon (i.e., a lunar month) is closer to 29.5 days. During this time, Earth has moved in its orbit around the sun, so the moon has to complete more than one full revolution around Earth to be opposite the sun again.

because of ice, how fast should the car go around the curve if the road is banked at an angle of 15°? Note: Even though the surface is inclined at an angle, choose component directions that are horizontal and vertical since the acceleration is horizontal (toward the center of the circle).

Problem 22.14: A 50-kg boy sits on the edge of a polished wooden disk. The disk has a radius of 3 m and the coefficient of friction between his pants and the disk is 0.3.
(a) What is the maximum friction force that can be applied by the disk?
(b) If the disk is rotated such that it takes 6 seconds for it to go all the way around, will the boy slide off?
(c) Does your answer to (c) depend upon the mass of the boy? Explain.

23. Predicting Rotational Motion

Puzzle #23: Why is it easier to start a hammer spinning (or stop a hammer from spinning) when spun around the hammer head rather than around the end of the handle?

Introduction

In this chapter, we apply the law of force and motion to rotation.

Throughout this book, every situation we encounter has some aspects that are similar to what we've seen before and other aspects that are different. The challenge in each case is to recognize that the differences aren't crucial and to use the commonalities to solve the problem.

With rotation, the problems may look different than what we've examined in parts A and B but the basic relationship between force and motion is the same in that a force imbalance is needed to change the motion. The difference is that with rotation we use rotation-specific terms for describing the rotational motion and how to change that rotational motion.

23.1 Torque and rotational motion equation

From the law of force and motion, we know that a force imbalance is needed to change an object's motion. What is needed to change an object's *rotation rate*?

To answer this, we can use something equivalent to the force and motion equation but with *rotational quantities* that correspond to velocity, force and mass.

From chapter 21, we already have the rotation-specific terms for describing rotational motion. In particular, we use terms like *angular* velocity and *angular* acceleration. In this chapter, we introduce the rotational equivalents of force and mass, namely *torque* and *rotational inertia*, respectively. Mathematically, then, instead of the force and motion equation,

$$(\text{change in velocity}) = \frac{(\text{net force})}{(\text{mass})}(\text{change in time})$$

we use the torque and rotational motion equation,

$$(\text{change in angular velocity}) = \frac{(\text{net torque})}{(\text{rotational inertia})}(\text{change in time})$$

Notice how the two equations are similar except that the second uses the rotational equivalents of velocity, force and mass.

To illustrate, let's consider the following situation.

> A circular plate has a rotational inertia equal to 100 $\text{kg} \cdot \text{m}^2$ about its center. If it starts at rest, how fast will it be spinning if a torque of 10 N·m is applied for 10 seconds?

According to the torque and rotational motion equation, the applied torque results in a change in angular velocity, just as in the force and motion equation an applied force results in a change in velocity. To find the change in angular velocity, we plug in the values that are given.

$$(\text{change in angular velocity}) = \frac{10\ \text{N} \cdot \text{m})}{(100\ \text{kg} \cdot \text{m}^2)}(10\ \text{s})$$

This gives a change in angular velocity equal to 1 rad/s.

There are two things I want to point out about what I've done.

First, notice that the units for angular velocity, torque and rotational inertia are *not* the same as the units for velocity, force and mass. In this chapter, I'll explain why they are what they are.

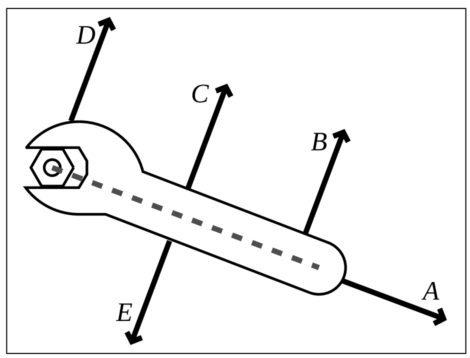
Second, while it is straightforward to plug values into an equation, that doesn't mean those terms have an meaning to us. In this chapter, I'll discuss what torque and rotational inertia mean so that you can get an intuitive feel for how they impact the rotational motion.

✓ *Checkpoint 23.1: (a) When predicting an object's change in angular velocity, what quantities do we use instead of force and mass? (b) Do those quantities have the same units as force and mass?*

23.2 Torque

23.2.1 Directions and the radial line

To make something spin faster or slower, you might think that we just need to exert a force on the object. However, it isn't as simple as that.



To see why, consider how a wrench is used to loosen a nut. A wrench is a tool designed to help loosen or tighten nuts. In the figure, the nut (the hexagon) sits on a bolt (the circle inside the hexagon). The arrows represent different forces that will be discussed shortly. All five forces have the same magnitude but not the same directions nor the same application point.

Suppose the nut is free to spin around the bolt (which, in turn, is fixed to some other object and not free to move). Which of the forces indicated would make the wrench unscrew the nut?

To answer that question, we first have to recognize that, to unscrew the nut, we need to get the nut rotating in the counter-clockwise direction.[i] To change the rotation rate in the counter-clockwise direction, we need to apply a force in the counter-clockwise direction.[ii]

For the situation shown in the figure, only the forces represented by arrows B and C would do what we want, which is to make the wrench rotate counter-clockwise. In comparison, the force represented by arrow E is acting in the

[i]Perhaps you have heard of the phrase "righty tighty lefty loosey" or something similar that people use to remember which way it has to be rotated.

[ii]If friction is present (which makes the nut hard to loosen) then that friction applies a clockwise torque, opposing the counter-clockwise torque that is being applied to loosen it.

opposite direction, clockwise around the rotation axis, and thus would act to tighten the nut.

WHAT ABOUT THE FORCE REPRESENTED BY ARROW A?

Arrow A represents a force that is not in the correct direction. Force A simply pulls the wrench off the nut, and doesn't change its rotation rate.

WHAT ABOUT THE FORCE REPRESENTED BY ARROW D?

Force D doesn't do anything because it is applied right at the nut. It would be like trying to open a door by pushing on the hinges.

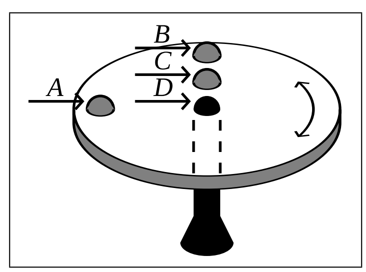
To determine whether a force will act to change an object's rotation rate, it is important to first identify the **rotation axis**, the imaginary line around which the object spins (or potentially spins). In this case, the rotation axis is located where the bolt is (in the center of the nut).

Forces acting right at the rotation axis (like force D) will not act to change the object's rotation rate.

For those forces not acting at the rotation axis, the force needs to act "around" the rotation axis. To determine whether a force is acting "around" the rotation axis, draw a line from the rotation axis to where the force is acting. That "radial" line is indicated by the dashed line in the figure.

Once the **radial line** is drawn we can identify whether the force is acting to change the object's rotation rate or not by comparing the direction of the force to the orientation of the radial line.[iii] For example, force A is directed parallel to the radial line and thus will not act to change the nut's rotation rate. Force B, on the other hand, is directed perpendicular to the radial line and thus will act to change the nut's rotation rate.

To illustrate the same concept in another context, consider the figure on the right, showing a platform that is free to rotate around the support at its center. Although the all the forces represented by arrows are directed rightward, only forces B and C act to change the platform's rotation rate.



[iii]The technical name for the radial line is the **radial arm**. I'm using radial "line" because it is too easy to misinterpret radial arm as meaning a real, physical thing rather than some imaginary line that is drawn from the axis to the point of force application.

The force represented by arrow A is directed *inward* (toward the rotation axis) and thus does not make the platform spin faster or slower. The force represented by arrow D is directed right at the rotation axis and likewise does not make the platform spin faster or slower.

Notice that the radial line orientation depends on where the force is acting. The radial line for force A extends from the platform center (rotation axis) to the left of the platform. In comparison, the radial line for force B extends from the platform center to the top of the platform.

Regardless, to get an object to rotate faster or slower, the force needs to be directed around the rotation axis (i.e., *perpendicular* to the radial line that is drawn from the rotation axis to the location of that force). Forces B and C are examples of forces that are directed around the rotation axis (rather than toward or away from the rotation axis).

✓ *Checkpoint 23.2: Suppose you had a 10-kg platform (like that in the figure) of radius 1 m.*
(a) Of arrows A, B and C, which are directed around the rotation axis?
(b) Of arrows A, B and C, which are directed perpendicular to the radial direction at each location, respectively?

23.2.2 Torque definition

We want to use the law of force and motion to predict when something will start rotating and when it won't (or, more generally, when something will spin faster or slower and when it won't).

In order to apply the law to rotating objects, though, we need to recognize that when dealing with rotation, it isn't the *force* that is of interest but rather something called the *torque*. This is because just knowing the force does not tell us if the rotation rate will change. It also depends on the *direction* of the force (as discussed in the previous section) and *how far* the force is applied from the rotation axis.

To illustrate the dependence on how far the force is applied from the rotation axis, compare the forces indicated by arrows B and C in the wrench figure (on page 387). Both forces are directed around the rotation axis (i.e., perpendicular to the radial direction), so both should act to speed up or slow down the rotation rate. However, it turns out that force B, by being further away from the axis of rotation, is more effective at making the nut spin faster or slower.

For the same reason, it is easier to open a door the further you push from the hinges. You can't open a door by pushing right where the hinges are.

❢ The torque depends on how far from the axis the force is being exerted.

Thus, it depends both on the force (assuming it is directed around the rotation axis) and on how far the force is applied from the rotation axis (i.e., the length of the radial line to that location).

✓ *Checkpoint 23.3:Door handles are usually placed far from the hinges of the door. In terms of torque, what might be the advantage of doing so?*

The **torque** represents the effect of *both* of these things: the component of the force around the axis (which I'll indicate as F_{circ}) and the distance from the rotation axis to the point where the force is applied (which we indicate as r).

Why does F_{circ} have a "circ" subscript?

As in prior chapters, I'll use a subscript "circ" to indicate a quantity that is directed "around" the circle rather than into toward the center (or away from the center). To exert a torque, the force must be directed around the rotation axis (rather in toward the axis or away from axis) and so the torque depends on F_{circ}.

❢ Torque is defined as the product of F_{circ} and r.

How does the torque depend upon F_{circ} and r?

The torque is simply the product of the two. In mathematical equations, we represent the torque by the Greek letter τ: [iv]

$$\tau = F_{\text{circ}} r \tag{23.1}$$

Why is torque represented by τ?

It is a lower-case Greek letter tau. As mentioned before, the convention is to use lower-case Greek letters for rotational quantities. In this case, the Greek letter is like a little Roman letter T.

[iv] The torque used to be called the *moment force*, where "moment" refers to the "tendency to twist" rather than a "moment in time." It is better to call it torque, since the units are different than force.

✍ The Greek letter τ also looks a little like the lower-case Roman letter r so be careful not to confuse the two.

What units does the torque have?

Torque is equal to the product of a force and a distance. We are using the SI units for force (N) and distance (m), so the SI unit for torque is N·m.

While torque is analogous to force in a way, it is not the same. Force by itself doesn't tell us whether the object's rotation rate will change. We also need to know how far from the axis of rotation the force is applied. For the same reason, the unit of torque is not just N but rather N·m.

For example, in the illustration of the wrench on page 387, there are three forces all in the same direction, indicated by arrows B, C and D. Of these three, the force represented by arrow B will exert twice the torque as arrow C because force B is twice as far from the rotation axis as force B. In other words, using a longer handle will produce a greater torque for the same amount of force. Force D, in comparison, doesn't produce any torque at all, since it is applied right at the rotation axis.

Example 23.1: Suppose you had a platform like that in the figure on page 388. If we exert a force of magnitude 10 N in the manner indicated by arrow B, at a point 0.8 meters from the rotation axis, what is the torque on the platform?

Answer 23.1: The force is applied a distance 0.8 m from the rotation axis so r is equal to 0.8 m. Since the force is directed around the rotation axis, we can use the entire magnitude of 10 N. By multiplying the magnitude of the force by r (see equation 23.1), we get an applied torque of $8 \text{ N} \cdot \text{m}$ (in the clockwise direction).

Notice how we didn't need to know the radius of the platform in the example. The torque depends on where the force is applied on the object, not the radius of the object.

✍ Note that I am using r to represent the length of the radial line (i.e., the distance from the rotation axis to the location where the force is being applied). It does not represent the radius of the object. In general, I'll use upper-case R to indicate the radius of an object, and lower-case r to indicate a general distance from the center (as with the radial line). Hopefully this makes it clear which I am referring to.

✓ *Checkpoint 23.4: Consider a platform like that in the figure on page 388. Suppose the platform had a mass of 10 kg and a radius of 1 m.*
(a) What is the torque on the platform when a 10-N force is applied in the manner indicated by arrow C in the figure, 0.4 meters from the rotation axis?
(b) What is the torque on the platform when a 10-N force is applied in the manner indicated by arrow A in the figure, 0.8 meters from the rotation axis? Note that the force (indicated by the arrow at A) is directed inward (i.e., in toward the axis of rotation).

23.2.3 Net torque

One consequence of the law of force and motion is that an object's velocity (speed and direction) will not change if the net force on the object is zero. This idea is known as the law of inertia.

When applied to rotation, it means that an object's rotation rate will not change if the net *torque* on the object is zero.

❢ For an object's rotation rate to remain constant, the net torque on the object must be zero. Consequently, the clockwise torques must balance the counterclockwise torques.

In other words, it is the net torque that is associated with whether an object's rotation rate will change or not. To determine the net torque, we need to add up all of the torques that are acting.

Do we add torques the same way we add forces?

Yes, as long as you pay attention to the direction of the torques, rather than the directions of the forces. They are not the same thing.

Torque, like force, is a vector and thus it has a direction.[v] Like adding any other vector, we can't simply add the magnitudes of the torques and expect to get the net torque.

On the other hand, torque directions are circular (like clockwise and counterclockwise) instead of straight (like north and south). We can still use positive and negative to indicate opposite directions but we need to keep in mind that the directions are *circular*, not straight.

To illustrate, consider the following scenario:

[v] For this reason, many people put an arrow on top, like $\vec{\tau}$.

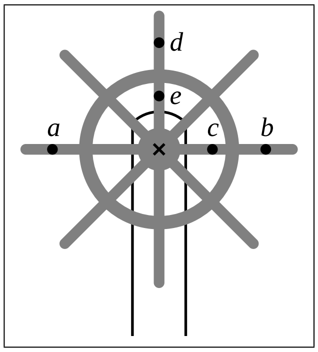


Figure 23.1: An illustration of a captain's or ship's wheel that is used to steer a sailing ship.

> A wheel like the one shown in Figure 23.2.3 is used to steer a ship. Suppose a downward force of magnitude 20 N is applied at the point indicated as a in the figure. What force would be needed to be applied at point b, equidistant to the rotation axis (indicated by the $\times$) but on the other side, so that zero net torque is applied to the wheel?

This is a rather simple case. What makes it simple is that the two forces are being exerted at locations that are the same distance from the rotation axis.

Since the distances are the same, the magnitudes of the forces must be the same in order to make magnitudes of the torques the same.

However, what about the directions?

It turns out that the force at b must also be downward.

To see why, keep in mind that torque directions are circular (like clockwise and counter-clockwise) instead of straight (like upward and downward). In this case, the force at a is applying a counter-clockwise torque about the rotation axis. You can see that by imagining what that force would do to the wheel if it, alone, was acting.

In order for the net torque to be zero, the force at b must be applying a clockwise torque (opposite the torque being applied at a). That means the force at b must be downward.

✍ Mathematically, we could treat one direction as positive and he other as negative, in which case one of the torques would be positive and the other negative.

Now let's suppose we want to apply a force at c (half the distance to the rotation axis) that counters the force at a. What magnitude and direction would be needed?

In that case, the direction would still be downward, to produce a clockwise torque, as before. However, since it is half the distance to the rotation axis, the force at c would have to have twice the magnitude (i.e., 40 N).

Does that mean the torque at c is 40 N?

No. Torque has units of newton-meters, not newtons. This is because torque is the product of the force directed around the pivot and the distance from the pivot to where the force is applied.

In order to figure out the torque, we'd have to know the distance from the force to the axis.

Suppose, for example, that point a is 40 cm from the rotation axis. With a downward force of magnitude 20 N applied at that point, the torque is 8 N·m (or 800 N·cm) in the counter-clockwise direction. Multiply 20 N by 40 cm to get the 800 N·m, or multiply 20 N by 0.4 m to get 8 N·m.

In comparison, with a similar downward force at point b, also 40 cm from the rotation axis, the torque would be 8 N·m (or 800 N·cm) in the clockwise direction.

✓ *Checkpoint 23.5:Suppose a downward force of magnitude 20 N is applied at the point indicated as a on the wheel shown in Figure 23.2.3. If point a is 40 cm from the rotation axis, identify the magnitude and direction of the force required at points d and e such that the net torque on the wheel is zero.*

23.3 Law of interactions (rotation version)

Before showing how torque is used with the force and motion equation, let's first see how it is used with the law of interactions. When applied to rotations, the law of interactions can be stated as follows: *you cannot exert a torque on an object without that object exerting an equal but opposite torque on you.*

Does this mean that whenever I spin a top in a clockwise direction, I should spin counter-clockwise in response?

Yes, but there are two reasons why you don't end up spinning every time you try to spin something like a top. For one, you are much bigger than the top so, for the same torque, the effect will be much greater on the top.

Another reason why you don't spin (even a little bit) is because of friction with the ground, which prevents you from spinning. If you were on a frictionless surface (like ice), however, you would. Indeed, if you stood on a platform that was free to rotate, we would see that you would not be able to make an object rotate without making yourself rotate also (in the opposite direction; assuming both are at rest initially).

This is more apparent if you take something bigger like a bicycle wheel. You'll feel it resisting any change. And, if you try to make it spin by exerting a torque on it, the wheel will exert a torque on you, making you spin in the opposite direction.

✍ Keep in mind that we are looking at torques. You need to apply a torque to start something spinning but you also need to apply a torque to stop something from spinning, or make something spin faster or slower.[vi]

✓ *Checkpoint 23.6: Suppose I stand at rest on a platform that is free to rotate and I hold a bicycle such that the wheel axis is vertical. If I make the wheel spin clockwise (as seen by someone above me looking down) by exerting a clockwise torque on it, in what direction will I spin (given that I started at rest)? I will spin in the direction of the torque exerted on me due to the wheel.*

23.4 Rotational inertia

WHAT HAPPENS IF THE NET TORQUE IS NOT ZERO?

If there is a torque imbalance, the object will either start to rotate (if it is not rotating initially) or it will rotate faster or slower.

[vi] You also need to apply a torque to reorient an object's axis of rotation. For example, if you hold a spinning bicycle wheel and then flip it over, it will be spinning in the "opposite" direction. Technically, its rotation rate is now different, the result of you applying a torque on it. In such a situation, the wheel is likewise exerting a torque on you, in the opposite direction. The effect is very noticeable – you should try it.

To determine *how much* faster or slower it will rotate, we need to use the **rotational inertia** of the object.[vii] The larger the object's rotational inertia, the *harder* it is to spin up and, similarly, the *harder* it is to spin down. On the other hand, the larger the object's rotational inertia, the *easier* it is to "keep the object spinning" against opposing torques, like friction.

The rotational inertia is basically the rotational analog for mass. Just as mass tells us how how easy or hard it is accelerate an object, the rotational inertia tells us how easy or hard it is to spin up or spin down an object. In addition, just as an object's mass is not dependent on whether it is moving or not, an object's rotational inertia is not dependent on whether it is spinning or not.

However, the rotational inertia is *not* just dependent on the object's mass. It also depends on how the mass is distributed about the rotation axis.

For example, a heavy object is harder to start or stop rotating than a light object, but how hard it is to start or stop an object rotating also has to do with *where* the mass is relative to the rotation axis. That is why it is easier to start a hammer spinning or stop a hammer from spinning when spun around the hammer head than around the end of the handle (see puzzle).

In section 23.4.3 I will show you how to determine the rotational inertia of an object (given its size and shape). For now, we'll assume that you are given the value of the rotational inertia.

23.4.1 Units

Since rotational inertia depends not only on the mass but also on how far the mass is from the rotation axis, rotational inertia will not only have mass units (e.g., kg) but distance units as well (e.g., m).

In terms of kilograms and meters, rotational inertia has units of kg·m^2.

The fact that the units include both mass units (kg) and distance units (m) helps us remember that the rotational inertia depends on both the mass of the object and its shape (relative to where the rotation axis is).

[vii]Some people call this the **moment of inertia**. In a similar manner, torque used to be called the *moment force*.

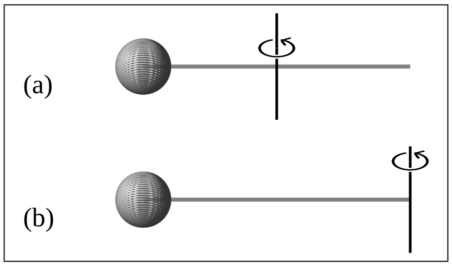


Figure 23.2: An sphere placed on the end of a massless rod and rotated about (a) the middle of the bar and (b) the end of the bar.

23.4.2 Notation

In equations, we use I as the abbreviation for rotational inertia. I know that is pretty weird. Fortunately, when we use the torque and rotation equation, all we have to remember is to use rotational inertia just like mass.

In other words, just as the regular mass represents an object's "resistance" to changes in its velocity (i.e., a massive object requires a large force to change its velocity), the rotational inertia represents the object's "resistance" to changes in its rotation rate.

23.4.3 Dependence on mass location

As noted above, the rotational inertia has units of both mass (kg) and distance (m) because rotational inertia depends on both the mass of the object and its shape (relative to where the rotation axis is).

To gain a better idea of what this means, consider the following:

> A sphere is connected to the end of a massless rod as shown in Figure 23.2. Suppose we want to spin the rod (with the sphere on it). Would it be easier to spin it about an axis in the middle of the rod (see part a of the figure) or about an axis coincident with one end of the rod (see part b of the figure)?

At first glance, you might think it wouldn't make any difference. However, it turns out that it is easier to spin the bar about its center (part a of the figure).

The difference has to do with different rotational inertias. The rotational inertia not only depends on the mass and but also how far that mass is from the rotation axis.

In both cases, the mass is the same, so the difference in rotational inertia is not due to the mass. Rather, it is due to where the mass is relative to the rotation axis.

The sphere (and its mass) is closer to the rotation axis in (a) than in (b). Consequently, the rotational inertia is smaller in (a). A smaller rotational inertia means it is easier to change the rotation. In other words, for the same net torque, spinning about the center of the rod would result in a larger change in the rotation rate.

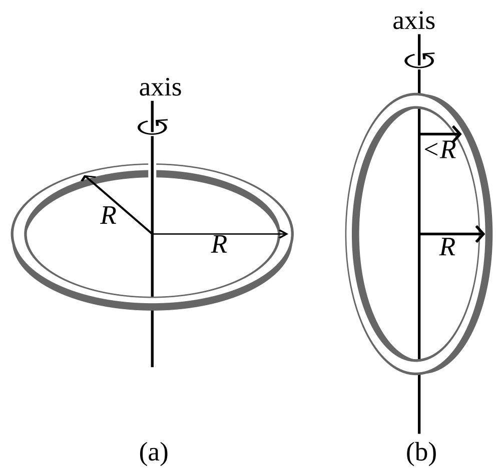
Notice how the rotational inertia does not depend on whether the object is spinning or not. It only depends on the object's mass and how close the object is to the rotation axis.

What if there is more than one object being spun or the mass is spread out among many points?

The answer depends on how the mass is distributed and how it is spun.

Consider, for example, the two hoops illustrated to the right. Suppose we want to spin each hoop.

Would it be easier (i.e., less torque) to spin the left hoop like a Frisbee about its center, as in part (a), or the right hoop like a top about an axis parallel to its diameter, as in part (b)?



It turns out it is easier to spin the left hoop like a top. The reason is that a hoop's rotational inertia is smaller when spun that way.

To see why, consider that when spun like a Frisbee, as in part (a), every part of the hoop is the same distance from the rotation axis. That distance is the radius R of the hoop.[viii]

In comparison, when spun like a top, as in part (b), each part of the hoop is <u>not</u> the same distance from the rotation axis. Only the parts at the middle

[viii] As mentioned on page 341, I use capital R to indicate the radius of an object, whereas I use a lower-case r to indicate a general distance from the center (as in the definition of torque).

are a distance R from the axis. The rest of the hoop is *closer* to the axis than R.

When spun like a top, then, the mass of the hoop is, in general, closer to the rotation axis and thus the hoop has a smaller rotational inertia. The lower rotational inertia when spun like a top means the hoop is easier to start or stop spinning when spun like a top.

✓ *Checkpoint 23.7: Suppose both hoops in the example were stationary. Would each hoop's rotational inertia around the axis (see the illustration) be zero in that case? If not, would they at least be equal?*

How do we compare rotational inertia when different parts are at different distances from the rotation axis?

You just have to break up the object into pieces, determine the rotational inertia of each piece and then add up all of the rotational inertias to get the total rotational inertia of the object.

For some objects this process requires calculus. However, even without calculus, one should be able to tell when the rotational inertia is greater or less, as in the case of the hoop rotated like a Frisbee or a top.

For example, consider a disk compared to a hoop:

> Suppose we had a hoop and a disk (like a coin or CD), both with the same radius and mass (note: with the same thickness, the disk would have to be less dense). Which is easier (i.e., requires less torque) to get spinning about an axis at its center, as shown in Figure 23.3?

It turns out that the disk is easier to start spinning (if the mass is the same) but its rotational inertia would be less.

To see why, consider that every part of the hoop is a distance R from the rotation axis, where R is the radius of the hoop. This is illustrated in Figure 23.3, where two points on the hoop are identified along with their distances from the rotation axis. Both distances are equal to the radius of the hoop.

In comparison, consider the disk. In Figure 23.3 I've also identified two points on the disk along with their distances from the rotation axis. Both distances

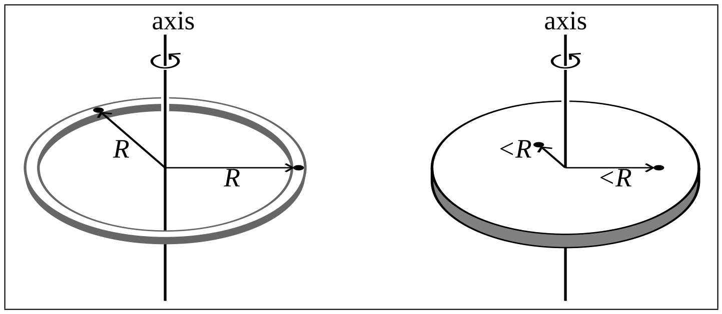


Figure 23.3: A hoop (left) and a disk (right) both spinning like a Frisbee (around an axis through their center).

are *less* than the radius of the hoop. Indeed, only the very edges of the disk happen to be a distance R from the rotation axis (not shown).

Consequently, the disk's rotational inertia is less than the rotational inertia of the hoop (when they are spun this way). This means the disk requires less torque to spin up or spin down.

A similar analysis can be done for cylinders, which are just a thick type of disk, and spheres. In fact, a similar analysis can be used to show that a hollow sphere will have a larger rotational inertia than a solid sphere if their mass and size are the same.

✓ *Checkpoint 23.8: (a) Suppose you had a hoop and a disk of the same size but the disk has a greater mass. Do they necessarily have the same rotational inertia when spun like a Frisbee? Why or why not?*
(b) Suppose you had two disks of identical size and mass, but one was spun like a Frisbee and one was spun like a top. Do they necessarily have the same rotational inertia? Why or why not?

23.5 Units revisited

The fact that the rotational inertia has units of mass (kg) and length (m) is consistent with the idea that the rotational inertia depends upon the mass and size of the object, as discussed before. But why is the length unit *squared*?

To understand why the length unit is squared, we need to examine the torque and rotation equation, written here using the abbreviations discussed earlier.

$$\Delta\omega = \frac{\tau_{\text{net}}}{I}\Delta t \tag{23.2}$$

where I is the rotational inertia. Notice that this has the same form as the force and motion equation:

$$\begin{aligned} \Delta\vec{v} &= \vec{F}_{\text{net}}\Delta t/m \\ \Delta\omega &= \tau_{\text{net}}\Delta t/I \end{aligned}$$

with ω instead of $\vec{v}$, τ_{net} instead of $\vec{F}_{\text{net}}$ and I instead of m.

❢ The law of force and motion remains the same for rotation except that the rotational analogs are used in place of the regular force, mass and velocity.

Before we get to actually using this equation, let's compare the units. On the left side, the original force and motion equation has the change in velocity, which has SI units of meters per second. The rotational version has the change in *angular* velocity, which has SI units of radians per second. Notice that the meters is missing.

On the right side, the units for Δt is the same for both expressions. However, torque has SI units of newtons times meters, so there is an "extra" meters. To counter the "extra" meters, plus remove an additional meters to match the left side, the rotational inertia must involve a *square* meters.

✍ One can also derive the rotational version from the original version but it is a little tricky. It is sufficient to recognize that we'd need to first re-write it for circular motion (i.e., Δv_{circ} and $F_{\text{circ,net}}$) then use the relationship between v_{circ} and ω that was shown on page 370 (see equation 22.1): $v_{\text{circ}} = r\omega$, which requires that the angle be in radians.

There are two important things to keep in mind.

First, since rotational inertia is more than just the mass, rotational inertia does not have the same units as mass does, just as angular velocity does not have the same units as velocity does and torque does not have the same units as force does.

Second, since the expression involves both angular and length units, we must use radians as the angular unit.[ix]

To illustrate how the units work together, consider the following scenario:

[ix] Otherwise the expression would involve additional constants, like π.

> A local playground has a large, uniform, circular platform (much like that shown on page 388) of radius 2 m and rotational inertia 1000 kg · m^2. If the platform starts at rest, how fast is it rotating at the end of 10 seconds if a constant force of 10 N is applied at its edge in a clockwise direction for the 10 seconds? Assume no friction at the pivot.

According to the law of force and motion, the platform's rotation rate will change because there is a non-zero net torque on it. The torque and rotation equation tells us how much the rotation rate will change given the torque.

In this case, the torque is not provided but we can figure it out given the force and the location where the force is applied (at the edge of the platform). The applied force has a magnitude of 10 N, is directed clockwise around the axis and is applied 2 m from the rotation axis. The torque, then, has a magnitude equal to the product of 10 N and 2 m, which is 20 N·m, and a direction that is clockwise around the axis.

Now we apply the equation of torque and rotation to find the change in angular velocity. Plugging in 20 N·m for the torque, 10 s for the time and 1000 kg·m^2 for the rotational inertia, I get a change in angular velocity equal to 0.2 rad/s.

Since the platform started at rest, the final angular velocity must be 0.2 rad/s.

How did you know that the angular velocity is in units of radians per second?

❢ When using the torque and rotation equation, angular quantities need to be in radians (such that the units work out).

If you keep track of the units, you'll find that the angular velocity has units of (m/m)/s or "(meters per meter) per second." The "meters per meter" part is equivalent to radians.

✍ As long as you use the same length unit for both torque (e.g., N·m) and rotational inertia (e.g., kg·m^2) then the change in angular velocity $\Delta\omega$ has units of rad/s. That is because the radian is essentially defined as a length per length (i.e., arc length divided by radius).

Note that in this case the platform started at rest, so the final angular velocity is equal to the change. As we know from other problems, the object need not start at rest. For example, suppose we have the same platform as before but the platform is initially spinning counter-clockwise (looking down) at a rate of 0.1 rad/s. How fast would it be rotating after the 10 seconds (with the same constant torque of magnitude 20 N·m clockwise applied)?

Since the torque, elapsed time and rotational inertia is the same as before, the change in angular velocity is the same magnitude as that calculated before (i.e., 0.2 rad/s clockwise). The only difference is that now it isn't at rest initially.

Given that it started with an angular velocity of 0.1 rad/s *counter-clockwise* and experienced a change in angular velocity of 0.2 rad/s *clockwise*, that means it must have an angular velocity of 0.1 rad/s *clockwise* at the end of the 10 seconds.

The process is pretty straightforward and matches the problems we've seen before with the law of force and motion. However, you need to be especially careful with units. Not only do the units have to be SI (like newtons, meters, kilograms and seconds) but the angle has to be in radians.

Example 23.2: Consider the same platform as in the previous examples (rotational inertia equal to 1000 $\text{kg} \cdot \text{m}^2$). Let's suppose, however, that it isn't frictionless. And, after getting the platform spinning clockwise at a rate of 0.3 rev/s, we find it takes a minute to slow down to a stop. What is the torque applied by friction?

Answer 23.2: In order to slow down, a torque is needed. The amount of torque is given by the torque and rotation equation.

However, we can't just plug in the elapsed time (1 min), the rotational inertia (1000 $\text{kg} \cdot \text{m}^2$) and the change in angular velocity (0.3 rev/s counter-clockwise; notice that it is counter-clockwise because it is slowing down). This is because the units don't all match.

First, we need to convert the time units into seconds. That is pretty easy, since 1 minute equals 60 seconds.

Second, we need to convert the angular units into radians. There are 2π radians in a revolution, so the initial rotation rate is

$$\begin{aligned} \omega_i &= (0.3\ \cancel{\text{rev}}/\text{s})(2\pi\ \text{rad}/\cancel{\text{rev}}) \\ &= (0.6\pi)\ \text{rad/s} \end{aligned}$$

which gives an initial rotation rate of 1.885 rad/s clockwise.

To find the net torque, then, we use the torque and rotation equation with an elapsed time of 60 s, a rotational inertia of 1000 $\text{kg} \cdot \text{m}^2$ and a change in angular velocity equal to 1.885 rad/s counter-clockwise. This gives a torque equal to 31.4 N · m counter-clockwise.

✓ *Checkpoint 23.9: A hoop with radius equal to 2 m and rotational inertia equal to* $0.5\ \mathrm{kg} \cdot \mathrm{m}^2$ *is originally at rest. Suppose we exert a clockwise force of 5 N on the hoop and, during the time the force is exerted, the hoop spins up to a rotation rate of 2 radians per second. How long does it take to reach that speed?*

Summary

This chapter examined how we apply the law of force and motion to rotating objects to predict whether a stationary object will rotate or not (or, if already rotating, whether it will rotate faster or slower).

The main points of this chapter are as follows:

- Torque is defined as the product of F_{circ} and r, where r is the length of the radial line (from rotation axis to where the force is applied) and F_{circ} is the force component perpendicular to the radial line.
- For an object's rotation rate to remain constant, the net torque on the object must be zero. Consequently, the clockwise torques must balance the counter-clockwise torques.
- The law of force and motion remains the same for rotation except that the rotational analogs are used in place of the regular force, mass and velocity.
- When using the torque and rotation equation, angular quantities need to be in radians (such that the units work out).

By now you should be able to use the torque and rotation equation, and interpret the meaning of torque and rotational inertia.

Frequently Asked Questions

Is torque the same thing as force?

No. See page 389.

How does the torque depend upon F_{circ} and r?

See page 390.

Why is τ used as the abbreviation for torque?

As discussed in chapter 21, the convention is to use Greek letters for rotational quantities. See page 390.

What units does the torque have? Is it the same as the units for force?

It has same units as a length times force. It is not the same as the units for force. See page 391.

Do we add torques the same way we add forces?

Yes. See page 392.

Does it matter which direction — clockwise or counter-clockwise — we define as positive?

No, as long as you specify which you are using and are consistent throughout a problem.

Does it matter whether we indicate the directions as clockwise and counter-clockwise or as positive and negative?

It doesn't matter for our purposes. However, if we really want to be careful, both are ambiguous since whether something is clockwise or counter-clockwise depends on one's perspective, and positive and negative are meaningless without being told what each corresponds to.

For example, consider someone holding a clock that is transparent except for the hands of the clock. Someone looking at the face of the clock would see the hands moving in a clockwise fashion. Someone looking at the back of the clock would see the hands moving in a counter-clockwise fashion.

Because of this ambiguity, it is more reliable to give the direction of rotation as the direction of the axis of rotation with the convention that a counter-clockwise rotation is directed "toward us" and a clockwise rotation is directed "away from us".

Why does the angular velocity have to be in radians per time when using the torque and rotation equation?

The angle in radians is equivalent to the ratio of the arc length to radius. That way, the units are consistent with the units used for torque and rotational inertia. See chapter 22 for more information.

What is the rotational inertia?

The rotational inertia represents the object's "resistance" to changes in its rotation rate. See section 23.4 for more information.

To be consistent, shouldn't we use a Greek letter for the rotational inertia?

Yeah, I guess, but then again maybe it is a Greek letter. I can't tell if it is the Roman letter "I" or the Greek letter "iota". They look the same.

Does the rotational inertia depends on the size of the object?

Yes. The further the mass is from the axis, the harder it is to accelerate the mass around the axis.

Is r the radius of the object?

As mentioned on page 341, lower-case r represents a distance and upper-case R represents the radius of the object. In this case, r is the distance from the rotation axis. For example, it may represent where the mass is or where a force is applied (as in the torque). In some cases, the value of r and the value of R may be the same but that doesn't necessarily need to be the case.

What units does the rotational inertia have?

The SI units for rotational inertia are $\text{kg}\cdot\text{m}^2$.

How does the rotational inertia depend upon where the axis of rotation is?

See section 23.4.3.

Does the the rotational inertia depend upon the radius of the object?

Yes and no. It depends on how far the object's mass is from the rotation axis. A larger object may have its mass farther from the rotation axis but that isn't necessarily the case.

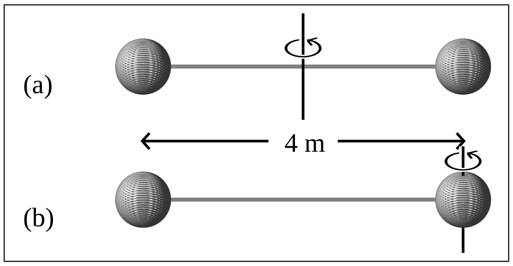


Figure 23.4: (a) Two objects, separated by 4 m (center to center) connected by a massless bar and rotated about a point midway between them. (b) The same two objects but rotated about an axis coincident with one of the objects.

Terminology introduced

Moment of inertia	Rotation axis
Radial arm	Rotational inertia
Radial line	Torque

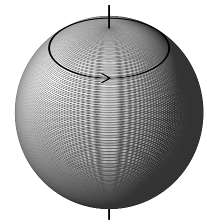
Additional Problems

Problem 23.1: A 0.2-kg hoop of radius 1.5 m is originally at rest. Suppose we exert a clockwise force of 5 N on the hoop and, during the time the force is exerted, the hoop spins up to a rotation rate of 2 radians per second. With a rotational inertia of 0.45 kg·m^2, how long does it take to reach that speed?

Problem 23.2: The rotational inertia has SI units of kg·m^2. This means that the distance (in meters) has more of an effect that the mass (in kilograms). Given this, consider the two identical 2-kg objects connected by a massless rod shown in Figure 23.4 such that the objects are 4 m apart (center to center). Suppose we want to spin the setup. Should it be easier to spin the rod about an axis midway between the two objects (see part a of the figure) or about an axis coincident with one of the objects (see part b of the figure)? Why?

Problem 23.3: Suppose a hoop had a radius of 0.5 m, a mass of 0.1 kg and was originally spinning like a Frisbee with a rotation rate of 2 rev/s. Given a rotational inertia of 0.025 kg·m^2, how much torque do we have to exert on the hoop to bring the hoop to a stop in 0.1 s?

Problem 23.4: Suppose a solid sphere, a disk and a hoop all had the same mass and radius. If spun around their centers, which of the three would have the smallest rotational inertia? Explain.



Problem 23.5: Suppose a solid sphere has a rotational inertia of 0.002 $kg \cdot m^2$ when set spinning at a rate of 2 rad/s counter-clockwise about an axis through its center (see figure to right). What torque would be needed to stop the sphere from spinning in 0.8 seconds?

Problem 23.6: A playground has a large, circular platform of radius 2 m and rotational inertia 1000 $kg{\cdot}m^2$ that can rotate around a frictionless pivot at its center.
(a) If the platform starts at rest, how fast is it rotating if a constant force of 10 N is applied at its edge and in a clockwise direction for 10 seconds?
(b) If the platform is spinning counter-clockwise (looking down) at a rate of 0.1 rad/s, how fast is it rotating if a constant torque of 20 N·m clockwise is applied for 10 seconds?
(c) Suppose there is friction and, after getting the platform spinning clockwise at a rate of 0.3 rev/s, we find it takes a minute to slow down to a stop. What is the torque applied by friction?

24. Balance

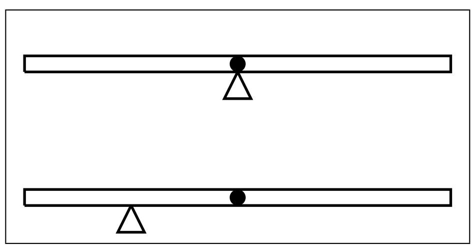
Puzzle #24: Why are trucks more likely than a car to tip over when going around a curve? And why do they tip over rather than just slide?

Introduction

We can explain the situation described in the puzzle by using ideas of torque and rotation from chapter 23 along with our knowledge of gravity. To start, I'll review a bit about torque and rotation by applying it to a see-saw. The difference between our analysis in this chapter and the analysis we did in chapter 23 is that in this chapter we'll examine torques applied by gravity.

24.1 Center of gravity

Try balancing a meter stick on a pencil, acting as a **pivot**. You should find that it balances best when the pencil is located at the center of the meter stick (see top illustration; the center of the meter stick is indicated by the dot and the pencil is indicated by the triangle).



To understand why the meter stick balances in the top case (centered) but not the bottom case (off center), let's first explain why the top case is balanced.

As discussed in chapter 23, for an object to not rotate (and remain that way), the net torque exerted upon the object must be zero. This doesn't mean there can't be *any* torques applied on the object. There can, but the counter-clockwise torques acting on it must equal the clockwise torques acting on it (such that the net torque is zero).

Since the meter stick has mass, there is a gravitational force on it (due to Earth). For the meter stick, the gravitational force on the side to the *right* of the pivot (pencil) is associated with a *clockwise* torque about the pivot. Conversely, the gravitational force on the side to the *left* of the pivot is associated with a *counter-clockwise* torque about the pivot.

HOW DO YOU KNOW THE DIRECTIONS OF THE TORQUES?

The torque direction can be determined by figuring out which way the meter stick would rotate if we only considered the gravitational force on one side. In this case, the gravitational force on the right side would pull the right side down, making the meter stick spin clockwise. The reverse is true for the gravitational force on the left side. Since the gravitational torque on each side is equal and opposite, the meter stick is balanced.

Now let's consider the off-center case.

In that case, there is more meter stick, and thus more mass, on the right side of the pivot. That means there is more gravitational force pulling down on one side vs. the other.

HOW DO WE KNOW WHICH SIDE HAS MORE MASS?

We first identify the spot where the object would be balanced if the pivot was located there. This spot (indicated by the dot in the illustration) is called the **center of gravity**[i].

In the top figure, the center of gravity lies directly above the pivot point and thus the meter stick is balanced in that case. That means that the gravitational force on the top meter stick does not apply a *net* torque about the pivot point. In other words, the clockwise torque applied by the gravitational force acting on the right half exactly counters the counter-clockwise torque applied by the gravitational force acting on the left half.

In the bottom figure, however, the pivot is shifted to the left of the center of gravity and thus is not balanced. This means that the gravitational force on the bottom meter stick <u>does</u> apply a net torque about the pivot point.

IS THE TORQUE IN THE BOTTOM CASE CLOCKWISE OR COUNTER-CLOCKWISE?

[i]This is also called the **center of mass**. There is a very slight difference between the two terms. As long as the object is not too big relative to the distance to the center of Earth, however, these two are the same.

Clockwise. This can be determined by figuring out which way the plank would rotate if it was released. In this case, the gravitational would pull the right side down more than the left (since there is more mass on the right side), making it spin clockwise.

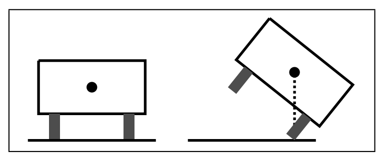
Is the center of gravity always in the middle of the object?

The center of gravity is only in the middle of the object if the object has its mass evenly distributed within it. For each meter stick in the figure, the center of gravity is exactly in the center of the plank, which suggests that the planks are uniform.

❢ For a uniform bar, the center of gravity is at its center.

This idea can be applied to how trucks are more susceptible to tipping than cars (when, for example, they go around curves).

Consider, for example, the figure to the right, where the two rectangles represent cars driving on a road (as seen from the perspective of someone ahead or behind the car).



In the right portion of the figure, the car is tipped such that its center of gravity is directly over the point where the tire contacts the road. If the car tips any further, the center of gravity will be to the right of that contact point and the car will tip over, rotating clockwise. Anything less than that and the car returns back to its original position, rotation counter-clockwise.

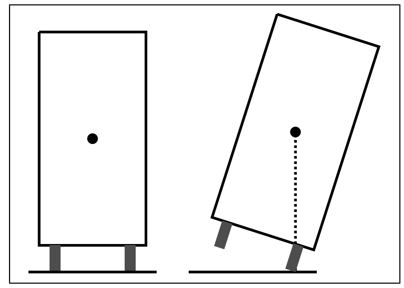
As you can see in the diagram, the car has to tip quite a bit to reach the point of no return, so to speak.

Why would the car tip at all?

As mentioned in chapter 20, when going around a curve, the car's inertia will carry it in a straight line unless friction acts on it. If no slipping occurs then the tires will curve but the inertia of the top part of the car may carry that part away from the center of the road (i.e., along the straight line path).

Why would the truck tip more?

The problem isn't necessarily that the truck tips more than the car but rather that the truck doesn't have to tip as much in order to reach the point where its center of gravity passes over the contact point with the road.



To see why, consider the figure to the right, where the two rectangles now represent trucks driving on a road. In the right portion of the figure, the truck is tipped such that its center of gravity is directly over the point where the tire contacts the road. In comparison with the car, the truck doesn't need to tip as far to reach that point.

✍ The problem is made worse if some of the load inside the truck shifts as the truck rounds the curve, because that will shift the center of gravity even further over the contact point with the road.

✓ *Checkpoint 24.1: Suppose you have a bunch of boxes that fill the entire space in a truck. Half the boxes are heavy and half are light. For maximum safety (to avoid tipping), how should the boxes be stacked: heavy ones on bottom and light ones on top, heavy ones on top and light ones on bottom, or heavy ones on one side and light ones on the other? Explain your choice.*

24.2 Determining torques when off center

Of course, we could apply the ideas discussed above to more than just a meter stick or automobile. In general, I'll consider a uniform **beam** or **plank**. It could be a meter stick, a piece of wood or a see-saw. Regardless, since it is uniform, the center of gravity is located at its center.

Can we balance a beam if the pivot is off center?

We can balance the beam if we apply a torque that counters the torque associated with the gravitational force on the beam. To know how much torque we need to apply, then, we first have to figure out the torque associated with the gravitational force on the beam.

❢ When determining the torque applied by gravity on the object, treat the entire gravitational force as though it was acting at the object's center of gravity.

The simplest way is to treat the object as though the gravitational force on it is acting solely at its center of gravity.

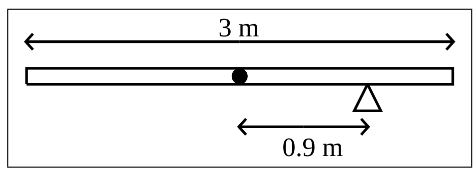
Consider, for example, a uniform 0.5-kg meter stick (one meter long) with the pivot shifted 0.25 m off-center, similar to the lower illustration on page 24.1.

To calculate the torque due to the gravitational force, we multiply the gravitational force on the meter stick by how far its center of gravity is from the pivot. For a 0.5-kg meter stick, the gravitational force is 4.9 N (multiply 0.5 kg by 9.8 N/kg) and the distance between the pivot and the center of gravity in our example is 0.25 m.

That means that the torque applied by the gravitational force is 1.225 N · m (multiply 4.9 N by 0.25 m) in the clockwise direction (since the center of mass is to the right side of the pivot, pulling that side down).

That means we'd have to exert a torque equal to 1.225 N · m in the *counter-clockwise* direction to balance the meter stick.

✓ *Checkpoint 24.2: The illustration to the right shows a uniform 2-kg plank, 3 m long, that is placed on a pivot 0.9 m from the plank's center. What is the torque about the pivot due to the gravitational force on the plank?*



How do we exert a torque equal to 1.225 N · m on the meter stick?

There are lots of ways.

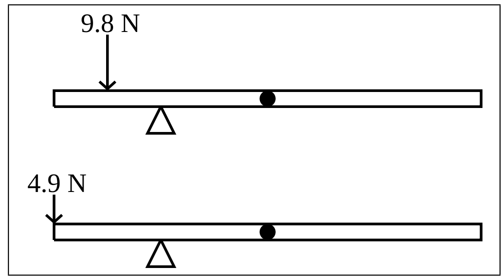
Recall that the torque τ depends on the magnitude of the force F_{circ} (around the pivot) and how far the force is applied from the pivot r:[ii]

$$\tau = F_{\text{circ}} r$$

As long as the product of F_{circ} and r is equal to 1.225 N · m counter-clockwise, the torque we apply will balance out the gravitational torque on the meter stick (1.225 N · m clockwise, as determined before).

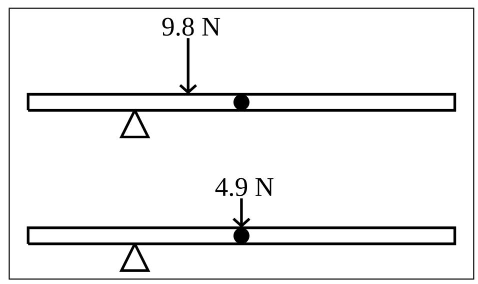
[ii]Be careful. The abbreviation for torque is the lower-case Greek letter τ (tau), which looks very similar to the lower-case Roman letter r.

Consider, for example, the two forces shown in the illustration to the right. Both produce a torque equal to 1.225 N · m counter-clockwise and thus will balance the meter stick. In the bottom case, the product of the force (4.9 N) and the distance from the pivot (0.25 m) gives a torque equal to 1.225 N · m, which is what we need to balance the meter stick.

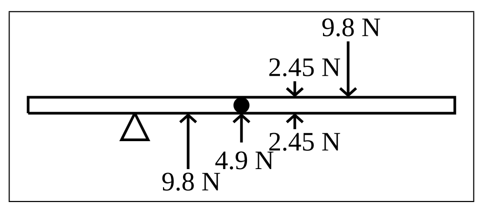


In the top case, the force is twice as large (9.8 N) and is located half the distance to the pivot (0.125 m), giving the same torque, and thus will likewise balance the meter stick.

On the other hand, the two forces shown in the illustration to the right will *not* balance the meter stick. Although each is associated with a torque equal to 1.225 N · m, the direction is clockwise and thus will not balance the meter stick (since the torque associated with the gravitational force is also clockwise).



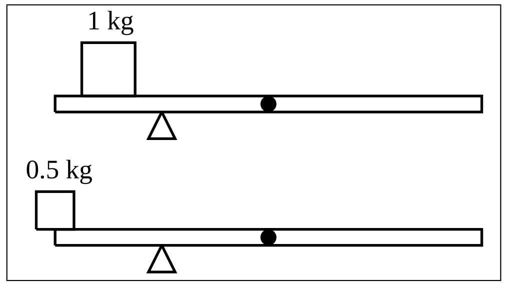
✓ *Checkpoint 24.3: The illustration to the right shows a uniform meter stick that is placed on a pivot 0.25 m from the meter stick's center. It also shows five forces. Which of those forces, by itself, would balance the meter stick?*



24.3 Balancing using additional objects

Rather than balancing an off-center plank by pushing on it, we can instead place objects on the plank.

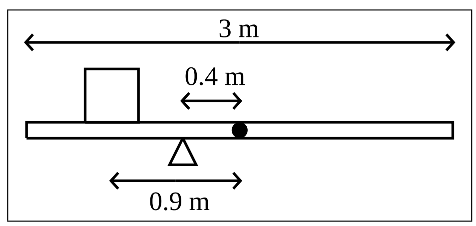
Consider, for example, the off-center meter stick discussed before. Rather than exerting a downward force of 4.9 N at a point 0.25 m from the pivot, we could instead place a 0.5-kg block at a point 0.25 m from the pivot (see bottom illustration).



The reason this works is because the gravitational force on the 0.5-kg block is 4.9 N (multiply 0.5 kg by 9.8 N/kg). That is the same force we needed before to produce a torque of 1.225 N · m counter-clockwise, which is what was needed to balance the torque due to gravitational force on the meter stick itself (clockwise).

Similarly, instead of pushing down at 9.8 N at a distance 0.125 m from the pivot, we could instead place a 1-kg block at that same distance (see top illustration). The gravitational force on a 1-kg block is equal to 9.8 N, after all.

✓ *Checkpoint 24.4: The illustration to the right shows a uniform 2-kg plank, 3 m long, that is placed on a pivot 0.4 m from the plank's center. Suppose we want to balance the plank by placing a box whose center is 0.9 m from the center of the plank (0.5 m from the pivot). What must the mass of the box be?*

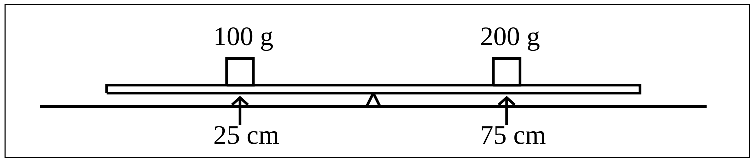


What if there is more than one object placed on the beam?

The process is the same. Simply calculate the torque associated with the gravitational force on each object separately.

To illustrate this, let's consider a meter stick that is placed on a pivot at its center. That way, the gravitational force on the meter stick does not exert a torque.

> A uniform meter stick is balanced on a pivot at the 50-cm mark. A 100-g block is placed at the 25-cm mark and a 200-g block is placed at the 75-cm mark (see figure on next page). Is the meter stick balanced?



Without doing any math, you might guess that meter stick is not balanced. Although the blocks are equidistant from the pivot, they do not have the same mass.

Let's double-check this conclusion by calculating the torque associated with each block. The gravitational force on the 100-g block is 0.98 N (multiply 0.1 kg by 9.8 N/kg). The gravitational force on the 200-g block is 1.96 N (multiply 0.2 kg by 9.8 N/kg).

To find the torques associated with each force, multiply each force by the distance from the axis to where the force is applied.

In this case, both forces are applied 25 cm from the axis, since that is where the blocks are. That means the two torques are:

$$\begin{aligned}\tau_1 &= F_1 r = (0.98\text{ N})(0.25\text{ m}) = 0.245\text{ N}\cdot\text{m}\\ \tau_2 &= F_2 r = (1.96\text{ N})(0.25\text{ m}) = 0.49\text{ N}\cdot\text{m}.\end{aligned}$$

where τ_1 and τ_2 are the torques associated with the two blocks.

The first torque is counter-clockwise and the second torque is clockwise. Each torque is in an opposite direction so they cancel somewhat but since they have different magnitudes the net torque is not zero and the meter stick is not balanced.

HOW DO WE KNOW WHICH IS COUNTER-CLOCKWISE AND WHICH IS CLOCKWISE?

Think about what the meter stick will do if only one torque is applied. For example, imagine the meter stick with only the left block on it. Which way will the meter stick start to rotate?

Since the block is to the left of the pivot (see figure), the meter stick will start to rotate counter-clockwise. That is why we say the torque due to the left block is counter-clockwise.

Conversely, suppose we only had the right block on the meter stick. The meter stick would start to rotate clockwise. That is why we say the torque due to the right block is clockwise.

What about the force applied by the pivot?

Since the pivot is touching the meter stick, the pivot does indeed apply a force (upward) on the meter stick. In fact, that force must be 2.94 N in order to balance the two downward forces (0.98 N and 1.96 N), due to gravity on each block.

However, the upward force due to the pivot does not apply a torque on the meter stick because the force due to the pivot is applied right at the rotation axis and forces at the rotation axis do not apply a torque (since r for such a force would be zero).

What if the distances to the pivot were different for each block?

To find out if the meter stick balances at some other distance, we use the same approach. Calculate the torque associated with each. If they are equal in magnitude and opposite in direction, the object is balanced.

❢ For an object's rotation rate to remain constant, the net torque on the object must be zero. Consequently, the clockwise torques must balance the counter-clockwise torques.

Could we balance the meter stick with more than two blocks?

Yes. The approach remains the same. Calculate the torque associated with each. Add up all the counter-clockwise torques and do the same for all the clockwise torques. If the total counter-clockwise torques equal the total clockwise torques then the object is balanced.

Could we balance the meter stick by placing additional blocks right at the pivot?

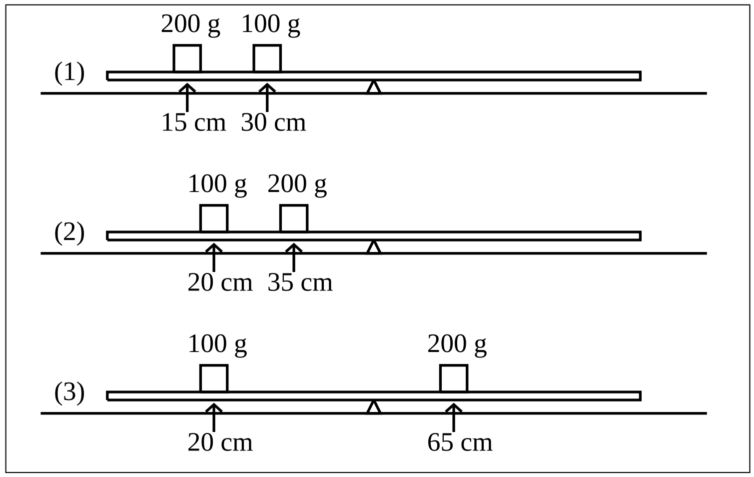
The pivot is right at the axis. For a force at the pivot, the distance to the axis (r) is zero, and so there no torque is applied about that axis.

You might want to try experimenting with a meter stick that is balanced on a pencil. Place a coin on one end of the meter stick to represent a block. How much of a force do you need to apply to the center of the meter stick such that it is balanced?

It turns out that you can't do it because no matter how much force you apply at the center there is no torque associated with that force.

✍ Remember: torque is not the same thing as force. The units are different and the directions are different (the direction of torque is given as a rotation around the pivot).

Figure 24.1: Three meter sticks, each balanced upon a pivot at its center. Two blocks are placed on the meter stick in each case but their locations vary. The locations indicate the distance from the *left* end of the meter stick.



✓ *Checkpoint 24.5:In figure 24.1, three 100-g meter sticks are shown, each placed on a pivot at the 50-cm mark. Two blocks are placed on each meter stick, with their locations indicated (according to the mark on the meter stick).*
(a) Without doing any math, which of the three do you think is balanced? Why?
(b) Check your answer by calculating the torque due to each block, in each case, and then adding them up to find the total torque exerted on the meter stick (in each case).

Summary

This chapter examined how we apply the law of torque and rotation to predict whether an object is balanced.

The main points of this chapter are as follows:

- For an object's rotation rate to remain constant, the net torque on the object must be zero. Consequently, the clockwise torques must balance the counter-clockwise torques.
- For a uniform bar, the center of gravity is at its center.
- When determining the torque applied by gravity on the object, treat the force as though it was acting at a single location called the *center of gravity*.

Frequently Asked Questions

How do we calculate the net torque applied by gravity?

See page 412.

For an object in balance, does it matter which point you use as the rotation axis?

No.

I tend to use the pivot as the rotation axis but actually any location can be used as long as the object is in equilibrium and the same point is used when determining the torques applied by each force.

However, if you use a different location as the axis, you need to include the torque applied by the pivot. The pivot exerts a force on the object (it has to, in order to balance the forces on the object). There is a torque associated with that force about any point *except* the point where the pivot is.

Instead of using clockwise and counter-clockwise to keep track of the torque directions, could we instead use positive and negative?

Yes. As mentioned in chapter 23, all angular quantities can be positive or negative depending on their direction (clockwise or counter-clockwise). You just have to remember which sign you assigned to each direction.

Terminology introduced

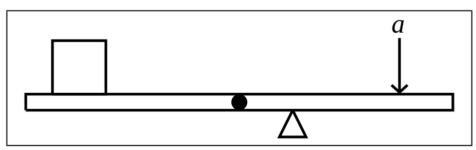
Beam	Pivot
Center of gravity	Plank
Center of mass	

Additional Problems

Problem 24.1: A uniform see-saw of length 4 meters is balanced on a pivot at its center. A small child of mass 15 kg sits on one end, a distance of 1.5 m from the pivot. How much force do I have to exert at each of the following

locations so that the see-saw is still balanced?
(a) A point *2.0 m* from the pivot on the side opposite the child
(b) A point *1.0 m* from the pivot on the side opposite the child

Problem 24.2: A uniform 50-kg 4-m-long see-saw has the pivot shifted 0.5 m off-center (see below). A 15-kg child (represented by the box) sits 0.5 m from the left end.
(a) How much force do I have to exert at a point 0.5 m from the other end (see arrow a) to balance the see-saw?
(b) If the 15-kg child sits on the very edge of the see-saw, 2 meters from the center (2.5 meters from the pivot), rather than 0.5 meters from the edge, how much force do we need to apply at location a, which is 1.0 m from the pivot, to keep the see-saw balanced?
(c) With the child still on the edge, suppose we shifted the applied force to the end of the see-saw, 1.5 m from the pivot. How much force needs to be applied to keep the see-saw balanced?
(d) Suppose the child gets off the see-saw. How much force do we need to apply at location a, which is 1.0 m from the pivot, to keep the see-saw balanced?



Part F

Problems Without Time

25. Work and Kinetic Energy

Puzzle #25: How do we predict the velocity without knowing the time?

Introduction

Up until now, everything has been about predicting an object's change in velocity given its mass, the forces acting on the object and the length of time those forces are acting.

The problem with our approach so far is that it depends on knowing the time. Often, we don't know the time. Instead, we given the displacement. In this chapter, we utilize our knowledge of force and motion to solve problems in a way that the time is not needed.

25.1 Force and displacement

Suppose we want to know how fast a ball is traveling after falling a certain distance. While it isn't *impossible* to figure out with the force and motion equation, it is quite difficult unless we are given the *time* it takes for the ball to fall that distance. The force and motion equation just isn't that helpful if we don't know the time.

Before introducing another way, let's see what the law of force and motion tells us about the motion. For example, even without the time, we can still make a qualitative prediction of whether the object will speed up or slow down. After all, we know that a force acts to speed the object up when the force is in the direction of motion, slow it down when the force is opposite the direction of motion, and make the object turn if the force is perpendicular to the direction of motion.

In other words, if all we know is the net force act on the object and the *direction* it has moved, we can at least predict whether the object has sped up, slowed down or turned. Let's consider some examples:

- When an object is dropped from rest, its displacement is *downward*. The only force on it is the gravitational force (due to Earth), which is also *downward*. Since the net force and the displacement are in the *same* direction, the object speeds up as it falls (ignoring air resistance).
- When a box is given a push along the floor, it may speed up when the push is in the direction of its displacement but it slows down once that force is removed because friction acts *opposite* its displacement.
- As discussed in chapter 20, an object moves in a circle without speeding up or slowing down only if the net force on the object is directed continually toward the center of the circle, *perpendicular* to the object's motion around the circle.

Notice that the net force needs to be *parallel* to the displacement to make it speed up (same direction as displacement) or slow down (opposite the direction of the displacement). If the net force is *perpendicular* to the displacement then the object moves at a constant speed (just turning).

What if the net force is zero?

If the net force is zero then the object has a constant speed (or is at rest). For example, when we walk across the floor at a constant speed, friction applies a forward force on our back foot (preventing it from slipping backward) and a backward force on our front foot (preventing it from slipping forward). The net force is zero so, despite our forward motion, we neither speed up nor slow down. While *starting* to walk (speeding up), the net force on us is forward but *while* walking the net force on us is zero.

✓ *Checkpoint 25.1: If the net force on an object is northward while the object is moving southward, is the object (a) speeding up, (b) slowing down, (c) turning, or (d) is there is not enough information to tell?*

What if the displacement is zero?

If the displacement is zero because the object is at rest and staying at rest then, of course, it isn't speeding up or slowing down. In that case, the net force must also be zero, so the result is the same as if the object was moving

– a zero net force means the object neither speeds up nor slows down. For example, when I lean against a wall, there is a force on me due to the wall but I remain at rest because the friction of the floor prevents me from sliding away from the wall.

However, an object *could* be moving and end up with a zero displacement, since a zero displacement just means the object ends up at the same place it started. For example, suppose you throw a ball up the air and then catch it. Its total displacement is zero, since the ball ends up in the same place it started (your hands) even though the net force on it was not zero.

If the net force on the ball is constant, though, then the ball will speed up on the way down by the *same* amount it slowed down on the way up. Consequently, its *total* change in speed is zero, since it ended up with the same speed it started with.

So, as a general rule, we can say that if the net force is constant then an object's *total* change in speed depends on the net force acting on it, its total displacement, and whether the net force and displacement are in the same direction or opposite. If either the net force or the displacement is zero then the object has the same speed at the end as at the beginning (object's total change in speed is zero). If the net force and displacement are in the *same* direction then the speed is *greater* at the end than at the beginning. And, if the net force and displacement are *opposite* in direction then the speed is *less* at the end than at the beginning.

✓ *Checkpoint 25.2: If a constant northward net force is applied to an object that is initially moving southward, such that the object slows down, stops then speeds up moving northward, where must it be, relative to its initial position, when it reaches a speed equal to its initial speed?*

25.2 Definitions

25.2.1 Work

The previous section examined how we can predict whether an object has a greater or less speed simply by examining the directions of the net force and displacement. Rather than look at the net force, we can look at each individual force and the same thing holds true – forces acting in the direction of the displacement act to speed up the object, forces acting opposite the direction of displacement act to slow it down, and forces acting perpendicular to the displacement only act to change the direction, not the speed.

Consequently, since the objective is to determine the object's speed, we only need to concern ourselves with those forces acting parallel to the direction of the displacement. We say that these forces are doing **work** on the object. Only forces acting parallel to the displacement do work on the object. Forces in the direction of the displacement do *positive* work, acting to make the object speed up. Conversely, forces opposite the direction of the displacement do *negative* work, acting to make the object slow down. And forces perpendicular to the direction of the displacement do *no* work, and do not make the object speed up or slow down.

The further the object moves while a force does work on it, the more it speeds up or slows down. Consequently, it probably comes as no surprise that *how much* the speed changes depends on both the force and the displacement. In fact, the work is defined as the *product* of the force and displacement (W represents the work done by the force F):

$$W = F_{\|}\Delta s \tag{25.1}$$

In this expression, Δs is the magnitude of the displacement and $F_{\|}$ is the component of the force that is parallel to the displacement. The reason we use $F_{\|}$ instead of F is because the object changes speed only when the force is parallel to the displacement, as discussed earlier.[i]

Example 25.1: Suppose we drop a 0.5-kg ball and it falls 0.4 m. How much work is done by the gravitational force on the ball as it falls the 0.4 m?

[i] Technically, Δs is the displacement of the point being acted on by the force, which is not necessarily the displacement of the center of mass. For example, for an accelerating car the displacement of the car's center of mass is in the direction of the friction on the tires but, technically, the friction does no work on the car because friction acts on the tire tread and the tire tread is stationary against the ground while friction is acting on it. This is discussed further in chapter 26, where it is noted that the car speeds up not because $W \neq 0$ but because energy is transferred. Work is technically non-zero only if the force acts as a conduit for the flow of energy.

Answer 25.1: Both the gravitational force and the displacement are downward, so we know that the work done by the gravitational force is positive. To find the value of the work done by the gravitational force, we multiply the magnitudes of the force (4.9 N) and the displacement (0.4 m) to get 1.96 N·m.

There are two things I want you to notice. First, the value we got for work has units of N·m because it is the product of force (in N) and displacement (in m). The units are not the same as force because work is not the same thing as force. A second thing to notice is that the gravitational force does *positive* work on the ball, making it speed up.

When would the work be negative?

The work is negative when the force acts to *slow* down an object, as in the next example.

Example 25.2: Suppose a box is sliding across the floor and slowing down because of friction. If the force of friction is 0.4 N and it takes 2 m to slow to a stop, how much work is done by the friction force on the box as it slides to a stop?

Answer 25.2: The friction force is opposite the displacement, which is why the box slows down, so we know that the work done by the friction force is negative. To find the value of the work done by the friction force, we multiply the magnitudes of the force (0.4 N) and the displacement (2 m) to get -0.8 N·m.

When would the work be zero?

The work is zero when the force is not acting to speed up or slow down the object. Examples of this could be when the force is just acting to turn the object (as in an object moving in a circle) or when there are other forces acting that prevent the object from moving. An example of the latter is when I push on a wall. The wall remains where it is, so I do no work on the wall.

If there are multiple forces acting on object then some of the forces could be doing positive work on the object while others are doing negative work or zero work. For example, suppose I pick up a book off the floor and place it on the table. The book is at rest initially (on the floor) and at the end (on the table), but I did positive work on the book (exerting an upward force on it while it was moving upward) while the gravitational force did negative work on the book (exerting a downward force on it while it was moving upward). The end result is no *net* work on the book and so its speed at the end is the same as it was in the beginning (zero).

Example 25.3: Suppose a 0.5-kg box is sliding across a horizontal floor and slowing down because of friction. It takes 2 m for the box to slow to a stop The gravitational force is 4.9 N downward. How much work is done by the gravitational force on the box as it slides to a stop?

Answer 25.3: The gravitational force does no work on the box as it slides to a stop. The reason it slows to a stop is because friction acts opposite its displacement. The gravitational force (downward) acts perpendicular to the displacement (horizontal) and so does no work on the box.

For the box sliding to a stop on a horizontal floor, the gravitational force (downward) and the surface repulsion force (upward in this case) do no work on the box because they are perpendicular to the displacement. Only the friction force does work on the box, and that work is negative, making the box slow down.

✓ *Checkpoint 25.3: A liter bottle of water is sitting on my desk. I exert a horizontal force to slide it across the table 50 cm rightward, where it comes to rest again (because friction slows it down). To figure out the work done on the bottle, could I multiply the gravitational force on the bottle by the 50 cm?*

25.2.2 Kinetic energy

Based on what we know so far, we can predict whether an object speeds up or slows down without needing the time. However, we still can't predict *how much* an object speeds up or slows down. To do that, we need an equation, much like the force and motion equation, that relates the motion with the work. That expression is as follows:

$$\Delta E_{\mathrm{k}} = W \tag{25.2}$$

The quantity on the left is called the change in **kinetic energy**. Notice that it is *not* the speed. It *can't* be, since the units on the right (N·m) are not the units of speed.

What is kinetic energy?

Kinetic energy has a property similar to speed in that it is larger when the object is moving faster (and smaller when the object is moving slower). It is for this reason that it is called the *kinetic* energy, since the the word "kinetic" refers to motion (much like the word "kinesthetic").

❢ The faster an object moves, the greater its kinetic energy.

This quantity is called kinetic *energy* because it happens to have units of energy, which we will explore in chapter 26.

✍ As mentioned in section 20.5, we can always come up with a reference frame for which an object's speed is zero. Consequently, an object's kinetic energy is likewise relative to a reference frame. That is why the *value* of the kinetic energy is not as valuable as the *change* in kinetic energy (and also why we are more concerned with the *change* in velocity rather than the *value* of the velocity).

The work is negative when an object slows down. Does that mean the kinetic energy can also be negative?

No. The kinetic energy, like an object's speed, is always positive. The *change* in kinetic energy can be negative, though, since the change in kinetic energy is equal to the work. A negative change in kinetic energy simply means the object slowed down (decreasing the kinetic energy).

✓ *Checkpoint 25.4: Suppose the work done on an object is negative. Does that mean the object's kinetic energy is negative?*

❢ By definition the kinetic energy is $E_{\mathrm{k}} = \frac{1}{2}mv^2$.

It turns out that an object's kinetic energy is equal to the following:[ii]

$$E_\text{k} = \tfrac{1}{2}mv^2 \qquad (25.3)$$

Notice that the kinetic energy, like the speed v, is always positive.[iii] And, just like speed, it doesn't have a direction, so an object moving upward at 5 m/s has the same kinetic energy as the same object moving downward at 5 m/s or horizontally at 5 m/s. It is for this reason that the definition has v (the speed) rather than $\vec{v}$ (the velocity).

❢ An object's kinetic energy has no direction and is always positive.

Example 25.4: A 4-kg object has a speed of 3 m/s. What is the object's kinetic energy?

Answer 25.4: The kinetic energy is obtained by applying the definition of kinetic energy (equation 25.3), with m equal to 4 kg and v equal to 3 m/s. The square of the speed is 9 m^2/s^2 (notice how I also square the units). Multiplying by 4 kg and then by one-half gives a kinetic energy of 18 $\text{kg}\cdot\text{m}^2/\text{s}^2$.

✓ *Checkpoint 25.5: A 1.5-kg object has a speed of 4 m/s. What is the object's kinetic energy?*

You may noticed that kinetic energy has units of $\text{kg}\cdot\text{m}^2/\text{s}^2$ whereas work has units of N·m. If work is equal to the change in kinetic energy then they must have the same units. They do. You may recall from before that a newton (N) is equal to a kg·m/s^2. Multiplying both by meters shows that N·m is equivalent to $\text{kg}\cdot\text{m}^2/\text{s}^2$.

❢ The SI unit for work and kinetic energy is the joule, abbreviated as J.

Since it can get unwieldy handling this combination of units, we often replace the whole group with just J, which stands for a **joule**[iv]. So, instead of N·m

[ii] One can do a hand-wavy type of derivation for this by starting with the force and motion equation arranged as $m\Delta v/\Delta t = F$, where the force and displacement are parallel. Multiply both sides by Δs then replace $\Delta s \Delta t$ by v_avg and, assuming a constant force, replace v_avg by $(v_\text{f}+v_\text{i})/2$. Since $(a-b)(a+b) = a^2-b^2$, we have that $(\Delta v)((v_\text{f}+v_\text{i}) = \Delta v^2$. You then end up with $m(\Delta v^2)/2 = F\Delta s$, and the left side of that is the change in kinetic energy.

[iii] Even if the speed could be negative, its square would still be positive.

[iv] The joule is named after James Prescott Joule, an English physicist, who lived from 1818 to 1889.

for work units and $\text{kg} \cdot \text{m}^2/\text{s}^2$ for kinetic energy units, we can instead use J for both work and kinetic energy (assuming we are working with SI units).

✓ *Checkpoint 25.6: A 1.5-kg object has a speed of 4 m/s. What is the object's kinetic energy in joules?*

25.3 Problems without time

Now that we can determine the work done, how that impacts the kinetic energy, and how the kinetic energy depends on the speed, we can solve problems that don't involve the time.

For example, consider the following situation:

> Suppose you have a 3-kg box on a horizontal, frictionless surface and you apply a constant horizontal force of 5 N on it. If it starts from rest, how fast is it going after traveling 6 m?

Because this problem provides the displacement and not the time, we'll first identify the work done by the 5-N force, then figure out the change in kinetic energy, then figure out the final speed.

Since the force must necessarily be in the same direction as the displacement (since it starts at rest), the work done is positive and is equal to the product of the force magnitude (5 N) and the displacement magnitude (6 m) to get 30 J. That means the box has gained 30 J of kinetic energy.

Since the box started at rest (zero kinetic energy), its final kinetic energy must be 30 J. From the definition of kinetic energy ($\frac{1}{2}mv^2$), we can solve for the speed, which gives 4.47 m/s.

What if the box did not start at rest?

Keep in mind that that the work tells us the *change* in the kinetic energy. It does *not* tell us the *final* kinetic energy.

If the accelerating force and the distance is the same, the work done would be the same so the change in kinetic energy would be the same (30 J). However, the *initial* kinetic energy would no longer be zero.

For example, if the box started with a velocity of 6 m/s in the direction of the force, then its initial kinetic energy would be 54 J. Add that to 30 J transferred by the 5-N force and you get 84 J. Solve that for the speed and you get 7.5 m/s.

✓ *Checkpoint 25.7: Suppose you have a 4-kg box on a horizontal, frictionless surface and you apply a constant horizontal force of 5 N rightward on it. If it is initially moving at a velocity of 2 m/s rightward, how fast is it going after traveling 2 m?*

What if the box in the previous example was initially moving opposite the direction of the force?

If that was the case, the work would be negative, the box would slow down and its kinetic energy would decrease not increase. In other words, we would subtract the change in kinetic energy from its initial kinetic energy, instead of adding it.

So, instead of ending up with a kinetic energy of 84 J (with a final speed of 7.5 m/s, faster than the initial speed of 6 m/s), it would end up with a kinetic energy of 24 J. Solve that for the speed to get a speed of 4 m/s (slower than the initial speed of 6 m/s).

✓ *Checkpoint 25.8: A 4-kg box is set sliding across the floor with an initial speed of 5 m/s. If the friction exerts a force of 8 N against the motion, how fast is the object moving after sliding 2 m?*

As mentioned above, we subtract from the kinetic energy if the object is slowing down. However, objects can't have a negative kinetic energy. This means there is a limit to how far an object can travel – it stops once the kinetic energy is zero.

For example, if the 3-kg box in the example was initially going 2 m/s instead of 6 m/s, its initial kinetic energy would be 6 J, not 54 J. If you subtract 30 J from that, you get a negative kinetic energy, which is not possible.

What this means is that the box never gets to the 6 m specified in the problem. It would actually travel less than 6 m before stopping and turning around.

What if it goes 6 m in the other direction?

If it was able to stop, turn around and then travel in the *other* direction, then we *could* ask how fast it is going when it reaches a point 6 m from its starting point, just not in the direction it was originally traveling. However, its displacement would then be in the *same* direction as the force and so the work done by the 5 N force would be positive, not negative. That would mean we'd *add* the 30 J to kinetic energy, not subtract it. Since the initial kinetic energy was 6 J, the final kinetic energy would be 36 J. A kinetic energy of 36 J corresponds to a speed of 4.9 m/s (set 36 J equal to $1/2mv^2$ and solve for v).

What is really interesting is that it doesn't matter which direction the object was initially moving in. All we need to know is how fast it was moving initially and where it ended up. For example, if you throw an object *upward* with an initial speed of 10 m/s, it would slow down, stop and then fall. If we asked how fast it was moving when it was 5 m below the starting point, the answer would be the same as what we'd get if we instead threw the object *downward* with an initial speed of 10 m/s. The displacement would be the same in both cases (5 m downward) and the gravitational force would be the same in both cases (same object) so the work done would be the same in both cases. And, since the initial kinetic energy is the same in both cases (same initial speed), the final kinetic energy would likewise also be the same in both cases.

✓ *Checkpoint 25.9: (a) Suppose you throw a 0.5-ball up in the air with an initial speed of 4 m/s and an initial height of 1 m above the ground. What is the ball's kinetic energy at the moment it hits the ground?*
(b) Suppose you threw the ball downward instead of upward. With the same initial speed and height, what is the ball's kinetic energy at the moment it hits the ground?

One nice application of this approach is to determine how far an object moves until it stops. For example, if there is a friction we could ask how far an object will slide on a surface before stopping, as in the following scenario:

> A 4-kg box is set sliding across the floor with an initial speed of 5 m/s. If the friction exerts a force of 8 N against the motion, how far does it slide before stopping?

The box slows down because the 8-N force applies negative work on it, equal to the decrease in kinetic energy. To stop, that decrease must equal the kinetic energy it had initially. Since the object started with a speed of 5 m/s, its initial kinetic energy was 50 J (from the definition of kinetic energy). Thus, the work done must have been −50 J. Since the work is equal to $F_{\|}\Delta s$, where $F_{\|}$ is equal to 8 N, we can set that equal to 50 J and solve for the distance. Dividing 50 J by 8 N, I get 6.25 m.

✍ In all of the cases involving friction, we wouldn't need to be given the friction force if, instead, we were given the friction coefficient. We know from chapter 17 that the magnitude of the friction force is equal to the friction coefficient times the magnitude of the surface repulsion force. That adds a step to the solution but, otherwise, the process is the same. See Problem 25.4 for an example.

✓ *Checkpoint 25.10: A 2-kg box is given a push so that it slides across a horizontal floor with an initial speed of 3 m/s. If the force of friction is 10 N, how far does it slide before coming to a stop?*

25.3.1 Problems without time or mass

In all of the problems up to now, we've needed the mass in order to get the speed from the kinetic energy value. It turns out that we don't need the mass if we are given the acceleration instead of the force.

This is because we can use the force and motion equation ($\vec{a} = \vec{F}_{\text{net}}/m$) to replace the $F_{\|}$ in $F\Delta s$ by $ma_{\|}$. Set that equal to change in kinetic energy and divide both sides by the mass to get:

$$\Delta\left(\frac{1}{2}v^2\right) = a_{\|}\Delta s$$

The left side is the kinetic energy *per mass* and the right side is the work done *per mass*. The SI unit for each is J/kg. Notice that the energy per mass has units of joules *per kilogram*, not joules.

Since objects in free fall experience a known acceleration, this approach is ideally suited for free fall problems. To illustrate, let's consider the following scenario.

> A rock of unknown mass is thrown downward with an initial speed of 8 m/s. How fast is it moving when it is 2 m below its starting point?

We know that the acceleration is 9.8 m/s^2 downward in free fall. Multiplying that by the distance it moves downward (2 m) gives 19.6 J/kg as the work per mass done on the rock.

This equals the kinetic energy per mass ($\frac{1}{2}v^2$). In this case, the rock starts with an kinetic energy per mass equal to 32 J/kg. With the added 19.6 J/kg, the final kinetic energy per mass is 51.6 J/kg. We can then solve for the final speed (from $\frac{1}{2}v^2$) to get 10.16 m/s.

✓ *Checkpoint 25.11: An object is thrown straight downward at an initial speed of 5 m/s. As it falls, it experiences free fall and speeds up such that its speed is 15 m/s just before it hits the ground. How far above the ground was it released?*

As long as we know the object's acceleration, we can still use the same approach, even if the acceleration isn't due to a gravitational interaction. For example, consider the following situation:

> An object is moving rightward with an initial speed of 5 m/s. If it is accelerating at 10 m/s^2 leftward, how far does it go before stopping (and turning around)?

In this case, the kinetic energy per mass is initially 12.5 J/kg (from $\frac{1}{2}v^2$). By stopping, it has decreased by 12.5 J/kg. That should equal the work per mass, which is the product of the acceleration (10 m/s^2) and the distance. Solve for the distance to get a distance equal to 1.25 m.

The process just described is basically how the U. S. Department of Transportation determines how far the average driver travels while stopping. Basically, they estimate a reasonable deceleration and then use that to determine how far the car travels (along with a 1.1-s reaction time; see page 122).[v]

[v]Compare this approach to the one we would've used in chapter 8 (see problem 8.2).

✓ *Checkpoint 25.12: A 4-kg box is set sliding across the floor with an initial speed of 5 m/s. If the box slows with a deceleration equal to 2 m/s^2, how far does the block slide before it stops?*

25.3.2 Problems without time, mass or acceleration

If you aren't given the time, mass or acceleration, you might think it is impossible to solve for any of the remaining quantities, like the distance, force or speeds. However, if you given a *comparison* situation, you can often solve the problem by taking advantage of the proportions.

> Suppose it takes 20 meters to stop when a car is initially traveling at 30 mph. How far would the car travel if initially traveling at 60 mph instead, assuming the same rate of deceleration?

We know that if v doubles then v^2 quadruples, and so the kinetic energy quadruples as well. That means that the 60 mph car has four times the kinetic energy of the 30 mph car. That means that the 60 mph requires four times the work to stop it. We know that work is equal to the product of the force and displacement (in the direction of the force). We can assume that the friction force is the same in both cases (same tires and road). That means that the distance must quadruple, to 80 meters.

✍ You might wonder how a car, originally driving twice as fast, can travel four times as far. Yes, it is only twice as fast, but it is moving for twice as long, since it takes twice the time to stop (at the same deceleration). The two factors combine to produce a distance four times longer.

✓ *Checkpoint 25.13: Two boxes slide across the floor with initial speeds of 5 m/s and 15 m/s. Assuming the same deceleration for both boxes, how much further does the faster box travel than the slower box?*

25.4 Problems in two dimensions

So far we've seen situations where forces are in the same direction as the displacement (doing positive work and speeding up the object), opposite the displacement (doing negative work and slowing the object), and perpendicular to the displacement (doing zero work and not impacting the object's speed). What do we do when the force is at an angle to the displacement?

In this case, we need to know the component of the force that is *parallel* to the displacement. In practice, it turns out to find the reverse: the component of the displacement that is parallel to the force. The product is the same either way: $F_{\|}\Delta s = F\Delta s_{\|}$. Let's see what this looks like for some of the two-dimensional situations we examined in part D.

25.4.1 Projectile motion

In projectile motion, an object is in free fall (gravitational force is the only force acting) but the object is moving both vertically and horizontally. An example of projectile motion is when a 0.5-kg rock is thrown horizontally with an initial speed of 8 m/s. In that situations, what will be the rock's speed at the moment it is 2 m below its original height?

The rock will be moving faster because it has moved downward and the gravitational force, being downward, does positive work on objects that move downward, speeding them up. Even though we don't know the total displacement (since we only know the vertical displacement, not the horizontal displacement), we can still figure out the work done because we know the gravitational force on the rock (4.9 N in this case; multiply 9.8 N/kg by 0.5 kg) and the component of the displacement in the direction of that force (2 m; the vertical displacement). The work, then, is equal to $F\Delta s_{\|}$, which is 9.8 J.

That means the rock's kinetic energy has increased by 9.8 J. The initial kinetic energy is $1/2mv^2$, where m is 0.5 kg and v is 8 m/s. That gives an initial kinetic energy of 16 J. Since the change is 9.8 J, the final kinetic energy is 25.8 J. Solve this for the final speed to get 10.2 m/s.

So we don't have to look at the rock's horizontal motion separate from the rock's vertical motion?

No. The gravitational force does positive work on an object when it moves downward, speeding it up, and does negative work on an object when it moves upward, slowing it down. It doesn't matter how far the object moves horizontally – the work done by the gravitational force only depends on how far the object moves vertically.[vi]

IF THE HORIZONTAL DISPLACEMENT DOESN'T MATTER, WOULD IT MATTER IF THE OBJECT WAS THROWN UPWARD OR DOWNWARD INSTEAD OF HORIZONTALLY?

No. Energy is a scalar, so it has no direction and thus we do not need to split up the rock's motion into two separate, perpendicular directions as we did in part D. Mathematically, that is a major advantage over the force and motion approach used before. In other words, even though the initial motion of the rock is horizontal, the kinetic energy is neither horizontal or vertical. It is just the kinetic energy.

As mentioned in section 25.3, as long as the initial speed is the same and the vertical displacement is the same then the speed at the end is independent of which way it was thrown. Certainly, it will take longer to fall the 2 m if it is thrown upward then, say, downward. However, at the moment it is at a point 2 m below its starting point, the speed at that moment is the same regardless of how it got there, assuming no air resistance.

After all, the work done by the gravitational force only depends on the gravitational force and the vertical displacement. If we aren't changing those values then we aren't changing the work done by the gravitational force and so the change in kinetic energy will be the same as before.

❢ The specific path taken (from the initial to the final position) is irrelevant for determining energy transfer.

After all, if the rock was thrown upward initially, it would eventually slow down and then come back down. By the time it returned to its initial position, it would have the same speed as before but downward instead of upward. By the time the rock fell an *additional* 2 m downward, it would reach the same speed it would have if it was just thrown downward initially. In other words, its change in kinetic energy only depends upon the difference in height.

✍ The *average* speed of the rock is greater if thrown downward, since it doesn't slow down to a stop during its flight, but the *final* speed is the same either way since the change in height is the same.

[vi] If an object is moving horizontally, the gravitational force may be responsible for changing the object's *direction* of motion, but not its speed.

✓ *Checkpoint 25.14: A 1-kg object is thrown straight downward at an initial kinetic energy 10 J. As it falls, it experiences free fall and speeds up such that its kinetic energy is 90 J just before it hits the ground.*
(a) What is its change in kinetic energy between when it was released and just before it hits the ground?
(b) What would be its change in kinetic energy if it was thrown straight upward with an initial kinetic energy of 10 J instead of straight downward, assuming the same release height and same ending height (at the ground)?

25.4.2 Inclines

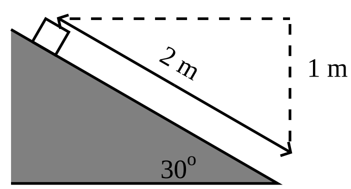
Another benefit of using work and kinetic energy is that we only need to consider forces that are doing work on the object. Even if there are other forces acting on an object besides the gravitational force, you may be able to ignore those other forces if those other forces don't act to speed up or slow down the object.

For example, when an object slides along a surface, the surface repulsion force is *perpendicular* to the motion and thus can't be directly responsible for speeding it up or slowing down.

❢ Forces acting perpendicular to the motion do not act to change an object's speed.

Thus, when using work and kinetic energy with inclines to solve for the final speed, we can ignore the surface repulsion force entirely. If the surface is frictionless and the only other force acting is gravity then we can treat the problem exactly like a free fall problem. That is so amazing.

Consider, for example, the scenario depicted to the right. A 1-kg box is on a frictionless inclined surface that is inclined 30° above the horizontal. If the box starts at rest, what is its kinetic energy after traveling for 2 meters along the ramp?



The gravitational force in this case is 9.8 N (multiply the 1-kg mass by 9.8 N/kg). The vertical displacement in this case is 1 m (as shown in the figure but which can be found by multiplying the 2-m displacement by the cosine of 60 degrees). Multiply the two together to get 9.8 J, which is the work done by the gravitational force and thus the change in kinetic energy. Since the kinetic energy started at zero (the box started at rest), this is also the final kinetic energy. Once you know the final kinetic energy, you can solve for the speed (4.42 m/s).

HOW DID YOU KNOW YOU NEEDED TO MULTIPLY THE DISPLACEMENT BY THE COSINE OF 60 DEGREES?

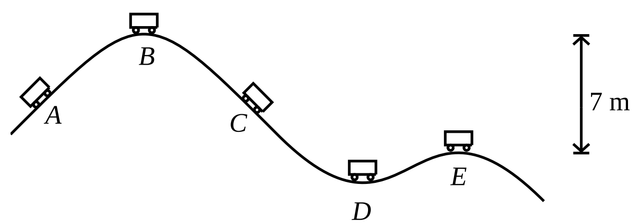
Because we need the vertical component of the displacement. The displacement is 60 degrees from vertical. It helps to draw a picture in order to see the geometry. You could have instead used the sine of 30 degrees.

✓ *Checkpoint 25.15: A 5 kg box is on a frictionless inclined surface that is inclined 20° above the horizontal. If the box starts with a kinetic energy of 20 J, what is its kinetic energy after traveling for 4 meters along the ramp.*

25.4.3 Roller coasters

It was mentioned before that we can ignore the surface repulsion force because it acts perpendicular to the motion. The example was that of a straight incline. However, it turns out that this is true even if the surface is not straight. As long as the surface is frictionless, we can make the surface as crazy as we want and the procedure is still the same as that for free fall.

To illustrate this, let's consider the roller coaster illustrated to the right, where a roller coaster cart moves to the right, from point A to point B and so on.



We'll assume the track is frictionless, so the only two forces acting on the cart are the gravitational force (directed downward) and the surface repulsion force due to the track. Now let's consider a problem with some numbers:

> Suppose a rightward-moving 1000-kg roller coaster cart passes over the top of a hill (point B) with a kinetic energy of 1.2×10^4 J. It then goes down the hill and over a smaller hill (point E), which is 7 m lower than the hill at B. What is the cart's kinetic energy at the moment it is at the top of the second hill?

As before, we can ignore the surface repulsion force entirely (since it doesn't contribute to changing the speed) and treat it exactly like a free fall problem.

The gravitational force in this case is 9800 N (multiply the 1000-kg mass by 9.8 N/kg). The vertical displacement in this case is 7 m. Multiply the two together to get 6.86×10^4 J, which is the work done by the gravitational force and thus the change in kinetic energy. Since the kinetic energy started at 1.2×10^4 J, the final kinetic energy must be 8.06×10^4 J (add the change in kinetic energy to the initial kinetic energy).[vii]

In the figure, the cart moves down to point D prior to moving up the next hill. Doesn't that matter?

No. The further it moves down, the faster it will go. For example, as the cart reaches point D, the kinetic energy will be greater. However, as it moves back up to point E it will slow down.

Doesn't it matter how far the cart travels horizontally?

No. We are assuming no other forces are acting other than the gravitational force and the surface repulsion force. If there was friction then the horizontal distance would make a difference, but there is no friction in our example.

So the change in kinetic energy will be the same if you simply dropped the cart and it fell 7 m rather than going from point B to point E?

Yes.

✓ *Checkpoint 25.16: Suppose the smaller hill is 5 meters below the top of the first hill (rather than 7 m). Assuming the cart passes over the first hill with a kinetic energy of* 1.2×10^4 *J, what is the 1000-kg cart's kinetic energy at the moment it is at the top of the second hill? Assume no friction between the cart and the track.*

What if the cart moves up instead of down?

As the cart moves up the gravitational force acts to slow it down instead of speed it up, and the kinetic energy decreases rather than increases.

[vii] Once we know the initial and final kinetic energies, we can obtain the initial speed (4.9 m/s) and final speed (12.7 m/s) from the definition of kinetic energy ($\frac{1}{2}mv^2$).

✓ *Checkpoint 25.17: Suppose the cart is moving to the* left *on the track illustrated on page 440 and the smaller hill (at point E) is 5 meters below the higher hill (at point B). Assume no friction between the cart and the track.*
(a) If the cart has a kinetic energy of 6.1×10^4 *J at point E, what is the cart's kinetic energy at the moment it is at point B?*
(b) Suppose the cart had a kinetic energy of 1.2×10^4 *J at point E. Does it make it over the hill at point B? Explain.*

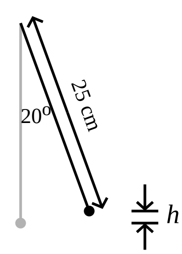
25.4.4 The pendulum

Let's now consider a pendulum, which consists of a ball swinging back and forth on a string. In this case, the force due to the string (directed along the string) is perpendicular to the motion and, as such, it does not do any work on the ball and does not impact its kinetic energy.

WHY IS THE FORCE DUE TO THE STRING PERPENDICULAR TO THE MOTION?

We'll assume that the ball does not move in such a way that the string shortens or lengthens. That means that the ball can't move in a direction parallel to the string. Since the force due to a string is always parallel to the string, the force must be perpendicular to the motion. Since the force acts perpendicular to the motion, it doesn't contribute to changing the speed and we can treat a problem involving a pendulum exactly like a free fall problem.

For example, consider a pendulum with a 100-g ball and a massless string of length 25 cm (as illustrated at right). If the pendulum is then displaced 20° from vertical and released from rest, what is the ball's kinetic energy when it reaches its lowest point (i.e., at the bottom of the pendulum motion), 1.5 cm below its initial position (indicated as h in the figure)?[viii]



[viii] This distance can be found using some trigonometry. Multiplying the length of string (25 cm) by the cosine of 20° gives 23.5 cm, which is how far below the pivot point the ball is initially. In comparison, when the pendulum ball is at the bottom of the motion, it is 25 cm below the pivot point (since the pendulum length is 25 cm). Subtract the two to find the total vertical displacement: 1.5 cm.

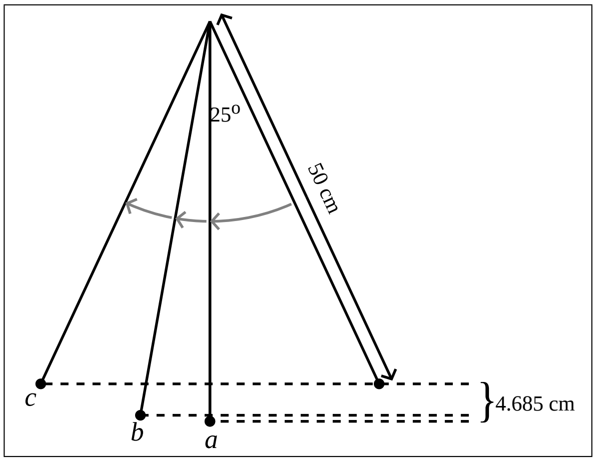


Figure 25.1: A pendulum of length 50 cm, with various positions marked.

To find the change in kinetic energy, we multiply the gravitational force by the vertical displacement to get 0.0147 J, which is the work done by the gravitational force. Since the kinetic energy started at zero (the pendulum started at rest), this is also the final kinetic energy.

✓ *Checkpoint 25.18: A pendulum is made up of a 1-kg ball and a massless string of length 50 cm. The pendulum is then displaced 25° from vertical (see Figure 25.1). Use the idea that we only need to know the vertical displacement (from initial to final position) to answer the following.*
(a) What is the ball's kinetic energy when it reaches its lowest point (at a, 4.7 cm below where the ball is released)?
(b) What is the ball's kinetic energy when it reaches a point where it is 25° from vertical on the side opposite where it started (at c)?

Summary

This chapter examined how use conservation of energy to quantify how much an object speeds up or slows down.

The main points of this chapter are as follows:

- The faster an object moves, the greater its kinetic energy.
- By definition the kinetic energy is $E_\mathrm{k} = \frac{1}{2}mv^2$.

- An object's kinetic energy has no direction and is always positive.
- The SI unit for work and kinetic energy is the joule, abbreviated as J.
- The specific path taken (from the initial to the final position) is irrelevant for determining energy transfer.
- Forces acting perpendicular to the motion do not act to change an object's kinetic energy. This includes the surface repulsion force (for surfaces that aren't moving) and the force of a pendulum string.

Terminology

Kinetic energy
Work

Additional problems

Problem 25.1: For each of the following, identify whether the force and motion equation or the work and kinetic energy approach is better suited for solving the problem. Explain.
(a) Finding the speed of a pendulum ball (constrained to move along a known path; applied forces are gravity and tension).
(b) Finding the direction a ball is traveling after falling off a ledge (applied force is gravity).
(c) Finding the speed of a roller-coaster cart (constrained to move along a known path; applied forces are gravity and the surface repulsion force).
(d) Finding how fast a ball is going after falling a distance of 10 m when released at rest.

Problem 25.2: A 2-kg ball has a velocity of 10 m/s rightward. Three seconds later, the ball has a velocity of 5 m/s rightward. What is the change in the ball's kinetic energy?

Problem 25.3: Is a joule a unit of force? If not force, what quantity is it used for?

Problem 25.4: A 2-kg box is given a push so that it slides across a horizontal floor with an initial speed of 3 m/s. If the coefficient of friction is 0.4, how far does it slide before coming to a stop?

Problem 25.5: Suppose a 5-kg object in free fall moves downward by 4 m. What would be the object's change in kinetic energy in each of the following cases? What would be the object's *final* kinetic energy in each case?
(a) Dropped from rest.
(b) Thrown down initially at 4 m/s.
(c) Thrown horizontally initially at 4 m/s.
(d) Thrown at an angle of 45 degrees at 4 m/s.
(e) Thrown at an angle of 45 degrees at 8 m/s.

Problem 25.6: Suppose you have a box that is initially moving at a velocity of 2 m/s rightward when it experiences an acceleration of 1.25 m/s^2 rightward. How fast is the box moving after traveling 2 m?

Problem 25.7: One way to measure human reaction time is by dropping, without warning, a vertically-oriented ruler between someone's thumb and fingers, and then recording how far the ruler falls before that person recognizes it is falling and stops it by squeezing their fingers together. Determine the person's reaction time if the ruler falls 30 cm before it is caught. First find the speed of the ruler just before it is caught, then use that value and the initial speed (zero) to get the average speed. Knowing the average speed, one can find how long it took to travel the 30 cm.

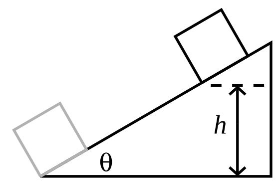
Problem 25.8: (a) A car is at rest at a stoplight. The light turns green and the car accelerates at a rate of 2 m/s^2 northward and continues to do so until it has traveled 100 m. How long does it take the car to travel the 100 m?
(b) Suppose, instead of starting at rest, the car starts with a velocity of 10 m/s northward. How long does it take the car to travel 100 m, assuming the same acceleration as before?
(c) Why can't we use the definition of average acceleration, by itself, to get the time?
(d) Why can't we use the definition of average velocity, by itself, to get the time?

Problem 25.9: A force of magnitude 10 N, acting in the same direction as an object's displacement, acts on an object while the object moves a distance of 2 m. With the object starting at rest, it experiences a change in speed equal to 10 m/s. What will be the change in speed if the magnitude of the force is tripled?

Problem 25.10: I throw a rock up in the air with an initial speed of 5 m/s. I throw a second rock up in the air with an initial speed of 10 m/s. How many times higher does the second rock go than the first rock?

Problem 25.11: A roller coaster cart passes over a particular hill at 5 m/s. Assume no friction between the cart and the track.
(a) How fast is the cart moving at the moment it reaches the top of a second hill that is 5 m lower than the first hill?
(b) How fast is the cart moving at the moment it reaches the top of a second hill that is 5 m *higher* than the first hill?

Problem 25.12: A 1.5-kg box slides down a frictionless, inclined surface that is inclined at an angle of 30° above the horizontal (i.e., $\theta = 30°$; see diagram to right). It is released, from rest, from a point that is 3 m above the floor (see black box; $h = 3$ m in the figure). What is the speed of the box the instant before it hits the ground (see gray box)?



Problem 25.13: Let's suppose the surface in previous problem was not frictionless. If the 1.5-kg box started at rest and slid down the surface (with $\theta = 30°$ and $h = 3$ m) such that the speed at the bottom was measured to be 3 m/s, what is the coefficient of friction between the box and the surface? Note: This is tricky, as the friction acts parallel to the surface. Thus, to find how that impacts the speed, you need to use the displacement along the surface, not the vertical displacement.

Problem 25.14: A 5 kg box is on a frictionless inclined surface that is inclined 20° above the horizontal. If the box starts with a speed of 3 m/s, determine its speed after traveling for 4 meters along the ramp.

Problem 25.15: A pendulum is made up of a 1-kg ball and a massless string of length 50 cm. The pendulum is then displaced 25° from vertical (see Figure 25.1). What is the ball's speed when it reaches a point where it is 10° from vertical (see point b in Figure 25.1)?

26. Conservation of Energy

Puzzle #26: What is energy and what use is it?

Introduction

Kinetic energy was introduced in chapter 25. The fact that there is *kinetic* energy, suggests that there are *other* types of energy besides kinetic, and indeed there are. In this chapter, we explore several types of energy, how they are all related, and how that gives us an alternative way of describing the relationship between force and motion.

26.1 The meaning of conservation

When a quantity is conserved, it means that the quantity cannot be created or destroyed. The only way a conserved quantity can change in one region or object is if some of it is transferred to or from an adjacent region or object. This is the fundamental physics definition of conservation.

It turns out that energy is conserved.

Does that mean the total amount of energy in the universe is constant?

Yes, the total amount of a conserved quantity in the universe cannot change, an idea we call **constancy**. However, while constancy is an interesting consequence of conservation, constancy by itself ends up not being particularly useful because requiring constancy of an object only means that if something disappears from your laboratory it may have appeared somewhere else in the universe, far away from your laboratory. Not only would that violate the fundamental definition of conservation, but that "somewhere else" may be so far away that one cannot know for sure whether it has appeared or not. Consequently, constancy by itself wouldn't be testable.

Does this have anything to do with, say, wildlife conservation?

Sort of. The Latin root of the word conservation means to keep or maintain. In that sense, wildlife conservation has to do with maintaining and protecting wildlife. In terms of the physics use of the word, however, a species' population is not something that is conserved, since a particular species' population can increase or decrease.

As always, we are not asking you to unlearn the vernacular definition of these words. However, you do have to learn the physics definition in order to properly apply the physics. Context matters.

✓ *Checkpoint 26.1: Since energy is conserved, does that mean your kinetic energy is constant? Why or why not?*

Is velocity conserved?

No, and we can show this quite easily by dropping a sandwich.[i] When you drop a sandwich, the sandwich eventually collides with Earth. By doing so, the sandwich slows down. For velocity to be conserved, Earth would need to gain the speed that the sandwich lost. Does it?

Of course not. Otherwise, we'd all notice Earth moving whenever someone dropped a sandwich.

How much an object slows down or speeds up depends on its mass. From the law of force and motion, we know that the same force will have less of an effect on a massive car than a light car. Consequently, the more massive Earth experiences a much smaller change in velocity than the much less massive sandwich. In fact, Earth's change in velocity is so small it is imperceptible.

Since Earth does not speed up by the same amount that the sandwich slowed down, the total "amount" of velocity between the both of them can't be constant and thus is not conserved.

✓ *Checkpoint 26.2: When a very heavy train collides with a relatively light car, the train keeps going and the car goes flying. (a) Is energy conserved in this situation?*
(b) Is velocity conserved in this situation?

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[i] I mean this as a thought experiment. Don't waste food.

26.2 Types of energy

While the kinetic energy and work approach is pretty useful, we can make it more powerful by utilizing energy conservation. To do so, we need to consider *all* types of energy, not just *kinetic* energy. Because energy can take on different forms, energy conservation can be difficult to grasp.

Keep in mind that we eventually want to use energy conservation to make prediction. To do this, we keep track of how each type of energy changes except for one type, allowing us to predict how that remaining type must change – it must change such that the total amount of energy remains the same. The important thing to remember is that, since energy is conserved, we cannot create any form of energy without using energy in some other form.

❢ By applying the principle of conservation of energy, one can predict changes in one type of energy if one can account for the changes in all of the other types of energy.

✍ If the system does not contain all of the objects that are interacting, it is called an **open system**. Much like an open door allows movement of material into or out of a room, an open system allows for the movement of mass, momentum or energy into or out of the system. If no mass, momentum or energy leaves or enters the system, we call that a **closed system**.

What types of energy are there?

Energy can be classified into two main groups: energy associated with motion and the energy associated with forces. Each group has several types. I'll examine a couple of each in a separate section.

✓ *Checkpoint 26.3: When a rock is dropped off a ledge, it speeds up as it falls. Suppose there are only two types of energy involved: kinetic energy and gravitational energy. If the kinetic energy is increasing in this situation, what must happen to the gravitational energy: decrease or increase? Explain how your answer is consistent with conservation of energy.*

Why do we need to turn off the lights to conserve energy if energy is conserved?

Some types of energy are useful to us and others which are not. So, when people outside of physics say you need to conserve energy, they mean that you need to maintain *useful* forms of energy.

In other words, non-scientists are using the word "conserve" in somewhat different way from us. While they still use it to mean "to save or maintain," they are only applying it to a subset of energy types. In our usage, energy is conserved whether we do anything about it or not. If anything, we can just move it around.

What are the useful forms of energy?

In general, the useful forms are those associated with chemical bonds (called chemical energy). Those forms are useful because, under various circumstances, the energy can be transformed into heat, light, sound and kinetic energy (like that associated with the movement of cars).

The energy that dieters fret about is the energy associated with the chemical bonds in carbohydrates, proteins and fat[ii]. The energy that we pay for to run our appliances is the energy mainly associated with the chemical bonds in fuels, like oil, coal and natural gas.[iii]

By bypassing these sources of energy and instead using the energy associated with solar radiation (called solar energy or radiation energy) or wind (essentially the kinetic energy of the wind), we can "conserve" the energy associated with the more traditional fuels.

In physics, though, energy is always conserved, which means that the total amount of energy remains constant and can only change somewhere if it is transferred to or from an adjacent region or object.

Stated another way, energy in a particular location can change only if the energy in some adjacent region changes in the opposite way. More specifically, since there are many different types, one individual type of energy can change if another type changes in the opposite way.

✓ *Checkpoint 26.4: Since energy is conserved, does that mean the total amount of energy in your house is constant? Why or why not?*

[ii] The more of that stuff in our body, the greater the chemical energy associated with the bonds.

[iii] Some regions of the country get energy from nuclear bonds in materials like uranium.

26.3 Power

As mentioned earlier, there are many types of energy. What is of interest is the *transformation* of energy from one type of another.

The *rate* at which energy is transformed from one type to another is called the **power**. Mathematically, this is written as follows:

$$P = \frac{\Delta E}{\Delta t} \tag{26.1}$$

where P is the power, ΔE is the change in energy (from one type to another) and Δt is the time.

In the SI system of units, energy is measured in joules (abbreviated as J; see chapter 25), regardless of which energy type it is.[iv] Power is the rate at which energy is transferred, so power has SI units of joules per second. Since power is used so often it is given its own SI unit: the **watt**,[v] where one watt (abbreviated as W) is equal to one joule per second (abbreviated as J/s).[vi]

❢ The rate at which energy is converted from one form into another is called the *power* (equation 26.1): $P = \Delta E/\Delta t$.

Example 26.1: Suppose a particular light bulb transforms 60 J of electric energy to heat and light energy every second. What is the power rating of the bulb?

Answer 26.1: The power dissipated by the bulb is 60 J/s or 60 W.

What is heat and light energy?

Heat (also known as thermal energy) and light energy are two types of energies. For example, when a bulb is attached to a battery via a wire, the bulb lights. In the process, the chemical energy (associated with the chemicals inside the battery) is transformed into heat and light energy.

[iv]Non-SI units of energy include the calorie (cal), the food calorie (Cal), and the British Thermal Unit (BTU).

[v]This watt is in honor of James Watt (1736-1819), a Scottish physicist who did work with steam engines.

[vi]Power can also be given in non-SI units. For example, the basal metabolic rate is typically given in Calories per day. Your basal metabolic rate represents the rate at which energy is transferred from chemical energy (see section 26.5.3) to other types of energy while refraining from doing any activity.

✓ *Checkpoint 26.5:How much energy does a 60-W bulb transform to heat and light energy in an hour?*

26.4 Kinetic Energy

❢ The *kinetic energy* represents the energy of motion.

As mentioned in chapter 25, **kinetic energy** is the energy associated with an object's motion. The energy associated with the motion of a car, for example, is kinetic energy.

Why is it called the kinetic energy?

The term "kinetic" refers to motion (much like the word "kinesthetic"). So, this is essentially the energy associated with the motion of the object. If the object isn't moving then the kinetic energy is zero and there is no energy associated with its motion.

26.4.1 Translational kinetic energy

What we've been calling kinetic energy is actually only one type of kinetic energy. When we are sitting at rest, the molecules inside us are still vibrating and moving, and that is kinetic energy also. To distinguish between the kinetic energy we've been considering, associated with the movement of an object we can see, and other types of the kinetic energy, we'll use **translational** kinetic energy to refer to the kinetic energy of an object we can see that moves from one place to another place.

✓ *Checkpoint 26.6: A 1.5-kg object has a speed of 4 m/s. If the speed increases, what happens to the kinetic energy: increase, decrease or stay the same?*

26.4.2 Vibrational kinetic energy

When we pluck a string on a guitar, the string vibrates. The string moves, and thus has kinetic energy, but it doesn't move like a car moving down the street. We thus refer to the kinetic energy of a vibrating string as the **vibrational** kinetic energy.

Other examples of vibrational kinetic energy include the energy associated with the vibration of air molecules as sound travels through the air – we call this *sound energy* (as a type of vibrational kinetic energy).

26.4.3 Rotational kinetic energy

When an object is spinning, there is kinetic energy associated with the rotation. We call that the **rotational kinetic energy** (or **rotational energy**, for short).

As we did in chapter 23 where we dealt with rotation, we describe the rotation in terms of rotational quantities. That means that the rotational kinetic energy can be expressed in terms of the rotational inertia (I) and angular velocity (ω) instead of m and v.

So, whereas the kinetic energy associated with an object moving with speed v is defined by equation 25.3 as

$$E_k = \tfrac{1}{2}mv^2,$$

with rotational quantities it is written as follows:[vii]

$$E_{\text{rot}} = \tfrac{1}{2}I\omega^2. \tag{26.2}$$

❢ Using rotational quantities, the rotational energy E_{rot} is equal to $\frac{1}{2}I\omega^2$.

✍ Notice that we don't replace E by the Greek equivalent because it still represents energy, with the same units as before (e.g., joules).

26.4.4 Thermal energy

Motion on the microscopic scale, like the motion of molecules, is also associated with kinetic energy. Since faster moving molecules are associated with a higher temperature of the object, we refer to this energy as **thermal energy**.

[vii] To show this, replace v with $r\omega$ (from the s equation on page 370) and replace mr^2 with I.

Is thermal energy the same thing as heat?

While some people refer to the thermal energy as **heat** (or **heat energy**), "heat" can be an ambiguous term. Sometimes it is used used as a verb (e.g., to heat something) and sometimes it is sometimes used to also describe the source of the heating, whether that is associated with thermal energy or something different, like light energy or sound energy. Now that you know the meaning of thermal energy, from now on I will use that term exclusively and only use "heat" as a verb (or "heating").

❢ Thermal energy is the energy associated with the temperature of objects (kinetic energy of the molecules).

A key property of thermal energy is that once energy is converted to thermal energy, it is hard to convert it back to some other form. So, when someone says that we must conserve energy, the problem is not that energy is disappearing. Instead, energy is being transformed into a form we can't "use" (e.g., the energy associated with the random motion of the molecules, which we call thermal energy).

Consider, for example, a block sliding along a table. If the table was frictionless, the block would continue to slide along the table without slowing down. With friction, on the other hand, the block slows down. The kinetic energy of the block is being removed and converted into thermal energy (i.e., the block and table warm up a little).

In general, there are two ways to find out how much energy is converted to thermal energy. One way is to figure out the work done by friction, as described in chapter 25. The other way is to just keep track of all the other energies and assume that any deficit in any of those energies must be "lost" to thermal energy (such that the total energy remains the same). This idea is illustrated in the following example.

Example 26.2: Suppose a hanging spring holds a block at rest. It is then extended by 5 cm and released. The block oscillates up and down. A total of 25 mJ (millijoules) is present (in the spring and block). With time, though, the amplitude of the motion decreases and only 4 mJ is present. How much energy was transformed to thermal energy (via friction)?

Answer 26.2: The energy present in the spring and block has decreased by 21 mJ. We assume that this "missing" energy has ended up in thermal energy (i.e., in an increase in temperature of the block, spring and/or air).

✓ *Checkpoint 26.7:Suppose we have two ramps of the same height. One ramp is frictionless, the other is not. When a 1.5-kg box is allowed to slide down the frictionless ramp, its kinetic energy at the bottom is 27 J. When sliding down the ramp with friction, on the other hand, its kinetic energy at the bottom is only 8 J. For the ramp with friction, how much energy is "lost" to thermal energy (during the time the box slides down the surface)?*

26.5 Potential energy

WHEN AN OBJECT SPEEDS UP, ITS KINETIC ENERGY INCREASES? WHERE DOES THAT ENERGY COME FROM?

As mentioned in chapter 25, we can change the kinetic energy by doing work. The work essentially transfer energy from a "storage tank" of energy. There are lots of sources of such stored energy, each corresponding to a particular attraction or repulsion between objects.

For example, a stretched rubber band has "stored" energy that is released when the rubber band returns to its original length. The loss of that stored energy corresponds to an increase in kinetic energy (perhaps the kinetic energy of the rubber band itself).

Stored energy acts like a "bank" from which energy can be transferred to and from. Just like a regular bank, we don't really care about how much energy is stored – we only care about how much is *transferred*. For example, with the rubber band, we don't need to know how much energy is being stored – we only need to know how much the energy is *transferred* as the rubber band is stretched or relaxed.

Because of the "bank-like" nature of stored energy, we call such energy **potential energy**. There are several types of potential energy, depending on the source of the energy. We'll examine a few in this section.

[vii]The word "potential" here refers to the energy being stored, with the potential to be transformed into kinetic energy. It does not mean that this energy is not "yet" energy.

26.5.1 Elastic energy

With a rubber band, the elastic force acts as the "agent" for transferring energy to and from kinetic energy. For this reason, we say that there is **elastic energy** stored in the stretched rubber band.

❢ Elastic energy is the energy associated with the stretching or compression of objects (like springs).

A spring is similar to a rubber band. Suppose you had two balls attached by a spring. If you stretch the spring, there is elastic energy stored in the spring. If you release the spring, that elastic energy will transfer into the kinetic energy of the two balls as the elastic force pulls the balls toward each other, making the objects speed up.

WHAT ABOUT A COMPRESSED SPRING?

The same idea applies. Every spring has an equilibrium position, which is its natural state. The greater the spring is stretched beyond the equilibrium position, the greater the elastic energy stored in the spring, as discussed earlier. However, a spring can also be compressed smaller than its equilibrium position. When compressed, there is again energy stored in this spring since energy is transferred to kinetic energy when the spring is released.

In general, if a force between two objects is attractive then the potential energy is larger when the two objects are stretched apart, whereas if the force is repulsive then the potential energy is larger when the two objects are pressed together.

✓ *Checkpoint 26.8:(a) For an attractive force between two objects, which situation is associated with a greater amount of stored energy: with the objects close together or with the objects are far apart?*
(b) For a repulsive force between two objects, which situation is associated with a greater amount of stored energy: with the objects close together or with the objects are far apart?

26.5.2 Gravitational energy

For objects that attract gravitationally, moving them farther apart is like stretching a rubber band, except that with gravity the energy is being stored in the "invisible springs" we call gravity. As such, we say that there is

gravitational energy associated with gravitational interactions. When two objects come together under their mutual gravitational attraction, the gravitational energy is transferred to kinetic energy (as the gravitational force acts to speed up the objects, much like what happens when an object falls to Earth).

Conversely, when gravity is responsible for slowing an object down (like when an object moves upward, away from Earth), we say that energy is being transformed *to* gravitational energy (from kinetic energy).

For example, consider the following situation.

> Suppose a rock is thrown vertically upward with an initial kinetic energy of 12 J. Half a second later, it has reached the top of its motion and is stationary for an instant and so its kinetic energy is zero. Where did the energy go?

In this case, the rock slowed down due to its gravitational interaction with Earth. The gravitational force is responsible for that slowing down, removing kinetic energy and converting it to gravitational energy.

Notice that the gravitational energy increases as the rock moves upward, away from Earth. In general, the energy associated with an attractive force (like gravity) is greater the farther apart the objects.

Does the gravitational <u>force</u> increase as the rock goes up?

No. The gravitational force remains essentially the same as the rock moves up. If anything, it decreases slightly. The gravitational *energy* is <u>not</u> the same as the gravitational *force*. Rather, the gravitational force is the mechanism by which energy is transferred to and from the gravitational energy "bank".

With what object is the gravitational energy associated?

The gravitational energy is associated with the *system* of objects that are interacting via the gravitational force, not with a particular object.[viii] In this case, the system is made up of the rock and Earth.

In terms of energy conservation, the decrease in kinetic energy of the objects (as they separate) is balanced by an increase in gravitational energy (as they

❢ The gravitational energy of two objects increases as the objects move apart.

[viii] For an object interacting with Earth, the change in gravitational energy is typically due to the object moving, not Earth, so it is common to mistakenly refer to the gravitational energy as residing in that object, when it is really associated with both objects.

separate). Conversely, when the rock falls, its kinetic energy increases. The increase in kinetic energy (as the rock and Earth move together) is balanced by a decrease in gravitational energy.

Example 26.3: A rock is thrown vertically upward with an initial kinetic energy of 12 J. Half a second later, it has reached the top of its motion and is stationary for an instant. What is the change in gravitational energy during this time?

Answer 26.3: To conserve energy, the gravitational energy must increase as the kinetic energy decreases. The rock's final kinetic energy is zero (at the top of the motion). Since the rock lost 12 J of kinetic energy on its way up, the system must have gained 12 J of gravitational energy, such that the *total* amount of energy is the same. Consequently, when the rock is at the top of its motion, there is 12 J *more* gravitational energy than when it started.

Keep in mind that we have only determined the *change* in gravitational energy, not "the value" of the gravitational energy. In other words, the gravitational energy at the top is not 12 J. Rather, we can only say it is 12 J *more* than what it was at the bottom.

Why can't we say it is 12 J at the top?

Because such a statement would imply that the gravitational energy was zero at the bottom. It wasn't. After all, there must be *some* gravitational energy at the initial position because we could've just as easily dropped the rock from there and the rock would then gain energy as it fell. Where else would its energy come from if not from the gravitational energy? Or, alternatively, as long as there is a gravitational force, the gravitational force must be pointing toward a *lower* gravitational energy.

Regardless of where it is, then, there is always some gravitational energy. That is why we focus on *changes* in the gravitational energy. In this case, the gravitational energy (associated with the rock and Earth interaction) *increases* by 12 J while the kinetic energy (associated with the rock's motion) *decreases* by 12 J.[ix]

Does Earth also move during this interaction?

[ix] Another way to say it is that when the rock is at the top of its motion, the gravitational energy, *relative to the initial position*, is 12 J.

Earth, being so much more massive than the rock, hardly moves at all.[x] Since Earth's speed hardly changes, neither does its kinetic energy. also.[xi]

WHAT IF THE ROCK HITS THE GROUND?

When the rock hits the ground, it stops, which means loses kinetic energy. Since the kinetic energy decreases, *some* kind of energy increases. It can't be gravitational energy, since gravitational energy increases only if the rock moves away from Earth, and that isn't happening here. Instead, it is the thermal energy that increases (the rock and ground warm up a little).

✓ *Checkpoint 26.9:Suppose a box is dropped from rest from a height of 3 m.*
(a) As the box fell, what happened to the gravitational energy of the box/Earth system: increase or decrease?
(b) As the box fell (and before it hits the ground), what happened to the kinetic energy of the box: increase or decrease?
(c) After the box falls and comes to rest on the ground, what happened to the gravitational, kinetic and thermal energies at the moment it hit the ground?

WHAT IF THE ROCK BOUNCES OFF THE GROUND?

Let's make it a bit more realistic and replace the rock with a rubber ball since, unlike a rock, a rubber ball bounces. Whereas a rock comes to rest when it hits the ground, the rubber ball compresses like a spring when it hits the ground, allowing it to bounce back up.

As noted earlier, when an elastic object like a rubber ball or a spring compresses, there is an **elastic energy** associated with it. Unlike thermal energy, which does not get returned to "useful" energy, elastic energy can be converted back to kinetic energy (or some other energy) when the object returns to its original shape.[xii] When the rubber ball expands back to its original

[x]As discussed in chapter 12, the magnitude of the force on Earth is the same as that on the rock but Earth, being more massive, experiences a much smaller change in velocity. In fact, Earth's mass is $5.9723 \times 10^{2}4$ kg, which is $5.9723 \times 10^{2}4$ more massive than the rock, so the rock experiences a change in velocity that is $5.9723 \times 10^{2}4$ times greater that of Earth.

[xi]Kinetic energy depends on both mass and speed but the speed is squared so it has more of an impact on the kinetic energy value.

[xii]The same is true for a rubber band, which you can demonstrate as follows. Take a rubber band and stretch it. While stretched, press the rubber band to your upper lip then

shape, the energy transforms from elastic energy to kinetic energy, sending the ball back upward.

✓ *Checkpoint 26.10:A 30-kg child bounces on a trampoline. At the bottom of a bounce (point A), the trampoline springs are stretched and the child is momentarily at rest. The child then moves upward, off the trampoline, reaching a maximum height (point B), where the child is again momentarily at rest. For each energy type, identify whether there was an increase, decrease or no change from point A to B.*
(a) The kinetic energy of the child,
(b) The gravitational energy of the child/Earth system, and
(c) The elastic energy of the trampoline and springs

With the falling rock, gravitational energy is transferred to kinetic energy. More precisely, it is transferred to *translational* kinetic energy. For an object rolling down an inclined surface, the energy is split between the translational kinetic energy and the *rotational* kinetic energy.

For example, consider the following scenario.

> A solid ball and a hoop are rolled down an inclined surface. Suppose both had the same mass and radius (so that the sphere, being solid, must be less dense). If both objects are released from rest from the same height above the bottom, which object will reach the bottom first?

This sounds complicated but, using what we know now, it is actually pretty easy to solve.

The transfer of energy is the same in both cases (since the drop in height is the same and the gravitational force on each is the same) so both objects must gain the same amount of kinetic energy as they roll down the incline.

However, a greater fraction of that transferred energy goes into rotating the object rather than moving the object down the surface. The larger the

release the rubber band and let it return to its original relaxed length. You should find that the rubber band is warmer when it is relaxed. That is because the elastic energy associated with the stretched rubber band has been converted to thermal energy.

rotational inertia, the more energy that goes into rotating the object and the less energy that is available to move the object down the surface.

Since they have the same radius and mass, the hoop will have the larger rotational inertia (see section 23.4.3). Consequently, more of its energy will be rotational rather than translational and so the hoop goes slower.

What if the masses were different?

It turns out that the mass is irrelevant. For example, consider two solid balls, a heavy one and a light one. If both objects are released from rest from the same height above the bottom, both will reach the bottom at the same time.

Wouldn't the lighter ball go faster since the heavy one has a larger rotational inertia?

Although the heavy ball has a larger rotational inertia (since it has more mass), the gravitational force on that one is also greater. It turns out the two effects cancel.

Does the size matter?

No. It turns out the size doesn't matter either. A larger ball will have a larger rotational inertia but it also doesn't need to spin as fast in order to move quickly. After all, a small ball has to rotate many times to "keep up" with one rotation of a large ball.

✓ *Checkpoint 26.11: (a) Suppose we had two cylinders, of the same density and length but different width. Which would roll faster down an incline; a narrow cylinder or a wide cylinder? Note that the narrow one will be less massive. Explain your choice.*
(b) Suppose we had two balls, both with the same mass, but one of which is hollow. If both balls are rolled down an incline, could we determine which ball was hollow? Explain.

26.5.3 Chemical energy

When we use fuel to run our car or heat our house, where does the kinetic energy of the car and the thermal energy of the house come from?

In those cases, the energy comes from something internal to the fuel. In particular, it comes from energy "stored" in the chemical bonds of the fuel molecules.

How do chemical bonds store energy?

As we will examine in Volume II, the atoms in a molecule are attracted to one another via the electric force. Just as there is more elastic energy when a spring is stretched, and there is more gravitational energy when two objects are further apart, there is more **chemical energy** when the molecules are "stretched", like a stretched spring. That energy is "released" when when the molecules are allowed to "relax" to a "closer" state (via a chemical reaction).[xiii]

Is the chemical energy released when the bonds are broken?

No. It is a common misconception to view molecules as being like *compressed* springs, rather than *stretched* springs. However, as will be discussed in Volume II, the atoms attract (via the electric force). Consequently, moving the atoms further apart (breaking the bonds) would *increase* the stored chemical energy, not decrease it. Chemical energy is only released (converted to kinetic energy) when "stretched" bonds are converted to "relaxed" bonds.

✍ Just as you have to provide energy to separate a rock from Earth (increasing gravitational energy), the environment has to provide energy to separate two atoms or molecules, as in evaporation. That is why you feel cold stepping out of a pool on a dry day, as energy from the environment (in the form of thermal energy) is lowered as the water molecules move apart (increasing chemical energy).

For example, we can think of the molecules in gasoline and oxygen (which is needed for combustion) as being "stretched" while the molecules in carbon dioxide and water (the products of combustion) as being "relaxed". When gasoline is burned, the transition from a stretched state to a relaxed state, just like with the rubber band, corresponds to a loss of stored chemical energy and an increase in kinetic energy (likely the translational kinetic energy of the car and the thermal energy of the environment).[xiv]

[xiii] Since the attraction is electrical, chemical energy is really a type of electric energy.

[xiv] Note that chemical energy is not the *result* of combustion. Rather, chemical energy is a type of stored, potential energy. The result of combustion is an increase in kinetic energy – the same kinetic energy that can result from a decrease in any other potential

The key point is that energy is conserved.

✍ Photosynthesis can be thought of as the opposite of combustion, where the fuel (and oxygen) are formed from carbon dioxide and water. When the process is run "backwards" like this, the stored chemical reaction increases. That increase is possible because of the plant's ability to utilize light energy. So, the light energy decreases and the chemical energy increases, consistent with energy conservation.

✓ *Checkpoint 26.12:Enzymes in your body convert the chemical energy of the "fuel" in your body to thermal and other energy. Given that about four-fifths of the chemical energy converted by your muscles goes into thermal energy (which explains why exercising makes you hot) and only one-fifth is used to "do" things (like move a bottle), how many food Calories do we need to "burn" to throw a 1.7-kg bottle of water with a kinetic energy of 21 J (about 5 m/s)? Note: 1 food Calorie is equal to about 4200 J.*

26.6 Work revisited

How does work fit into all of this?

In chapter 25, work is done on an object when a force acts to increase the object's kinetic energy. In a sense, work as a "conduit" for energy transfers.

❢ The amount of energy transferred by a force is equal to the work done by that force.

When a force acts to transfer energy to/from the interacting objects we say that positive/negative work is being done on the object. So, when I make a box start to slide along the floor by pushing on it, energy is being transferred from me (chemical energy) to the box (kinetic energy). We say that positive work on the box, due to the force on the box (due to its interaction with me), but we could just as well say that the force is acting as the agent for transferring the energy from chemical to kinetic. The amount of energy transferred in this case can be determined by using the work definition provided in chapter 25 (equation 25.1 (i.e., the product of the force on the box and the distance the box moved).

energy. We don't have different names for the kinetic energy that results from chemical reactions vs. the kinetic energy that results from gravitational interactions.

As another example, when a rock falls, the gravitational force can be considered to be an "agent" that acts to transfer energy from the stored gravitational energy (associated with the rock-Earth interaction) to the kinetic energy of the rock. The amount of energy transferred is equal to the work done by gravity and can be determined by multiplying the gravitational force on the rock by how far it fell, consistent with equation 25.1 in chapter 25.

What about when I hold a book? Is work being done on the book?

Just because a force is acting doesn't mean there is a transfer of energy. When you hold a book at rest, you are exerting an upward force on it but you are not adding any energy to the book. Similarly, gravity is exerting a downward force on the book but it is not adding any energy to the book. On the other hand, if you drop the book then the gravitational force does transfer energy (from gravitational energy to kinetic energy).

As you may recall from chapter 25, work is only done when the object undergoes a displacement in a direction parallel to the force. For example, when I lean against a wall, there is a force on the wall (due to the wall's interaction with me) and there is a force on me (due to my interaction with the wall) but there is no energy transfer between the wall and me.[xv] The same is true if I hold a book steady. Physicists say the same thing by the phrase "the force does no work on the object."

In the example of the sliding box, the force I apply does work on the box and acts as a mechanism for transferring energy from the stored chemical energy inside me to the kinetic energy of the box. On the other hand, while there is a gravitational force on the box, the gravitational force acts downward while the box moves horizontally (assuming a level floor). That means the gravitational force does no work on the box and the gravitational energy doesn't change.

Now consider a car that is accelerating from rest. During this time, energy is being transferred from stored chemical energy (associated with gasoline and oxygen) to the translational kinetic energy of the car. Even though there are outside forces acting on the car (friction force, gravitational force, surface repulsion force), none of those forces are doing work on the car – all of the energy transfer is internal to the car itself.

[xv] I may get warm but that is because the energy transfer is internal to me, where the stored chemical energy associated with the food I ate is converted to thermal energy.

Does the friction force do work on the car since the friction force is horizontal and the car moves horizontally?

While friction certainly acts on the tires, there is no energy transfer from the road to the car (rather it comes from the car itself) and so friction *in this case* is not the agent for transferring energy to the car. To be consistent, we must say that friction does no work on the car in this case.

In order to avoid double-counting energy transfers, we need to be a bit more precise in our definition of work. It turns out that when we calculate the force, the distance we use is not how far the *object* moves but rather how far the *point of contact* moves (i.e., where the force is acting).

In the cases we examined in chapter 25, there is no difference. However, it makes a difference when different parts of the object are moving separately. If a tire doesn't slip then the point of contact between the road and tire doesn't change and the friction force due to the road does not transfer energy to the car. The car's kinetic energy instead comes from the conversion of chemical energy (associated with the chemical bonds in the fuel) as the fuel burns.

✍ When you push off a wall, there is a force on you due to the wall, but that force does no work on you since the point of contact (your hand against the wall) doesn't move. Your increases kinetic energy comes from the decrease in chemical energy inside you.

What about when a box slides to a stop because of friction between the box and the floor?

In that case, the point of contact (the bottom of the box) does indeed move, so friction does do work (negative) on the box in that case. And, as you are probably aware, friction in that case leads to warming (which is why we rub our hands together to get warm). There rise in thermal energy corresponds to the decrease in translational kinetic energy of the box, and friction acts as the agent for that transfer.

✓ *Checkpoint 26.13: (a) A car decelerates to a stop without slipping. Does friction between the tires and the road do work on the car while it slows down? If so, is it positive work or is it negative work?*
(b) A car skids to a stop (so that the tires slide against the road). Does friction between the tires and the road do work on the car while it slows down? If so, is it positive work or is it negative work?

Summary

This chapter applied the idea of conservation to energy.

The main points of this chapter are as follows:

- By applying the principle of conservation of energy, one can predict changes in one type of energy if one can account for the changes in all of the other types of energy.
- The rate at which energy is converted from one form into another is called the *power* (equation 26.1): $P = \Delta E/\Delta t$.
- The *kinetic energy* represents the energy of motion.
- Using rotational quantities, the rotational energy E_{rot} is equal to $\frac{1}{2}I\omega^2$.
- Thermal energy is the energy associated with the temperature of objects (kinetic energy of the molecules).
- Elastic energy is the energy associated with the stretching or compression of objects (like springs).
- The gravitational energy of two objects increases as the objects move apart.
- The amount of energy transferred by a force is equal to the work done by that force.

By now you should be able to use conservation of energy to make predictions about energy changes.

Frequently Asked Questions

What does it mean for energy to be conserved?

When we say that the total energy is conserved, we mean that the total (if we could add up all of the various types) has the same value all the time and any decrease of one type of energy must correspond to an equivalent increase in some other energy type (or types) in some adjacent place.

Can mass be transformed into energy?

While mass can be considered a type of energy[xvi], there is practically no change in mass during any of the situations we are examining in this book.[xvii]

[xvi] Mass can be represented as the rest energy, E_0, which can be calculated according to the expression $E_0 = mc^2$.

[xvii] This is because the rest energy of an ordinary object is enormous compared to the

Is kinetic energy conserved?

Since kinetic energy is just one type of energy, it is not conserved by itself. For example, take your hands and clap them together once. As your hands move together, they each have some kinetic energy. After you clap, they come to a stop. The kinetic energy has been transformed into sound and thermal energy. The *total* energy remains constant but the *kinetic* energy does not, and thus is not conserved.

✍ Kinetic energy may be constant if there is no transformation of energy into other types, like thermal, light and sound.

How can the gravitational energy be higher when the objects are far apart? Isn't the gravitational force less when the objects are far apart?

Force and energy are not the same thing. You can think of the gravitational force as pushing the object (or objects) toward a lower gravitational energy configuration. Thus, when the objects are close (after the gravitational force has pushed them together), the gravitational *energy* is less.

With what object is the gravitational energy associated?

See page 457.

How do we determine the difference in gravitational energy?

From the definition of work, the work done by the gravitational force on an object is the product of the gravitational force and the vertical displacement of the object. Since gravity does positive work when energy is converted from gravitational energy to kinetic energy, the gravitational energy decreases when the work done by the gravitational force is positive.

Is chemical energy released when bonds are broken?

When chemical bonds are broken, the molecules/atoms move apart, corresponding to an *increase* in chemical energy. That increase in chemical energy

rest of the energy (since the c^2 in mc^2 is a very large number). Consequently, changes to the other energies are tiny relative to the rest energy, which means the mass remains approximately the same as well. For example, two pounds of fat has a rest energy equal to the energy released in about 500 billion atomic bombs like the kind dropped on Hiroshima, Japan. The energy released during exercise is tiny compared to that. With nuclear reactions, on the other hand, the energy involved can not be ignored compared to the rest energies.

must correspond to a decrease in some other energy, like thermal energy. Most people would say that energy is "absorbed" during this type of reaction. Basically, the energy of the environment goes down and the internal energy (chemical in this case) goes up.

Terminology

Chemical energy	Internal energy	Rotational kinetic energy
Elastic energy	Kinetic energy	Thermal energy
Heat	Potential energy	
Heat energy	Rotational energy	

Additional problems

Problem 26.1: Suppose we only had three types of energy to worry about: kinetic, gravitational and thermal. Further, suppose the kinetic energy increases by 100 J, while the gravitational energy decreases by 120 J. By how much must the thermal energy change? Did the thermal energy increase, or decrease?

Problem 26.2: What happens to the following types of energy as a ball falls in free fall: increase, decrease or stay the same? Explain your choices.
(a) The ball's kinetic energy
(b) The Earth's kinetic energy
(c) The total energy of the Earth/ball system
(d) The total kinetic energy of the Earth/ball system

Problem 26.3: A 1-kg package slides from rest down a portion of a mail chute. If the top is 6 m above the ground and the speed at the bottom is 6 m/s, how much energy is lost to thermal energy as the box slides down the chute?

27. Conservation of Momentum

Puzzle #27: In chapter 6, we were able to predict the change in motion when two cars collide by using the law of interactions and the force and motion equation. However, in every case we examined the final velocity of one of the cars was already known. What if we don't know the final velocity of either car?

Introduction

In this chapter, we take the idea of conservation, introduced in chapter 26, and apply it to a quantity called momentum. In so doing, we are able to do more with collisions than we were able to do in chapter 6. Namely, we are able to predict the motion of *both* cars after a collision.

27.1 Terminology

Before discussing what it means for momentum to be conserved, we first have to define momentum. As you may recall from chapter 6, we could solve collision problems by first calculating the product of an object's mass and change in velocity, $m\Delta\vec{v}$. That product is defined as the object's **momentum**. From the force and motion equation, that quantity is equal to the product of the net force and the time, $\vec{F}_{\text{net}}\Delta t$, which we define to be the net **impulse** acting on the object.

Using these terms, the 3-step process of chapter 6 can be stated as follows:

1. Find the change in momentum for one object and use the force and motion equation to determine the impulse on that object
2. Use the law of interactions to determine the impulse on the other object

3. Use the force and motion equation to determine the other object's change in momentum and then divide that by the mass of the other object to find its change in velocity

❢ An object's momentum ($\vec{p}$) is defined as the product of the object's mass and velocity.

Note that the process requires us to find the *change* in momentum. Since the change in momentum is the product of the mass and change in velocity, it probably is no surprise that momentum is the product of mass and velocity. Mathematically, we use p to indicate momentum, so

$$\vec{p} = m\vec{v} \tag{27.1}$$

❢ The direction of the momentum is the same as the direction of the velocity.

Since velocity is a vector, momentum is also a vector, which is why I put the arrow on top of the p. The direction of the momentum is the same as the direction of the velocity.

Why do we use p to indicate momentum?

When momentum was introduced (by Gottfried Wilhelm Leibniz in 1691), it was referred to as **progress**.

What units does momentum have?

Since momentum is equal to the product of mass and velocity, and we are using the SI units for mass (kg) and velocity (m/s), the SI units for momentum are kg·m/s.

❢ Since momentum is equal to the product of mass and velocity, the SI units for momentum are kg·m/s.

Example 27.1: A 1000-kg car is traveling at a speed of 30 m/s eastward. What is the car's momentum?

Answer 27.1: Multiply the car's mass (1000 kg) by its velocity (30 m/s eastward) to get a momentum of 3×10^4 kg · m/s eastward.

✓ *Checkpoint 27.1: A 3000-kg truck is moving at 20 m/s northward. What is the truck's momentum?*

27.2 Meaning of momentum conservation

As mentioned in chapter 26, when a quantity is conserved, that means the quantity cannot be created or destroyed. The only way a conserved quantity can change somewhere is if it is transferred to or from an adjacent region or object. It turns out that momentum, like energy, is conserved.[i]

How do we know that momentum is conserved?

One can show this by utilizing what we learned in chapter 6. To recap, let's suppose that we have two objects, A and B, that collide. The following is one way to write the force and motion equation for each object:

$$\begin{aligned} m_{\mathrm{A}}\Delta\vec{v}_{\mathrm{A}} &= \vec{F}_{\mathrm{net,A}}\Delta t_{\mathrm{A}} \\ m_{\mathrm{B}}\Delta\vec{v}_{\mathrm{B}} &= \vec{F}_{\mathrm{net,B}}\Delta t_{\mathrm{B}} \end{aligned}$$

The left side is the object's change in momentum and the right side is the impulse acting on the object. In words, we have:

$$\Delta(\mathrm{momentum}_{\mathrm{A}}) = \text{impulse on A}$$
$$\Delta(\mathrm{momentum}_{\mathrm{B}}) = \text{impulse on B}$$

In a collision, each object experiences a force due to that interaction. Since it is the same interaction for each object, the magnitude of the force on each must be the same (as described by the law of interactions). The only difference is the direction of the forces – they are opposite to one another.

The forces are also exerted for the same length of time, which means the product of force and time for each object has the same magnitude but opposite direction. Since impulse is the product of force and time, that means the impulses have the same magnitude and opposite direction.

❢ Conservation of momentum follows from the law of interactions, the law of force and motion, and the fact that the time of interaction is the same for both objects.

Since right side of the two expressions is equal to the impulse, and the values are identical except for the direction, that means the left side of the two expressions must likewise be identical except for the direction. Using positive and negative to indicate opposite directions, we have:[ii]

$$\Delta(\mathrm{momentum}_{\mathrm{A}}) = -\Delta(\mathrm{momentum}_{\mathrm{B}}) \tag{27.2}$$

In other words, object A must "gain" whatever momentum object B "loses" and visa-versa. Since that is true, the *total* momentum must always remain

[i] As it turns out, energy and momentum conservation are even more universal than the law of force and motion.

[ii] As noted earlier, in equations we use $\vec{p}$ to represent momentum. I am writing it out just to be clearer.

the same – the total momentum of the two objects before the collision must equal the total momentum of the two objects after the collision. Momentum is just transferred via their interaction. That is conservation of momentum.

✓ *Checkpoint 27.2: Assuming two objects are interacting with each other and no other objects, which of the following is NOT necessarily true when two objects interact?*
(A) The force on each object has the same magnitude but opposite direction
(B) Each object experiences the force for the same length of time
(C) Each object experiences a change in velocity of the same magnitude but opposite direction.
(D) Each object experiences a change in momentum of the same magnitude but opposite direction.

It is important to note that no assumptions were made other than that the only interaction was between the two objects. If more objects are involved in the interaction, the momentum could be transferred among any or all of the objects that are involved.

Because of this, sometimes it seems that momentum is not conserved when it actually is. One such situation is when you are so focused on the interactions among one group of objects that you neglect to consider that there might also be interactions with objects outside the group you are considering. Momentum is still conserved, of course, but it isn't constant for a given system of objects if momentum is being transferred to or from objects outside that that system of objects. This is illustrated by the following example.

Example 27.2: In the hallway, I slide a 1-kg block of wood on the floor. It then collides with another 1-kg block of wood that is initially at rest on the floor. The first piece was initially traveling at 10 m/s but, due to friction, it is only going 5 m/s when it collides with the second block. After the collision, the blocks move a bit but eventually both come to rest. At the end, when both blocks are at rest, the total momentum of the two blocks is zero. Before the collision, the total momentum of the two blocks is not zero (it has a magnitude of 10 kg·m/s). Is momentum conserved?

Answer 27.2: At the beginning, the first block has a momentum of 10 kg·m/s. That momentum then decreases to 5 kg·m/s just before it collides with the second block of wood. Eventually, both blocks come to rest and so the total momentum is zero. If we consider the system to be just the two blocks, then the system's momentum is not constant (it has decreased). However, momentum is still conserved (i.e., the system's loss of momentum is countered by an equal gain in momentum outside the system).

The reason why the two-block system's momentum is not constant in this case is because the frictional force is due to an interaction with an object **external** to the two-block system. Just as the momentum of a single object is not constant when it interacts with another object, so is the momentum of a system not constant when the system interacts with something outside the system. Only if the system is **isolated** from the surroundings (i.e., no momentum is exchanged between the two-object system and the surroundings) will the momentum of that system be constant.

If we include the Earth (floor) as part of our system then, yes, the momentum of the system will be constant. The momentum lost by the blocks will be gained by Earth. Earth, of course, has a huge mass and so the little bit of momentum it gains results in an insignificant velocity change.

Another approach is just to consider the velocities of the two blocks immediately before and immediately after the collision. During this short time interval, the force of friction may be small in comparison to the internal[iii] forces and so momentum of the system is roughly constant.

✓ *Checkpoint 27.3: A 1-kg piece of clay is thrown against a wall. The velocity of the clay is 5 m/s before it collides with the wall. After the collision, the clay sticks to the wall. What happened to the momentum of the clay?*

27.3 Applying conservation to momentum

To show how momentum conservation can be applied to collisions, let's consider the following collision, which is one we've previously examined in chapter 6:

[iii] Internal forces are forces associated with the interactions among the objects within the system.

> A red 4-kg cart rolls at 3 m/s eastward. It then collides with a blue 2-kg cart at rest. After the collision, the red 4-kg cart is observed to be rolling at 1 m/s eastward. How fast is the blue 2-kg cart moving after the collision?

In chapter 6, we solved this via a 3-step process. With conservation of momentum, we can simplify the process. Basically, we just need to figure out how much momentum the red cart lost and then add that to the momentum of the blue cart.

First I'll determine the momentum of each cart before and after the collision by multiplying its mass by its velocity:

Momentum	**red 4-kg cart**	**blue 2-kg cart**
before collision	12 kg·m/s eastward	0
after collision	4 kg·m/s eastward	?

We can see that the red 4-kg cart has lost 8 kg·m/s (eastward). The blue 2-kg cart must have gained that momentum. Since the blue cart started at rest, its final momentum be 8 kg·m/s (eastward).

Now that we know the blue cart's final momentum, we can figure out its velocity. Since the mass of the blue cart is 2 kg, we can divide its final momentum by its mass to get a final velocity of 4 m/s (eastward).

✓ *Checkpoint 27.4: A 600-kg car traveling eastward at 30 m/s collides with a heavier 1200-kg truck that is at rest. Immediately after the collision, the 600-kg car is find to be moving 5 m/s eastward. What is the 1200-kg truck's velocity immediately after the collision? Assume no other forces are acting to speed up or slow down the car or truck during the collision.*

The process of transferring momentum is similar to the process of transferring energy. There are just two big differences. One is that the amount of momentum transferred depends on the force and *time* whereas the amount of energy transferred depends on the force and *distance* (the object moves in the direction of the force).

The other big difference is that momentum is a vector whereas energy is not. For momentum that means that we have to pay attention to the directions.

For example, consider the recoil that happens when you shoot a gun, which we discussed in chapter 6.

At first glance, it seems that both objects are gaining momentum as the bullet accelerates within the gun barrel and the gun recoils. How is this consistent with the conservation of momentum?

The key is to consider the vector nature of momentum. Before you shoot the gun, both the gun and the bullet are at rest, so the total momentum is zero. When you shoot a gun, momentum is imparted to the bullet but an equal and opposite amount of momentum is imparted to the gun. That way, the *total* amount of momentum remains the same as before, since the sum of the two will add up to zero.

Consider, for example, the following situation, which we also examined in chapter 6:

> A 70-kg father and his 30-kg daughter are facing each other at rest on ice skates (father on the left, daughter on the right). With their hands, they push off against one another. Afterwards, the daughter is moving rightward at 3 m/s. With what speed is the father moving?

Since both objects start at rest, they each have zero momentum initially. The daughter gains a momentum equal to 90 kg·m/s rightward (multiply the daughter's mass and velocity together). For the total momentum to be conserved, the father's momentum must have "lost" 90 kg·m/s rightward, which is the same thing as gaining 90 kg·m/s <u>leftward</u>.

Since the father's mass is 70 kg, we divide his change in momentum by 70 kg to get a change in velocity of 1.3 m/s leftward. Since he started at rest, this must be his final velocity.[iv]

✓ *Checkpoint 27.5: An 80.0-kg astronaut pushes herself away from a 1200-kg space shuttle at a velocity of 3.00 m/s. Find the recoil velocity of the space shuttle (i.e., its speed after the astronaut pushes off).*

[iv] As with other examples, we could have used positive and negative for opposite directions, like positive for rightward and negative for leftward.

27.4 Angular momentum

Conservation of momentum is not just useful for solving collision problems more efficiently. It also makes it easier to solve any interaction problem. To illustrate, consider the situation examined in section 23.3, where it was pointed out that if it wasn't for friction, I would spin in the opposite direction whenever I tried to spin a top. It was also pointed out that my much higher mass meant my change in rotation rate would be much smaller.

This is very similar to the recoil experienced when shooting a gun. The difference is that with the gun and bullet they experience equal and opposite momentum whereas with me and the top we experience equal and opposite **angular momentum**.

What is angular momentum?

❢ Angular momentum is the product of rotational inertia I and angular velocity ω.

Angular momentum is the rotational equivalent of momentum. It has the same form as momentum except that instead of mass m and velocity $\vec{v}$ we use rotational inertia I and angular velocity ω:[v]

$$\text{angular momentum} = I\omega. \tag{27.3}$$

For reference, I'll call $\vec{p}$ the **linear momentum** to distinguish it from the angular momentum.

Is angular momentum conserved, just like linear momentum?

Yes. Angular momentum, like linear momentum, is conserved. In terms of angular momentum, we can say that if one object gains an angular momentum in one direction, the other object must gain an angular momentum in the opposite direction (or lose angular momentum in the same direction).

Suppose, for example, that I spin a top such that it has an angular velocity of 10 revolutions/second. If there is no friction and angular momentum is conserved, what should be my angular velocity as a result (assuming I started at rest)?

[v] Angular momentum is usually represented by L in equations. I am writing it out just to be is clearer, since we are only going to use it in this section. It seems that the choice of L comes from how the atomic orbital angular momentum is related to the azimuthal quantum number, which is represented by the lower-case ℓ. The first use of L was in quantum mechanics papers around 1930, with ℓ first being used just a few years prior.

To figure it out, we need to determine the angular momentum that was imparted to the top. A typical top[vi] has a rotational inertia of around 10^{-6} kg · m^2. Its angular momentum is the product of its angular velocity and its rotational inertia. That product is equal to 10^{-5} kg · m^2 · rev/s (the strange units is because I am using revolutions instead of radians).

I must gain an opposite angular momentum (i.e., 10^{-5} kg · m^2 · rev/s in the opposite direction).

To figure out my angular velocity, I need my rotational inertia, which I estimate[vii] is around 1.4 kg · m^2. Since angular momentum equals $I\omega$, I can divide my angular momentum by my rotational inertia to get an angular velocity of 7×10^{-6} rev/s.

As you can see, the angular momentum involved is very small.

However, even though the angular momentum is small, that doesn't explain why we don't spin in the opposite direction even a little bit. The reason for that is friction with the ground. If I was on a frictionless surface (like ice) it might be more noticeable. Indeed, if I stood on a platform that was free to rotate, we would see that I would not be able to make an object rotate without myself rotating in the opposite direction, as discussed in section 23.3 with the bicycle wheel.

✓ *Checkpoint 27.6: Suppose I stand on a platform that is free to rotate and I hold a bicycle wheel with its axis oriented vertically. My rotational inertia is* 1.4 kg · m^2 *about my center and the wheel's rotational inertia is* 0.25 kg · m^2 *about its axis.*
(a) If I make the wheel spin clockwise (as seen by someone above me looking down) at a rate of 5 rev/s, how fast and in what direction will I spin as a result?
(b) Do you have to convert the rotation rate into units of rad/s? Why or why not?

One of the more interesting aspects of angular momentum conservation has to do with the fact that the rotational inertia depends on where the object's

[vi] The rotational inertia of the top is roughly based on the top being a sold cylinder of mass 10 g and a radius smaller than a centimeter.

[vii] My rotational inertia is based on my mass being 70 kg and with my body similar to a solid cylinder of radius 20 cm.

mass is relative to the axis of rotation (see section 23.4.3). After all, an object's rotational inertia isn't fixed. If the object's mass moves toward or away from the rotation axis, that changes the rotational inertia.

This leads to some interesting effects.

In particular, let's suppose there is nothing *external* to our system applying any torques on it. From conservation of angular momentum, that means that the object's angular momentum won't change.

❢ In the absence of external torques, a system's total angular momentum is conserved (both in magnitude and direction).

However, angular momentum is equal to the product of rotational inertia and angular velocity. That means that even with no external torque acting, we can still change how quickly an object spins if we can change the rotational inertia of the object. In particular, the angular velocity will go up if the rotational inertia goes down (and visa-versa).

One example of this is a skater that spins faster when their arms are brought in toward the body. The rotational inertia gets smaller when the arms are brought in (because rotational inertia depends on how far the mass is from the axis of rotation). In order for the angular momentum to be conserved (i.e., L of the skater to remain the same), ω must increase (i.e., the rotation rate must increase).

Can it be used for anything that spins, like a hurricane?

Yes. The air in a hurricane is forced toward the center or eye of the hurricane (because atmosphere pressure is lower there). As the air gets closer to the center, its rotational mass decreases (because the air is closer to the axis of rotation) and, conserving angular momentum, its rotational velocity increases.

Why would the speed increase? Is there a force pushing on the air?

Yes, there is a force pushing on the air, but it is directed in toward the center of the hurricane, not *around* the hurricane. As the air moves toward the center, that force makes it go faster.

Do hurricanes always spin counter-clockwise in the northern hemisphere?

Yes.

Why?

Because, when looking down on the northern hemisphere, Earth is seen to spin counter-clockwise (at a rate of one revolution every 24 hours). Everything on Earth, including the air, the trees and you, is likewise spinning counter-clockwise. Since the air is already spinning counter-clockwise, when it approaches the center of the hurricane it just spins faster counter-clockwise.

How come I don't notice this spinning now?

We don't notice it because Earth is spinning at the same rate we do. Consequently, relative to the Earth, we are not spinning at all. We only notice when something spins faster (or slower) than Earth.

So if I stand on a patch of "frictionless" ice and pull my arms in I should start spinning counter-clockwise?

Probably not. Earth is spinning very slowly – one revolution every 24 hours is twice as slow as the hour hand on a clock. You'd have to pull your hands in a great deal more (like on the order of the size of a hurricane) for you to notice the change in rotation rate.

✓ *Checkpoint 27.7: (a) When looking down on the* southern *hemisphere, Earth is seen to spin* clockwise. *Based upon the discussion above, which way do you think a hurricane would spin in the southern hemisphere? Explain your reasoning.*
(b) A typical sink is very small in comparison to the size of a hurricane. Based on the discussion above, should water go down the drain a particular way in the northern hemisphere? Explain your reasoning.

27.5 Sticky collisions

In chapter 6 and again in section 27.3, we examined collision where we were given the initial velocities of the two objects and *one* of the object's velocities afterwards, and then were tasked with finding the velocity of the *other* object. The difference is that using conservation of momentum, as in section 27.3, provides a more straightforward way of solving for the velocity.

It turns out that momentum conservation has another advantage, in that we can use it in cases were we don't know the velocity of *either* object after the collision and can predict the motion of *both* objects after the collision.[viii] To do so, we need some *other* piece of information, like how much energy is lost to thermal energy during the collision.

For example, if we know that no energy is lost to thermal energy during the collision (a case known as a perfectly **elastic** collision, like two rubber balls bouncing off each other) then it turns out that the speed at which the objects move apart is equal to the speed at which they come together, and we can use that along with conservation of momentum to figure out the velocities of both objects after the collision.

❢ If two objects stick together that means their velocities after the collision must be the same.

Conversely, if the two objects stick together during the collision (a case known as a perfectly **inelastic** collision, like two clay balls that stick together after colliding) then we can use the knowledge that the velocities must be the same after the collision to figure out what it must be.

Mathematically, the elastic case is significantly more difficult to solve than the inelastic case so we'll illustrate the process with just the inelastic case.

> A blue 1000-kg car (car 1) traveling eastward at 20 m/s collides with a red 1200-kg car (car 2) traveling westward at 5 m/s. If the cars stick together after the collision, how fast do they travel after the collision? (assume no external forces acting)

Before doing any math, let's first discuss what we expect will happen. First, we expect that the blue car will slow down (because the red car is in its way) and the red car will speed up (because it is being pushed from behind), and the blue car will slow down a bit more than the red car will speed up (because the blue car is a bit lighter).

Now let's calculate the momentum of each car. Since the blue 1000-kg car is moving 20 m/s eastward, it has a momentum equal to $20,000 \text{ kg} \cdot \text{m/s}$ eastward (multiply the mass and the velocity together). Since the red 1200-kg car is moving 5 m/s westward, it has a momentum equal to $6,000 \text{ kg} \cdot \text{m/s}$ westward (or $-6,000 \text{ kg} \cdot \text{m/s}$ eastward). Adding them together, we get the total initial momentum of the system: $14,000 \text{ kg} \cdot \text{m/s}$ eastward.

[viii] When forensic physicists try to reconstruct an accident to determine if the cars were speeding before the collision, they are doing an equivalent but reverse calculation. They ascertain the velocities of both objects after the collision (based on how far the cars moved after the collision) and then use conservation of momentum to determine the velocities of both objects before the collision.

Because momentum is conserved, this must equal the total momentum after the collision. The question then becomes: what velocity must they have such that the total momentum is $14,000$ kg · m/s eastward?

Since their total mass is 2200 kg (add the two masses together), we can get the velocity by dividing the final momentum ($14,000$ kg · m/s) by the mass (2200 kg) to get 6.36 m/s eastward. This is the velocity of each car after the collision, *since they are stuck together.*

IN ALL OF THE CAR COLLISIONS I'VE SEEN, THE CARS ARE ALWAYS AT REST AFTER THE COLLISION. WHY IS THIS?

This is because there are external forces acting on the system, most likely friction. In the situations examined here, we've assumed there are no forces external to the system. If there are external forces, momentum will be "transferred" to or from objects outside the system.

✓ *Checkpoint 27.8:A 10-kg object is moving at 10 m/s rightward on a frictionless surface. It then collides with a 5-kg object that was at rest. If the objects stick together after the collision, how fast do they travel after the collision?*

When solving problems with momentum, whether the objects stick together or not, remember that momentum is a vector. This is illustrated in the following example.

Example 27.3: Two 1-kg balls of clay roll toward each other. One has a velocity of 1 m/s rightward and the other has a velocity of 1 m/s leftward. When they collide, they stick together and remain at rest. Calculate the total momentum of the system (of two balls) before and after the collision. Was momentum conserved?

Answer 27.3: If we set the rightward direction as positive, then the ball moving rightward has a momentum of (1 kg)×(+1 m/s) = +1 kg·m/s before the collision. The ball moving leftward has a momentum of (1 kg)×(−1 m/s) = −1 kg·m/s before the collision. The total momentum before the collision is zero (i.e., the momentum of one ball is opposite the momentum of the other). After the collision, the balls are at rest so the total momentum is zero then as well. Yes, momentum was conserved.

Don't both balls lose momentum in this situation?

That depends on what you mean by "lose." Yes, they both lost momentum in the sense that their momentums are zero after the collision so they must have lost whatever momentum they had previously. However, one lost rightward momentum and the other lost leftward momentum. Alternately, you could say that one lost rightward momentum and the other *gained* rightward momentum.

✓ *Checkpoint 27.9: Two identical balls of clay are rolled toward each other with the same speed. Can you determine the total momentum of the system (of two balls) before the balls collide without knowing the masses or speeds? If so, what is it? If not, why not? Hint: Try it first with some numbers (remember that the two balls have the same mass and the same speed and that momentum is a vector).*

Summary

The chapter introduced a quantity called momentum and discussed what it means for the momentum to be conserved during a collision.

- An object's momentum ($\vec{p}$) is defined as the product of the object's mass and velocity.
- The direction of the momentum is the same as the direction of the velocity.
- Since momentum is equal to the product of mass and velocity, the SI units for momentum are kg·m/s.
- Conservation of momentum follows from the law of interactions, the law of force and motion, and the fact that the time of interaction is the same for both objects.
- Angular momentum is the product of rotational inertia I and angular velocity ω.
- In the absence of external torques, a system's total angular momentum is conserved (both in magnitude and direction).
- If two objects stick together that means their velocities after the collision must be the same.

Frequently Asked Questions

Why do we use p to indicate momentum?

See page 470.

What units does momentum have?

See page 470.

How do we know that momentum is conserved?

See page 471.

If the total amount of momentum remains the same, does that mean the total amount of velocity remains the same?

No. Velocity is not conserved, unless the masses of the two objects are equal.[ix]

Is momentum always conserved?

Yes. However, the total momentum of a particular system may not be constant. It is constant only if the net force *on the system* as a whole is zero. In other words, the momentum of the system is constant if the forces acting to slow down, speed up or change the direction of motion of the objects in the system are only due to interactions with other objects within the system.

✍ We can treat momentum as always being constant if we take our system to be large enough. After all, the momentum of the *universe* is constant. Certain parts might gain momentum but then other parts must lose momentum.

Why is it called the angular momentum instead of the rotational momentum?

As mentioned in chapter 21, rotational quantities are frequently called angular quantities. In this case, I am using *angular* momentum instead of *rotational* momentum only because it is conventional to do so.

[ix] One can easily show from equation 27.2 that if the masses are equal then the change in velocities must be equal as well (and opposite in direction, since the forces are opposite).

Terminology

Angular momentum	Impulse	Linear momentum
Elastic	Inelastic	Momentum
External	Isolated	Progress

Additional problems

Problem 27.1: For the same two cars as in the example (600-kg car traveling east at 30 m/s colliding with a 1200-kg car at rest), suppose the cars stick together and are traveling at 20 m/s to the east immediately after the collision. Would that be consistent with momentum conservation? Why or why not?

Problem 27.2: Two 1000-kg cars, each traveling at 20 m/s, come directly toward each other. Suppose they deform and stick together as a result of the collision. How fast do they travel immediately after the collision?

Problem 27.3: A 600-kg car traveling eastward at 30 m/s collides with a heavier 1200-kg truck that is at rest. As a result of the collision, the two vehicles stick together. How fast are they traveling immediately after the collision?